ENGINEERING TRIPOS PART IIB

Friday 27 April 2012 2.30 to 4.00

Module 4C9

CONTINUUM MECHANICS

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C9 datasheet (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

vsd03

1 The Kronecker delta and the permutation tensor are denoted by the symbols δ_{ij} and e_{ijk}, respectively.

(a) By direct expansion or otherwise calculate the values of the following:

(i) $e_{ijk}e_{kij}$; [10%]

(ii) $e_{ijk}a_j a_k$, where a_i is an arbitrary vector. [10%]

(b) Show that $B_{ij} = e_{ijk}a_j$, where a_i is an arbitrary vector that is skew-symmetric (i.e. $B_{ij} = -B_{ji}$). [20%]

(c) Consider the symmetric stress tensor σ_{ij}. The principal directions n_i and the principal values λ are defined via the relation $\sigma_{ij}n_j = \lambda n_i$. Using this definition show that:

(i) the principal values λ of σ_{ij} are given by the relation $\det(\sigma_{ij} - \lambda\delta_{ij}) = 0$; [20%]

(ii) if the principal values of σ_{ij} are distinct, then the principal directions are mutually orthogonal. [40%]

2 (a) Show that the symmetric stress tensor σ_{ij} may be decomposed into a hydrostatic and deviatoric part in only *one* way. [30%]

(b) Figure 1 shows an elastic cantilever OAB in the form of a triangular plate of uniform rectangular cross-section. The thickness t of the plate may be assumed to be much smaller than any other dimension. Side OA is horizontal, side AB is built into a rigid support and the angle $AOB = \alpha$. Side OA carried a uniformly distributed pressure of magnitude p. Employ polar coordinates with the origin at O and the angle θ measured downwards from OA. A suitable Airy stress function for this problem is

$$\phi = \frac{Cr^2}{\tan\alpha - \alpha}\left[\alpha - \theta + \frac{\sin 2\theta}{2} - \tan\alpha\cos^2\alpha\right]$$

where C is a constant.

(i) Determine the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ at any point in the cantilever. [20%]

(ii) State the boundary conditions along OA and OB and show that these boundary conditions are satisfied. Hence determine the value of C in terms of the applied pressure p. [20%]

(iii) If the angle α is small, show using simple beam theory, that at point A at the root of the cantilever, the stress $\sigma_{rr} = 3p/\alpha^2$. Does the value of σ_{rr} derived from the above Airy stress function agree with the simple beam theory prediction? [30%]

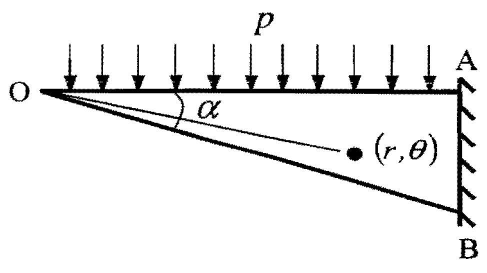

Fig. 1

3 A rigid, ideally-plastic solid with tensile yield strength σ_Y yields in accordance with the von Mises criterion. This solid is manufactured into a long hollow cylinder of inner radius a and outer radius b. The cylinder is constrained against axial lengthening and is subjected to an increasing internal pressure until it attains a collapse pressure p.

(a) At plastic collapse, a trial velocity field is proposed in terms of a cylindrical, polar co-ordinate system (r, θ, z), such that z is along the axis of the cylinder, r is the radial distance from this axis and the angle θ is in the hoop direction. This velocity field is given by $u_r = A / r$, $u_\theta = u_z = 0$ where A is a constant of proportionality. Derive expressions for the strain rate components in terms of (A, r), and show that incompressibility is satisfied. [35%]

(b) Obtain an expression for the plastic work rate per unit volume $\dot{W}$ in terms of (A, r) and determine the upper bound collapse pressure p. [30%]

(c) Determine the hydrostatic stress field throughout the cylinder, and comment upon whether the above trial solution is exact. [35%]

END OF PAPER

Numerical answers

1. (a) (i)	6
(ii)	0