ENGINEERING TRIPOS PART IIB

Friday 27 April 2012 2.30 to 4

Module 4F1

CONTROL SYSTEM DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Formulae sheet (3 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed
Supplementary pages: Two extra copies of Fig. 1 (Question 3).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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1 (a) Explain what is meant by a two-degree-of-freedom control system. State but do not prove the necessary conditions which apply to the transfer function from reference input to plant output. [15%]

(b) A simple model of a car for which a cruise control is to be designed is:

$$m\dot{v} = F_{\text{engine}} + F_{\text{hill}}$$

where m is the mass of the vehicle, F_{engine} and F_{hill} represent the forces due to the engine and terrain and v is the forward speed. The transfer function relating F_{engine} to throttle position u is given by $G_a(s) = \dfrac{B}{s+\alpha}$, $B > 0$. A proportional plus integral controller is proposed:

$$\hat{u}(s) = \left(k_p + \frac{k_i}{s}\right)\hat{e}(s)$$

where $e = v_{\text{ref}} - v$ and v_{ref} is the reference speed.

(i) Draw a block diagram of the system. [10%]

(ii) Find conditions on the parameters for the control system to be closed-loop stable. [15%]

(iii) Show that the transfer function from v_{ref} to e has a double zero at the origin. [10%]

(iv) Assume the closed-loop system is in steady state for $t < 0$ and that v_{ref} is a unit step at $t = 0$. By considering the definition of the Laplace transform, or otherwise, show that

$$\int_0^\infty e(t)dt = 0.$$

Hence explain why $v(t)$ must experience overshoot if v_{ref} is a step input. [15%]

(v) Calculate the step response of $R(s) = \dfrac{2}{(s+1)(s+2)}$ and show that it never exceeds one. Hence, or otherwise, find a pre-filter that gives no overshoot in the step response from v_{ref} to v. [20%]

(vi) Suppose the car is travelling on the level at a constant speed $v = v_{\text{ref}} = v_0$. Find the initial slope and final value of v if the car suddenly encounters a hill of constant angle θ. [15%]

2 (a) A control engineer is presented with the transfer function

$$G(s) = \frac{s-1}{s-2}$$

by his employer. It is explained that a plant has already been built and this transfer function model takes account of a preliminary selection of the sensor and actuator.

(i) Sketch the root-locus diagram of $G(s)$ for both positive and negative values of feedback gain k. Find the range of stabilising gain. [15%]

(ii) Explain the practical difficulties which would be experienced if a constant gain controller was used to control this plant. [20%]

(iii) What advice should the control engineer give to his employer? [15%]

(b) Consider a plant with transfer function

$$G(s) = \frac{1}{s-2}.$$

Suppose a stabilising controller is required to achieve the following specifications:

A: $|T(j\omega)| \leq 2$ for all ω,

B: $|T(j\omega)| \leq \varepsilon$ for all $\omega \geq 1$,

where $T(s)$ is the complementary sensitivity function and $\varepsilon > 0$.

(i) Explain how $T(s)$ is constrained at a particular value of s. [10%]

(ii) Find a positive lower bound for ε. [You may restrict attention to controllers which have no zeros satisfying $\mathrm{Re}(s) > 0$.] [40%]

3 Figure 1 is the Bode diagram of a system $G(s)$ which has no poles satisfying $\mathrm{Re}(s) > 0$. A compensator $K(s)$ in the standard negative feedback configuration is to be designed.

(a) (i) Determine the number of poles of $G(s)$ at $s = 0$. [5%]

(ii) Sketch on a copy of Fig. 1 the expected phase of $G(j\omega)$ if $G(s)$ were minimum phase. [10%]

(iii) What does this plot suggest about any possible all-pass factor in $G(s)$? [10%]

(iv) Are there any limitations on the achievable crossover frequency of the system? [5%]

(v) Sketch the complete Nyquist diagram of $G(s)$, paying close attention to any required indentations of the Nyquist D-contour. Determine the range of k for which the closed-loop system is stable with $K(s) = k$. [15%]

(b) The following loop-shaping specifications are sought for the return ratio $L(s) = G(s)K(s)$:

A: $|L(j\omega)| \geq 10$ for $\omega \leq 0.1$ rad s^{-1};

B: $|L(j\omega)| \leq 0.01$ for $\omega \geq 10$ rad s^{-1};

C: Phase margin of at least 45°.

