ENGINEERING TRIPOS PART IIB

Tuesday 8 May 2012 2.30 to 4.00

Module 4F2

ROBUST AND NONLINEAR SYSTEMS AND CONTROL

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

jmg02

1 (a) Figure 1 shows the input-output characteristic defined by

$$f(e) = \begin{cases} 2e + a & \text{if} \quad e < -a \\ 0 & \text{if} \quad -a \leq e \leq a \\ 2e - a & \text{if} \quad e > a. \end{cases}$$

Show that the describing function of this nonlinear characteristic is

$$N(E) = \begin{cases} 2 - \frac{4}{\pi} \sin^{-1}\left(\frac{a}{E}\right) & \text{if} \quad E > a \\ 0 & \text{otherwise.} \end{cases}$$

You may use the results $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$ and $\sin(2\sin^{-1} x) = 2x\sqrt{1 - x^2}$. [40%]

(b) Sketch the graph of $N(E)$. [20%]

(c) The nonlinearity defined in part (a) is placed in a negative feedback loop with a linear dynamic system with transfer function

$$G(s) = \frac{2}{(s+1)^3}$$

as shown in Fig. 2. Show that the describing function method predicts that there will be no limit cycle oscillation. [20%]

(d) Discuss whether the describing function prediction in part (c) is likely to be accurate. [20%]

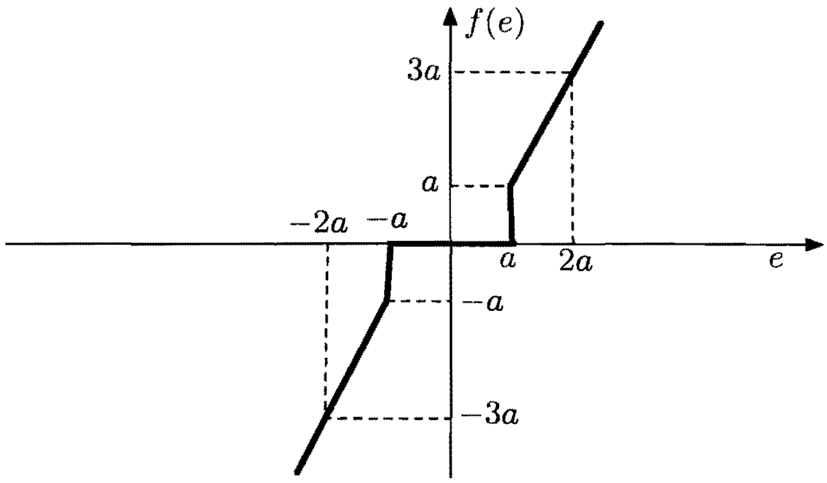

Fig. 1

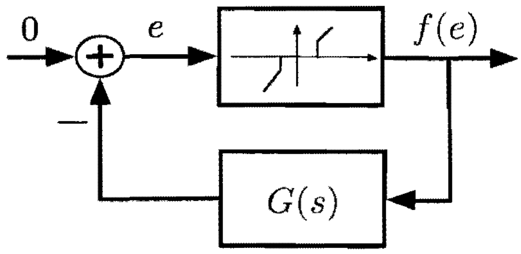

Fig. 2

2 (a) Define what it means for an equilibrium point of a dynamical system $\dot{x} = f(x)$ to be *stable*. [25%]

(b) Explain how the stability of an equilibrium point can be investigated by studying a linearised system. [25%]

(c) In a continuously-stirred chemical reactor, x_1 and x_2 denote the concentrations of two products; r_1 and r_2 are the feed rates of these products; u is proportional to the flow through the reactor, which can be used as a control input; and k is a rate constant. The product concentrations evolve according to:

$$\begin{aligned}\dot{x}_1 &= u(r_1 - x_1) - kx_1^2 \\ \dot{x}_2 &= u(r_2 - x_2) + kx_1^2\end{aligned}$$

with $u > 0$, $r_1 > 0$, $r_2 > 0$ and $k > 0$.

(i) Noting that the concentrations x_1 and x_2 cannot be negative, show that if u, r_1 and r_2 are kept constant, then the equilibrium is unique, if an equilibrium exists. [20%]

(ii) Verify that if $r_1 = 2$ and $u = r_2 = k = 1$, then the equilibrium is at $(x_1, x_2) = (1, 2)$. [5%]

(iii) Show that the equilibrium in part (c)(ii) is stable. [25%]

3 (a) Find

$$\left\| \frac{s+5}{s+7} \right\|_\infty$$

[10%]

(b) Let $G(s) = \frac{3}{s+4}$. Show that

$$\left\| \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{1+G(s)} \begin{bmatrix} 1 & G(s) \end{bmatrix} \right\|_\infty = \sqrt{2}$$

[40%]

(c) Calculate a normalised coprime factorisation $N(s)/M(s)$, for $G(s) = \frac{3}{s+4}$. [25%]

(d) Suppose that

$$G_\Delta(s) = \frac{N(s) + \Delta_N(s)}{M(s) + \Delta_M(s)},$$

where N and M are calculated in part (c) and $\left\| \begin{bmatrix} \Delta_N(s) & \Delta_M(s) \end{bmatrix} \right\|_\infty < \varepsilon$. Find an upper bound on ε that guarantees robust stability for all the systems $G_\Delta(s)$ stabilised by the controller $u = -y$, where u and y are the input and output of $G_\Delta(s)$, respectively. [25%]

4 Consider the system in Fig. 3. Systems K, P_1 and P_2 are multi-input multi-output unless stated otherwise.

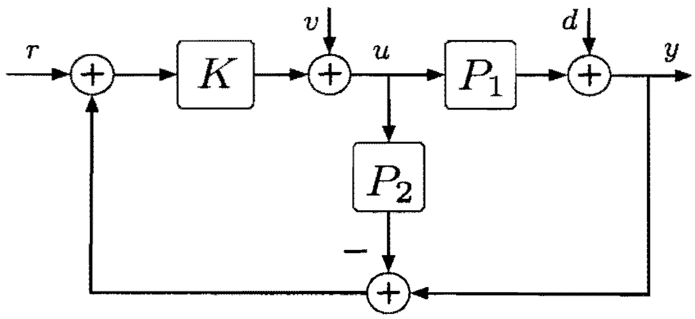

Fig. 3

(a) Show that the transfer functions from the inputs r, v and d to the output y are equal to

$$P_1\,(I-K(P_1-P_2))^{-1}K, \quad P_1\,(I-K(P_1-P_2))^{-1} \quad \text{and} \quad (I+P_2K)(I-(P_1-P_2)K)^{-1},$$

respectively. [*Hint*: Note that $y = d + P_1 u$.] [30%]

(b) Assume that $P_1 = P_2$ are stable. If K is unstable, can the closed-loop system be internally stable? Explain your answer. [10%]

(c) Let $P_1 = P_2 = \frac{1}{s-1}$. Is it possible to find K such that the closed-loop system is internally stable? Explain your answer. [10%]

(d) Assume that P_1, P_2, K are all stable. Show that the closed-loop system is stable if there exists a $c < 1$ such that

$$\sigma_{max}[P_1(j\omega) - P_2(j\omega)]\ \sigma_{max}[K(j\omega)] \leq c, \quad \text{for all } \omega$$

[30%]

(e) Show that the condition in part (d) is not necessary for closed-loop stability by finding a suitable counter-example. [20%]

END OF PAPER

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