ENGINEERING TRIPOS PART IIB

Monday 23 April 20122.30 to 4

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

Answer not more than three questions.
All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Attachments: 4D6 Data sheets (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 A two-storey sway frame is shown in Fig. 1(a). The mass of each storey is 2000 kg and all columns have a flexural stiffness of $E I=100 \mathrm{kN} \mathrm{m}$ 2and an elastic shear capacity of 3 kN . There is a pin-joint at point A; all other connections are fixed.
(a) The natural frequencies of the system are $\omega_{1}=3.06 \mathrm{rad} \mathrm{s}^{-1}$ and $\omega_{2}=7.5 \mathrm{rad} \mathrm{s}^{-1}$. Find the mode shapes.
(b) The building has 5\% damping and experiences an earthquake with the response spectra shown in Fig. 1(b).
(i) Determine the maximum ground acceleration and displacement.
(ii) Determine the maximum column shear. Would the structure remain elastic?
(c) Now assume that the structure in Fig. 1(a) is being designed using the inelastic design spectra in Fig. 1 (c). The expected earthquake has a peak ground acceleration of 0.5 g .
(i) Considering only the first mode, determine the required ductility factor $\mu$.
(ii) State the three most important factors that may cause the actual inelastic response during an earthquake to differ from that calculated using the ductility factor $\mu$ and using the procedure in part (i). Where appropriate, state whether these factors would cause this design to be conservative or unconservative.


Fig. 1(a)

Damping, $\xi=0,2,5,10,20 \%$


Fig. 1(b)


Fig. 1(c)

2 A steel, equal angle section $200 \mathrm{~mm} \times 200 \mathrm{~mm}$ is welded at right angles to a vertical column, forming a cantilever. The length $L$ of the cantilever from the root to its tip is 3 m . The mass per unit length of this angle section is $54.2 \mathrm{~kg} \mathrm{~m}^{-1}$. Other properties of the section can be found in the Structures Data Book. The mode shape of the vibration was determined from the static deflected shape to be;

$$
\bar{u}(x)=\frac{1}{3 L^{4}}\left[x^{4}-4 x^{3} L+6 x^{2} L^{2}\right]
$$

You may assume that the welded joint connecting the equal angle to the vertical column is rigid in parts (a), (b) and (c).
(a) Determine the natural frequency of oscillation for the first mode of flexural vibration of the equal angle section.
(b) A vertical force $f_{1}(t)$ is applied at point P which is 3 m from the welded joint i.e. at the tip of the cantilever as shown in Fig. 2(a). This force varies with time in a triangular manner, as shown in Fig. 2(b). Estimate the maximum dynamic deflection that will occur at point $P$ due to this load.
(c) Estimate the maximum dynamic deflection at the point $Q$ shown in Fig. 2a, due to the load $f_{1}(t)$ acting at point P .
(d) Without further calculations, discuss how the deterioration in the fixity of the welded joint can:
(i) affect the estimation of the natural frequency;
(ii) affect the estimation of the maximum deflection of point $P$.


Fig. 2(a)


Fig. 2(b)

3 (a) Explain briefly how coupled analysis can be carried out using the finite element method to account for the solid and the fluid phases of soil.
(b) A new concrete water tank is to be constructed in a seismic area. The concrete tank has outer dimensions of $4 \mathrm{~m} \times 4 \mathrm{~m}$ and has a wall thickness of 0.25 m . The water tank is to be fully buried into saturated sand such that the top of the tank coincides with the ground surface and the external depth of the tank is 2.25 m . The base slab is 0.25 m thick. The unit weight of concrete is $24 \mathrm{kN} \mathrm{m}^{-3}$ and that of the saturated sand is $19.5 \mathrm{kN} \mathrm{m}^{-3}$. The voids ratio and the Poisson's ratio of sand may be taken as 0.9 and 0.3 respectively. The water table is at the surface of the sand layer. By considering a reference plane 1.0 m below the lower surface of the base of the water tank, calculate the horizontal, vertical and rotational stiffness of the tank-soil system.
(c) The centre of mass of the empty tank can be taken as being 0.75 m above the lower surface of the base of the water tank. Calculate the natural frequencies for the horizontal, vertical and rocking modes of vibration of the tank-soil system for a small magnitude earthquake event. Assume that the whole system will rock about a point on the reference plane directly below the centre of the tank.
(d) A strong earthquake hits this region and the saturated sand bed liquefies. Estimate the excess pore pressure in kPa that you would expect to record on the reference plane below the midpoint of the base slab when full liquefaction has occurred.
(e) Describe how the empty water tank may suffer damage following the strong earthquake and liquefaction of the ground. Comment on what would have happened if the water tank had been full at the time of the earthquake.

4 A suspension bridge has a main span of 400 m . The deck is a thin box section, uniform along the length. The width $B$ of the deck is 15 m , the deck has a mass of $20,000 \mathrm{~kg}$ per metre and a mass moment of inertia of $320,000 \mathrm{~kg} \mathrm{~m}^{2}$ per metre.

Finite element analysis reveals a pure torsional mode of the deck which involves only the main span. The natural frequency of the mode is 0.5 Hz , and it has a half-sine-wave shape, having maximum rotation at the centre of the main span, and zero rotation at the towers. You may assume that this mode has structural damping of $0.5 \%$ of critical.
(a) Describe briefly the process of finite element modelling, including any special considerations that may be involved in modelling a suspension bridge.
(b) Estimate the mode-generalised inertia and stiffness of the torsional mode.
(c) Wind tunnel tests on a short static section of the bridge reveals the steady pitching moment per unit length (in $\mathrm{Nm} \mathrm{m}{ }^{-1}$ ) to be $M=3.3 \rho U^{2} B^{2} \alpha$, where $U$ is the wind velocity $\left(\mathrm{m} \mathrm{s}^{-1}\right), \rho$ is the density of air $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ and $\alpha$ is the angle of attack (in radians). Estimate the wind velocity at which static divergence will occur.
(d) It is proposed to add light but solid vertical screens along each side of the bridge to protect vehicles from side-winds. Further wind tunnel tests on short bridge sections undergoing prescribed torsional harmonic motions reveal that there are induced pitching moments at the same frequency as the prescribed motions, and in phase with $\dot{\alpha}$, the rate of change of angle of attack. The amplitude of this harmonically-varying moment per unit length is $C_{2} \rho U B^{3} \dot{\alpha}$, where the flutter derivative $C_{2}=+0.2$ and -0.7 for the cases with and without the windshields respectively. Estimate the wind velocity at which the bridge with windshields would undergo single degree-of-freedom torsional flutter in the 0.5 Hz mode.
(e) Describe briefly how your calculations in part (d) are relevant to the collapse of the 1940 Tacoma Narrows Bridge.

