# ENGINEERING TRIPOS PART IIA ENGINEERING TRIPOS PART IIB

Tuesday 8 May 2012 2.30 to 4

Module 4D8

PRESTRESSED CONCRETE

Answer one question from Section A and two questions from Section B.

Questions from Section A carry twice as many marks as questions from Section B.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

There are no attachments

STATIONARY REQUIREMENTS Single-sided script paper Graph Paper SPECIAL REQUIREMENTS Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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### SECTION A

#### Answer one question

1 An existing prestressed concrete bridge (ABCD) is to be repaired by the addition of external prestressing cables located above and below the top flange. The additional cables extend for 10 m on either side of the two internal supports, B and C, as shown in Fig. 1. Three of the tendons are placed 200 mm above the top of the section while three are placed 600 mm below it. The total force in all the cables is 36,000 kN.



Fig. 1

The expression from which the secondary moments can be calculated, using the notation in the lecture notes, is

$$\sum_{j} Q_{j} \int \beta_{j} \beta_{i} dx = \int \beta_{i} Pe_{s} dx \qquad i = 1, 2, \dots, n$$

(a) Determine the change in the stress caused by the new cables, in the top and bottom fibres, ignoring the effect of secondary moments.

(b) Determine the distribution of secondary moments. [60%]

[30%]

(c) Thus calculate the change in stress in the top and bottom fibres due to the secondary moments. [10%]

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2 The French designer Guyon introduced the concept of a "Critical Span". Above this span, the relatively large fixed dead-weight compared with the relatively small variable live load means that the minimum prestress, as defined by the Magnel diagram, would have to be placed outside the section.

A simply-supported rectangular beam of span L, depth d, and breadth b made with concrete with a density of  $\rho$ , has to carry a live load that varies from zero to qper unit length. The permissible stress in the concrete is  $f_c$  in compression and zero in tension. It may be assumed that the beam's dead load starts to act as soon as the prestress is applied. Ignore any loss of prestress with time, or variation along the span, and assume that the beam is post-tensioned.

Investigate the idea of a "Critical Span" for this beam by doing the following;

(a) Consider the range of moments that the section must carry at mid-span due to the variable live load, and hence find the limiting section dimensions. Thus find the corresponding minimum weight of the beam, w, as a function of the span/depth ratio L/d.

(b) Sketch a typical Magnel diagram, and determine the minimum possible prestressing force, and its corresponding eccentricity, on the assumption that no account has to be taken of the limits on the eccentricity caused by the section size.

(c) Combine the results of (a) and (b), together with a limit on the eccentricity of 0.4d, to find the Critical Span. Show that this span is independent of the live load if the span/depth ratio and the strength of the concrete are defined. If the span/depth ratio is 20, the density of concrete is 24 kN m<sup>-3</sup>, and  $f_c$  is 20 MPa, what is the Critical Span?

[40%]

[35%]

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[25%]

## SECTION B

#### Answer two questions

3 A simply-supported prestressed concrete beam of prismatic cross-section is to span 25 m. It is to carry a vertical point load of 500 kN which can be applied anywhere along the beam, and also a uniformly distributed dead load (including self-weight of the beam) of 50 kN m<sup>-1</sup>. The stress in the concrete must lie in the range -2 MPa to +10 MPa.

(a) Determine the minimum required elastic section modulus for two alternative designs;

- (i) the prestressing tendon has constant eccentricity,
- (ii) the eccentricity of the tendon is allowed to vary.

[40%]

(b) For the beam with constant tendon eccentricity, find suitable values of the parameters  $b_1$  and  $b_2$  for the flanged section shown in Fig. 3 (all dimensions are in m). [30%]

(c) Without doing detailed calculations, show how a suitable prestressing force and eccentricity can be found by drawing appropriate Magnel diagram(s) or otherwise. [30%]



Fig.3

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A rectangular beam of depth 1000 mm and breadth 500 mm is prestressed with a single tendon of cross sectional area 1000 mm<sup>2</sup> at an eccentricity of 200 mm. The stress,  $\sigma$ , in the prestressing tendon is related to the strain,  $\varepsilon$ , by

$$\sigma = 2200 \left( 1 - e^{-125\varepsilon} \right)$$
 MPa

The prestressing steel carries a stress at transfer of 800 MPa. The initial elastic compressive strain in the concrete at transfer is small and may be ignored.

(a) Calculate the ultimate moment capacity in sagging bending if the limiting
strain in the concrete in compression is 0.0035 and the stress in the concrete in the
compression zone at the ultimate load is 20 MPa everywhere. [75%]

(b) If, due to a flaw in manufacture, the strain capacity of the tendon is limited to 0.01, what is the revised moment capacity of the section? [25%]

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- 5 (a) Discuss qualitatively how creep of concrete is affected by;
  - (i) increasing the age of the concrete at loading,
  - (ii) the thickness of the concrete,
  - (iii) the humidity of the surrounding air,
  - (iv) the water-cement ratio.

(b) A concrete section, made from concrete with a cube strength  $f_{cu}$ , is to be post-tensioned along its centroidal axis with a steel tendon of yield stress  $f_y$ . The maximum initial prestress in the concrete is to be  $f_{cu}/4$ , and the minimum prestress in the concrete, after time-dependant losses, is to be  $f_{cu}/6$ . The steel can be prestressed initially to  $0.7 f_y$  and it has a Young's modulus of 200 GPa. Assume that concrete has a maximum shrinkage strain of 0.0005, the Youngs's modulus of concrete is related to its cube strength by  $E_c = 9000 (f_{cu})^{0.33}$ , where  $f_{cu}$  and  $E_c$  are in MPa, and the maximum creep strain is three times the initial elastic strain. Assume that the steel loses 2.5% of its initial stress due to relaxation.

(c) Comment on the results you have derived above in relation to the failures of the early attempts at prestressing concrete. [20%]

### **END OF PAPER**

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[20%]