ENGINEERING TRIPOS PART IIB

Wednesday 25 April 2012 2.30 to 4

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

jaw06

1 The schematic layout of a $10\,\mu\text{m}$ thick polysilicon surface-micromachined accelerometer is shown in Fig. 1. The proof mass has electrodes extending from either side to implement a differential capacitive sensing arrangement. The length of both fixed and movable electrodes is $750\,\mu\text{m}$ and the width of each electrode is $10\,\mu\text{m}$. The gap spacing between electrodes is $2\,\mu\text{m}$ and the electrode unit cell is replicated over a region that is $2.5\,\text{mm}$ in overall length with a spacing of $10\,\mu\text{m}$ between successive unit cells. The spring constant for the accelerometer is $10\,\text{Nm}^{-1}$ and the damping constant for the packaged device is $10^{-4}\,\text{Ns}\,\text{m}^{-1}$. The accelerometer operates at a temperature of 300 K.

(a) If the effective mass along the sensitive axis is $10^{-7}\,\text{kg}$, estimate the resonant frequency for this device. Hence, or otherwise, calculate the static deflection of the proof mass for an input acceleration of $1g$ ($1g = 9.81\,\text{ms}^{-2}$) along the sensitive axis. [20%]

(b) Estimate the total sense capacitance for the accelerometer. [20%]

(c) Calculate the fractional change in capacitance for an input acceleration of $1g$ along the sensitive axis. [20%]

(d) Calculate the rms thermo-mechanical noise-equivalent acceleration. [20%]

(e) Comment on the process and design parameters that may be tuned to obtain a lower noise-equivalent resolution for this device. [20%]

Note that: permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{F}\,\text{m}^{-1}$

Boltzmann constant $k_B = 1.381 \times 10^{-23}\ \text{JK}^{-1}$

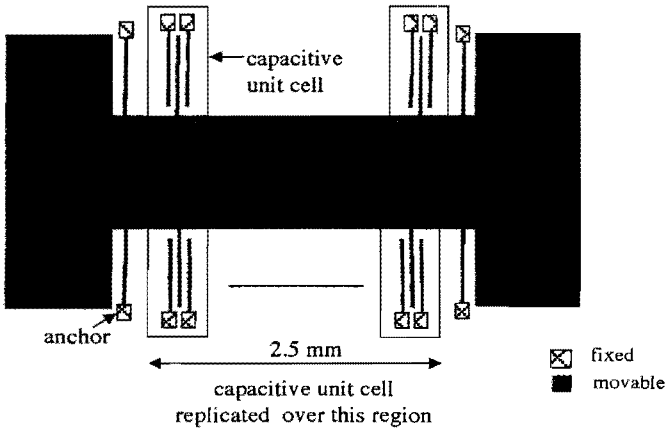

Fig. 1

2 A spring-supported proof mass is driven by a combination of a comb-drive actuator and a parallel plate actuator as shown in Fig. 2. At rest, each actuator has nominal overlap area A and nominal electrode gap spacing g_0. You may assume that the structural thickness for the entire device is t and spring constant for the system is k. The damping in the system is negligible and the dielectric constant for air can be taken to be 1.

(a) Derive an expression for the actuation force when the comb-drive and parallel-plate actuators are both driven by a common voltage V relative to the proof mass. [20%]

(b) Derive an expression for the electrical spring constant for voltage-controlled actuator operation. [20%]

(c) Derive an expression for the pull-in voltage and the corresponding value of the actuator displacement for voltage-controlled actuator operation. [50%]

(d) Describe the relative advantages/disadvantages for this system of actuation as compared to the case of employing either the comb drive actuator or the parallel-plate actuator independently. [10%]

Hint: The real solution to a cubic equation of the form $x^3 + px = q$ is given by:

$$x = \left(\frac{q}{2} + \sqrt{\frac{p^3}{27} + \frac{q^2}{4}}\right)^{1/3} + \left(\frac{q}{2} - \sqrt{\frac{p^3}{27} + \frac{q^2}{4}}\right)^{1/3}$$