(i) Explain why it is not possible to achieve these specifications using a compensator with one pole and one zero. [Hint: you may find it helpful to treat the cases of a lead and lag compensator separately.] [20%]

(ii) Design a compensator to achieve the specifications. Sketch the compensator and the final return ratio on a copy of Fig. 1. [Hint: you may find it helpful to first design a lead compensator to satisfy specification A together with a phase margin of say $55 - 60°$.] [35%]

Two copies of Fig. 1 are provided on separate sheets. These should be handed in with your answers.

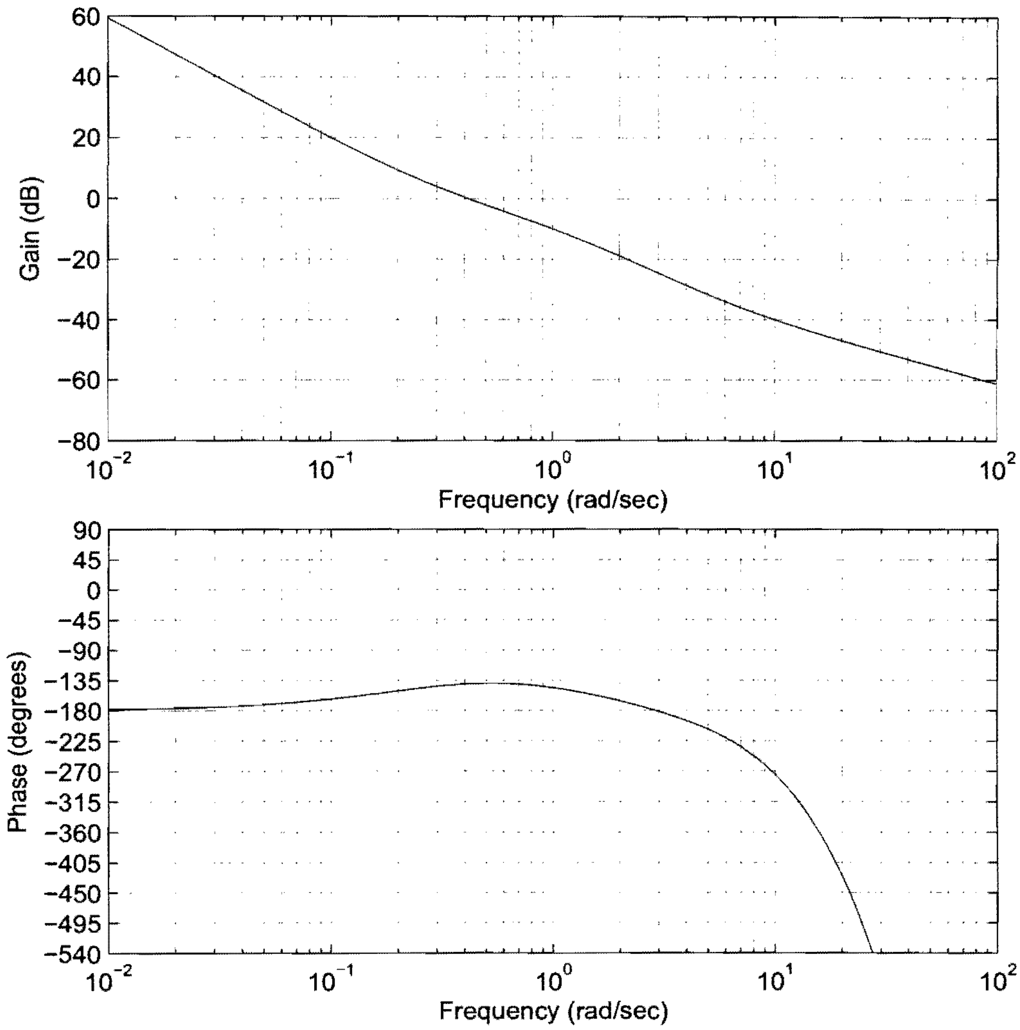

Fig. 1

END OF PAPER

Module 4F1: Control System Design, 2012 exam answers

1. (a) -
   (b) i. -
      ii. $\alpha > 0$, $k_p > 0$, $k_i > 0$ and $\alpha > \frac{k_i}{k_p}$.
      iii. -
      iv. -
      v. -
      vi. $v(t) \to v_0$, $\dot{v}(0) = -g\sin(\theta)$.
2. (a) i. $-2 < k < -1$.
      ii. -
      iii. -
   (b) $T(2) = 1$.
   (c) $\epsilon \geq 0.748$.
3. (a) i. -
      ii. -
      iii. All-pass term is a time delay of $\approx$0.28 seconds.
      iv. -
      v. $0 < k < 16$.
   (b) i. -
      ii. -

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