ENGINEERING TRIPOS   PART IIB

Monday 30 April 2012   9 to 10.30

Module 4M13

COMPLEX ANALYSIS AND OPTIMIZATION

Answer not more than ***three*** *questions.*

The questions may be taken from any section.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Attachment:
4M13 data sheet (4 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

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SECTION A

1 (a) Describe all of the singularities of the following functions, including the residues at any poles

(i) $z^2 \ln\left(z^2+1\right)^{1/3}$ [20%]

(ii) $\dfrac{(z-i)}{z^2+iz+2}$ [20%]

(iii) $\dfrac{z^{1/3}}{\sin\left(\dfrac{z^2-1}{z-1}\right)}$ [20%]

(b) Calculate the inverse Fourier transform $f(t)$ of the function

$$F(\omega)=\frac{\omega+1}{(\omega-2i)^2}$$

where

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} F(\omega)\,d\omega .$$ [40%]

2 (a) Calculate the Laurent series expansion up to the constant term of the series for the function

$$f(z) = \frac{\ln z}{(z-1)^2}$$

about the point $z = 1$. What is the residue at $z = 1$? [40%]

(b) Evaluate the Principal Value (P.V.) integral I as given by

$$I = \text{P.V.}\int_0^{2\pi} \frac{2}{(2\cos t - 1)}\, dt$$ [60%]

SECTION B

3 The principle of stationary potential energy states that the equilibrium states of structural and mechanical systems are characterised by minima of the potential energy of the system. This principle can be used to find the equilibrium displacements of a structural system under load.

Consider the symmetric two-bar truss shown in Fig. 1. The structure is subject to a load W at node C. Under the action of this load node C moves to a point C′.

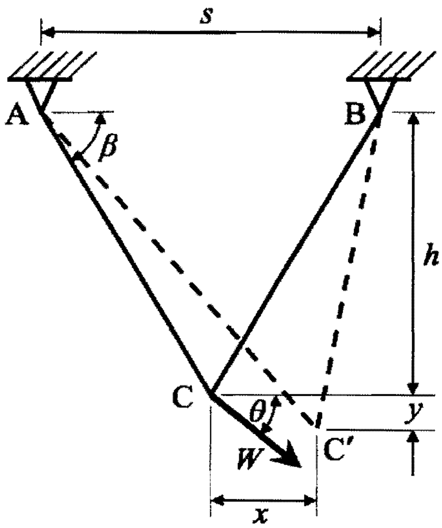

Fig. 1

Assuming small displacements, the total potential energy of the system is given by:

$$P(x,y)=\frac{EA}{2L}\left[(x\cos\beta+y\sin\beta)^2+(-x\cos\beta+y\sin\beta)^2\right]-W\left[x\cos\theta+y\sin\theta\right]$$

where E is Young's modulus of the material from which the truss is made, A is the cross-sectional area of the bars, L is the original length of the bars, x and y are the horizontal and vertical displacements of node C, and the angles β and θ are defined in Fig. 1.

(a) Show that for the case where $A = 10^{-5}\ \text{m}^2$, $h = 1.0\ \text{m}$, $s = 1.5\ \text{m}$, $\theta = 30°$, $W = 10\ \text{kN}$ and $E = 200\ \text{GPa}$, the potential energy of the system is given by:

$$P(x, y) = 5.76\times10^5 x^2 + 1.024\times10^6 y^2 - 8.66\times10^3 x - 5\times10^3 y$$

h and s are defined in Fig. 1. [15%]

(b) Find the displacements x and y that minimize P using standard optimality criteria. [20%]

(c) Starting from the Taylor series expansion given on the 4M13 data sheet for the value of a multivariate function $f(\mathbf{x})$ near a point $\mathbf{x}_k$, show that the step size α_k that minimizes the function in a search direction $\mathbf{d}_k$ from $\mathbf{x}_k$ is given by:

$$\alpha_k = -\frac{\nabla f(\mathbf{x}_k)^T \mathbf{d}_k}{\mathbf{d}_k{}^T H(\mathbf{x}_k)\mathbf{d}_k}$$ [15%]

(d) Starting from a point $(x_1, y_1) = (0, 0)$ and using the result in (c) execute two iterations of the Steepest Descent Method on $P(x, y)$. [35%]

(e) Comment on the performance of the Steepest Descent Method observed in (d) in light of the result in (b). How would you expect Newton's Method and the Conjugate Gradient Method to perform on this problem? [15%]

4 An engineer is designing a shell-and-tube heat exchanger. The design variables are the diameter D (in metres) and length L (in metres) of the heat exchanger. To provide sufficient heat transfer the total length of tubing T within the heat exchanger must be at least 100 m. For the design under consideration, T is related to D and L by the equation:

$$T = 15D^2L$$

The cost of the installation (in arbitrary units) has three components:

1. The cost of the tubes: $C_T = 150D^2L$
2. The cost of the shell: $C_S = 25D^{2.5}L$
3. The cost of the floor space occupied by the heat exchanger: $C_F = 20DL$

(a) Formulate the task of optimizing the design of the heat exchanger to minimize the cost of installation as a constrained minimization problem in standard form. [10%]

(b) Assuming that the constraint on the length of tubing is active at the optimum, use the Lagrange multiplier method to show that the optimal heat exchanger design has $D = 1.368$ m and $L = 3.562$ m. There is no need to check the second-order optimality conditions. [40%]

(c) If an additional constraint is introduced requiring that the floor space $A = DL$ occupied by the heat exchanger must not exceed 4 m^2, use the Kuhn-Tucker multiplier method to find the new optimal design. [50%]

END OF PAPER