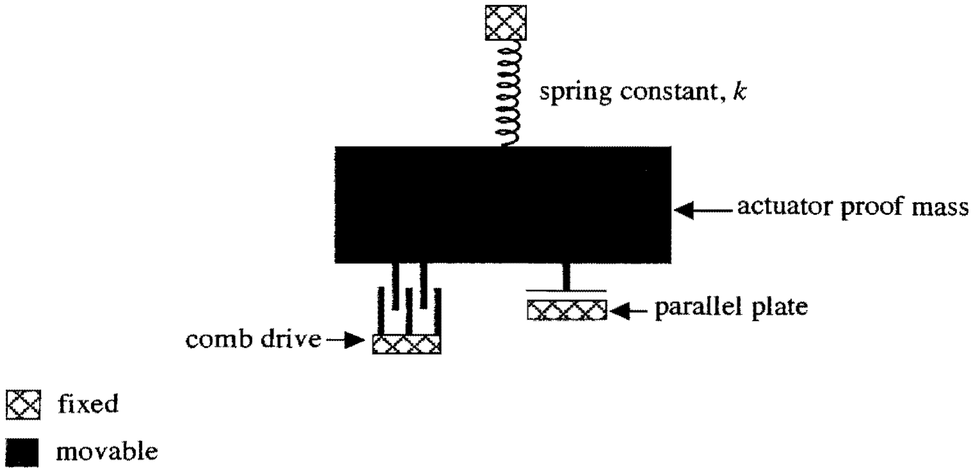

Fig. 2

3 (a) Explain briefly what is meant by the 6-12 Lennard-Jones relation expressing the interaction energy of two atoms. [15%]

(b) If two parallel plane surfaces of the same material are brought into close proximity then, considering only the *non-retarded* Lennard-Jones terms, the interaction energy per unit area $U(z)$, can be written as

$$U(z) = -\frac{A}{12\pi z^2}$$

where A is the Hamaker constant and z is the normal separation of the planes. If the two surfaces come into intimate atomic contact then $z \rightarrow h_0$ where h_0 is the equilibrium atomic spacing. The work of adhesion w can then be defined as the work done per unit area in increasing the separation of two such surfaces from h_0 to ∞. Confirm that w and A are related by the expression

$$w = \frac{A}{12\pi {h_0}^2} \quad .$$

[20%]

(c) Explain briefly what is meant by the *Derjaguin approximation* when one of the surfaces is curved and of a radius which can be considered large in relation to the separation of the surfaces. [15%]

(d) Figure 3 shows a rigid *cylindrical* surface of radius R held at a minimum distance d above a plane surface. By considering the forces of attraction operating at position x and approximating the profile of the cylinder to the expression $y = x^2 / 2R$ derive an expression for the force of adhesion P' per unit length acting between the two solids in terms of the quantities w, h_0, R and d. [40%]

(e) Comment on the feasibility of actually carrying out an experiment to validate your estimate of adhesion force. [10%]

