ENGINEERING TRIPOS PART IIB

Monday 7 May 2012 9 to 10.30

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) In order to analyse discrete images we express our array of sampled values, $g_s(u_1,u_2)$, as a 2D continuous image, $g(u_1,u_2)$, multiplied by a *sampling function*, $s(u_1,u_2)$;

$$g_s(u_1,u_2) = g(u_1,u_2)\, s(u_1,u_2)$$

Write down the mathematical form of the sampling function $s(u_1,u_2)$, for both rectangular and diamond sampling. [15%]

(b) For a rectangular sampling grid with spacings Δ_1 and Δ_2 in the u_1 and u_2 directions respectively, express the periodic sampling function $s(u_1,u_2)$ as a Fourier series and hence find the Fourier transform of the sampled image, $G_s(\omega_1,\omega_2)$, in terms of the Fourier transform of the continuous image, $G(\omega_1,\omega_2)$. Using this expression for G_s, explain the phenomenon of aliasing in images. [30%]

(c) Consider a continuous image $g(u_1,u_2)$, sampled on the hexagonal grid shown in Fig. 1, to produce a sampled image $g_s(u_1,u_2)$.

(i) If each hexagon has side of length d, write down an expression for this sampling grid as the sum of four rectangular sampling grids, (s_1, s_2, s_3, s_4) (each with a horizontal spacing of $3d$ and a vertical spacing of $\sqrt{3}d$), and give the Fourier series expressions for each of the s_i. [30%]

(ii) Hence show that the spectrum of the sampled image, $G_s(\omega_1,\omega_2)$, takes the form

$$G_s(\omega_1,\omega_2) = f(d) \sum_{p_1=-\infty}^{\infty} \sum_{p_2=-\infty}^{\infty} G(\omega_1 - \alpha_1 p_1, \omega_2 - \alpha_2 p_2)[1+Z]$$

where $f(d)$ is a real function of d, α_1, α_2 are constants and Z is a complex valued function of p_1 and p_2. Find f, α_1, α_2 and Z. [25%]

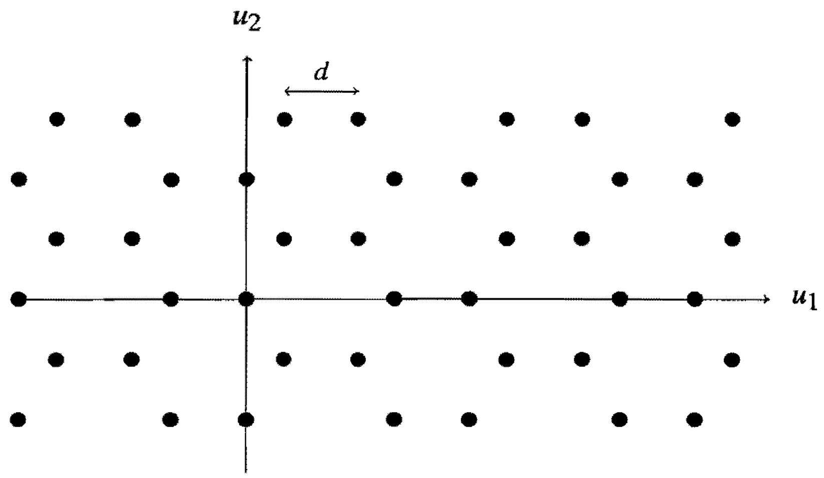

Fig. 1

2 (a) The process of *histogram equalisation* is the application of a transformation to an input image x to produce an output image y via $y = g(x)$, in such a way that the probability of the occurrence of various greylevels is constant.