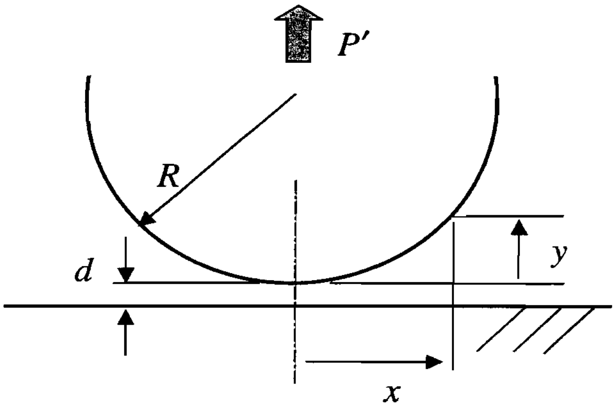

Fig. 3

4 A free-free beam polysilicon micro-resonator is shown in schematic layout in Fig. 4. It is driven into vibratory motion by a parallel-plate actuator of identical width and thickness. A separate parallel-plate electrode is symmetrically arranged on the opposite end of the micro-resonator to implement capacitive sensing of the motional current. The length L of the micro-resonator is $180\,\mu\text{m}$ and the width w of the device is $10\,\mu\text{m}$. The process-defined structural thickness h is $8\,\mu\text{m}$ and the nominal gap spacing g between the electrodes and the resonator is $1\,\mu\text{m}$. The Young's modulus E and density ρ of polysilicon may be assumed to be 160 GPa and $2330\ \text{kg}\,\text{m}^{-3}$ respectively.

(a) The effective stiffness k and effective mass m for the fundamental mode of operation are given by

$$k = \frac{\pi^2}{8}\frac{Ewh}{L} \quad \text{and} \quad m = \frac{\rho whL}{2}\ .$$

Estimate the resonant frequency for the fundamental mode and sketch the mode shape. [20%]

(b) If the Quality Factor of the micro-resonator is 10^5, calculate the amplitude of vibration at resonance for an applied micro-actuator DC voltage of 50 V and an applied AC voltage of 1 V. For this case, estimate the amplitude of the motional current for a DC voltage of 50 V applied between the sense electrode and the micro-resonator. [30%]

(c) Write down expressions for motional resistance, motional inductance and motional capacitance for the micro-resonator and calculate these values for a DC voltage of 100 V applied between the micro-resonator and the electrodes. [30%]

(d) Sketch the dependence of the motional resistance on the applied DC voltage and the resonator gap. Comment on how the motional resistance may be minimised for this device. [20%]

Note that the permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}\ \text{F}\,\text{m}^{-1}$.

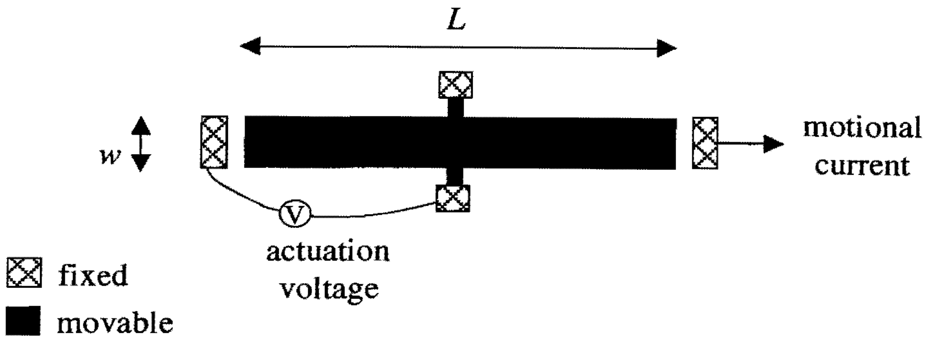

Fig. 4

Hertzian point contact under load P

Reduced radius R given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$

Contact modulus E^* by $\frac{1}{E^*}=\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}$

Radius of contact circle $a=\left\{\frac{3PR}{4E^*}\right\}^{1/3}$

Maximum contact pressure $p_0=\frac{3P}{2\pi a^2}=\left\{\frac{6PE^{*2}}{\pi^3R^2}\right\}^{1/3}$

Mean contact pressure $\bar{p}=\frac{2}{3}p_0$

Approach of distant points $\delta=\frac{a^2}{R}=\left\{\frac{9P^2}{16RE^{*2}}\right\}^{1/3}$

Maximum shear stress is of magnitude $0.31p_0$ and at depth $0.48a$.

Smooth surface adhesion: $p(h)=\frac{8w}{3h_0}\left\{\left(\frac{h}{h_0}\right)^{-3}-\left(\frac{h}{h_0}\right)^{-9}\right\}$

Elastic contact with adhesion, JKR: $\frac{4E^*a^3}{3R}=P+2\sqrt{2\pi wE^*a^3}$

END OF PAPER