(i) Using the above definition, show that for a mapping which has a greylevel range of 0 to L and N_L levels, the kth mapped greylevel in the histogram equalisation process is given by

$$y_k = \sum_{i=1}^{k} L \frac{N_i}{NM}$$

where N_i is the number of pixels in the ith original level, N and M are the dimensions of the image and k runs from 1 to N_L. State carefully any assumptions and approximations that are made in the derivation. [20%]

(ii) Consider the 6×6 image shown in Fig. 2 which has 18 greylevels from 1 to 18. Sketch and comment on the form of the histogram of this image. [10%]

(iii) Perform histogram equalisation on the image in Fig. 2 by finding the set of transformed values $\{y_k\}$, $k = 1, .., 6$, onto which the original greylevels are mapped. Sketch the new image histogram and comment on how well the process has worked. [20%]

(b) The linear Wiener filter is a classic tool in deconvolving noisy, distorted images. In a Bayesian derivation of the Wiener filter we maximise the posterior, $P(\mathbf{x}|\mathbf{y})$, which is given by

$$P(\mathbf{x}|\mathbf{y}) \propto e^{-\frac{1}{2}[(\mathbf{y}-L\mathbf{x})^T N^{-1}(\mathbf{y}-L\mathbf{x}) + \mathbf{x}^T C^{-1}\mathbf{x}]}$$

where $\mathbf{x}$ is the true image, $\mathbf{y}$ is the observed image and L is the linear distortion.

(i) In this expression for the posterior, identify the *prior* and the *likelihood* and hence the nature of the matrices N and C. [10%]

(ii) If the posterior is written as $P(\mathbf{x}|\mathbf{y}) \propto e^{-\frac{1}{2}[(\mathbf{x}-\hat{\mathbf{x}})^T M^{-1}(\mathbf{x}-\hat{\mathbf{x}})]}$ so that $\mathbf{x} = \hat{\mathbf{x}}$ maximises $P(\mathbf{x}|\mathbf{y})$, derive M in terms of L, C and N.

The Wiener filter, W, is thus given by $\hat{\mathbf{x}} = W\mathbf{y}$. Find W in terms of L, C and N. [30%]

(iii) If we allow different priors on our image, we can obtain non-linear filters which can show improved performance over the Wiener filter. Give an example of such an image prior commonly used in image deconvolution. [10%]

17	3	15	4	16	1
5	17	2	18	1	12
6	2	18	1	18	13
7	18	1	18	2	14
16	1	18	2	17	16
1	3	4	15	3	17

Fig. 2

3 (a) Draw a block diagram of a basic image coding system, showing the three main encoder blocks and the three equivalent decoder blocks. Explain the purpose of each block. [25%]

(b) Briefly discuss the two main classes of image transform which are in common use and describe their important characteristics, mentioning the types of coding distortion that each type tends to produce. [25%]

(c) The Haar transform employs the matrix

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Show how this can be used to transform a 2×2 block of pixels into a 2×2 block of transform coefficients, and how it may also be used to perform the inverse transform operation. Explain how this transform preserves energy between the pixel domain and the transform domain. [25%]

(d) A 2-level Haar transform, when applied to a monochrome image of 1024×768 pixels, gives the following entropies for each subband:

Level 1 : Hi-Lo : 1.2 bit; Lo-Hi : 1.2 bit; Hi-Hi : 0.8 bit
Level 2 : Hi-Lo : 2.5 bit; Lo-Hi : 2.5 bit; Hi-Hi : 1.8 bit Lo-Lo : 5.6 bit

Estimate the number of bits needed to code this image. [25%]

4 (a) Sketch a two-band analysis filter bank, formed from filters $H_0(z)$ and $H_1(z)$, and the equivalent reconstruction filter bank, formed from filters $G_0(z)$ and $G_1(z)$. Include appropriate downsamplers and upsamplers which are used to avoid redundancy in the two systems. [20%]

(b) Show that if $\hat{y}(n) = y(n)$ when n is even and $\hat{y}(n) = 0$ when n is odd, then the z-transform of $\hat{y}_n$ is given by

$$\hat{Y}(z) = \frac{1}{2}[Y(z) + Y(-z)]$$

[20%]

(c) Hence derive the anti-aliasing and perfect reconstruction conditions for the filters in the 2-band filter bank system of part (a). [25%]

(d) By considering the lowpass product filter $P(z) = H_0(z)G_0(z)$, outline the steps which are necessary to design a wavelet transform system that has good image compression properties. If $P(z)$ can be factorized into a relatively smooth lowpass filter and a somewhat less smooth lowpass filter, discuss how $H_0(z)$ and $G_0(z)$ should be chosen in order to achieve good performance in an image compression system. [35%]

END OF PAPER