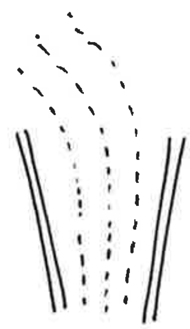
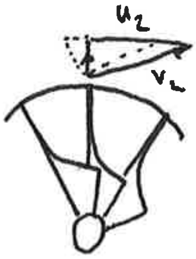
Q1

a)





The flow can only follow the blades when the cross passage pressure gradient is balanced. A exit the blade load drops to zero so the flow bends. [15]

The number of blades has a strong influence on slip. [5]

b) $\dot{m} = 1\,\mathrm{kg\,s^{-1}}$
$T_{01} = 300\,\mathrm{K}$
$P_{01} = 105\times10^3\,\mathrm{Pa}$

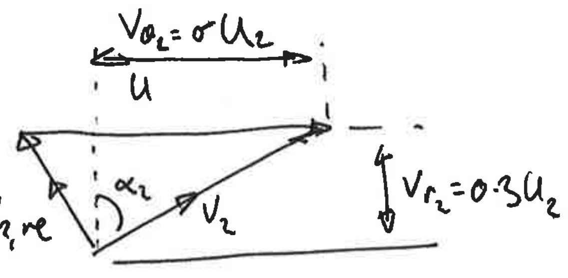
$\dot{w}_x = \sigma U_2^2$ as no inlet swirl.

$= c_p \Delta T_0$

$$\text{so } \Delta T_0 = \frac{0.92(\Omega r_2)^2}{1005} = 361.4\,\mathrm{K}$$

$$\text{so } T_{02} = 661.4\,\mathrm{K} \qquad (U_2 = 628.3\,\mathrm{m\,s^{-1}})$$ [15]

c) Exit velocity triangle



$$\alpha_2 = \tan^{-1}\left\{\frac{\sigma U_2}{0.3U_2}\right\} = 71.94^\circ$$ [10]

$$M_2 = \frac{V_2}{\sqrt{\gamma R T_2}} \qquad T_2 = T_{02} - \frac{V_2^2}{2c_p} \qquad V_2^2 = V_{r_2}^2 + V_{\theta_2}^2 = U_2^2(\sigma^2 + 0.3^2)$$

$$\Rightarrow T_2 = 477.5\,\mathrm{K}$$

$$V_2 = 608.0\,\mathrm{m\,s^{-1}}$$

c) continued

$$M_2 = \frac{608}{\sqrt{1.4 \times 287.1 \times 477.5}} = 1.39$$

$$M_{2,rel} = \frac{V_{2,rel}}{\sqrt{1.4 \times 287.1 \times 477.5}}$$

$$V_{2,rel} = \sqrt{\left\{(U_2 - \sigma U_2)^2 + (0.3 U_2)^2\right\}}$$

$$= U_2 \sqrt{\left\{(1-\sigma)^2 + 0.3^2\right\}}$$

$$= 195.1 \text{ ms}^{-1}$$

$$\Rightarrow M_{2,rel} = 0.445$$

[15]

d) $\eta_{TT} = 0.91$

$$T_{02_s} = \eta_{TT}(T_{02} - T_{01}) + T_{01} = 0.91(361.4) + 300$$

$$= 628.87\,\text{K}.$$

$$P_{02_s} = P_{01}\left\{\frac{T_{02s}}{T_{01}}\right\}^{\frac{\gamma}{\gamma-1}} = 105 \times 10^3 \times \left\{\frac{628.87}{300}\right\}^{\frac{1.4}{0.4}}$$

P_{02} $\quad T_{02s} = \eta_{TT}\left(T_{02} - T_{01}\right) + T_{01}$

$$= 0.91 \times \left(361.4\right) + 300$$

$$T_{02s} = 628.87\,K.$$

$$P_{02} = P_{a}\left(\frac{T_{02s}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 140.03\ kPa$$

[10]

$$M_2 = 1.39 \quad \Rightarrow \quad \frac{\dot{m}\sqrt{c_p T_{02}}}{A_2 P_{02}} = 1.1546$$

$$A_2 = 2\pi r_2 b \cos\alpha_2$$

$$\Rightarrow A_2 = \frac{1 \times \sqrt{1005 \times 661}}{1.1546 \times 140 \times 10^3}$$

$$\Rightarrow b = 0.05\ m$$

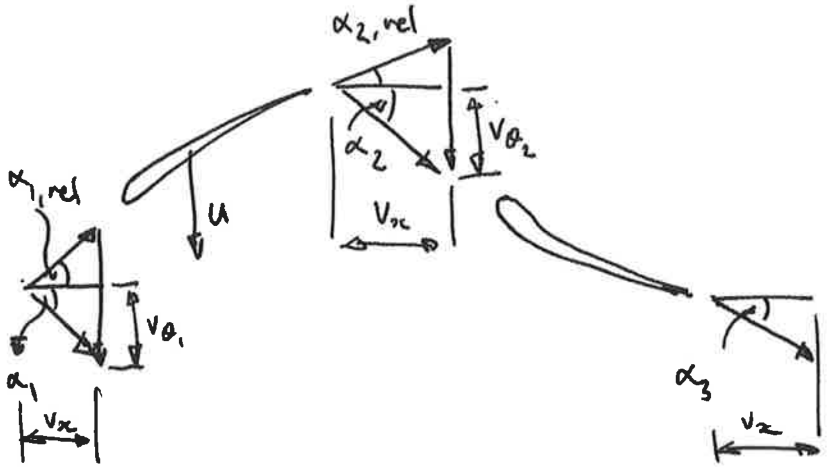
[15]

e) SPECIFIC SPEED IS A STRONG FUNCTION OF ϕ, ψ
LOW SPECIFIC SPEED MACHINES HAVE LOW ϕ AND HIGH ψ
CENTRIFUGAL MACHINES HAVE LOWER SPECIFIC SPEED.
CENTRIFUGAL COMPRESSORS HAVE LOWER MASS FLOW AND GREATER PRESSURE RISE.
CENTRIPETAL TERMS RAISE PRESSURE OF FREESTREAM AND BOUNDARY LAYER EQUALLY.

[15]

Q2

a)



+ve angles in direction of rotation

Euler $\Delta h_0 = U(V_{\theta_2} - V_{\theta_1})$

$V_{\theta_1} = V_x \tan\alpha_1$

N.B. -ve angle

$V_{\theta_2} = V_x \tan\alpha_2 = V_x \tan\alpha_{2,rel} + U$

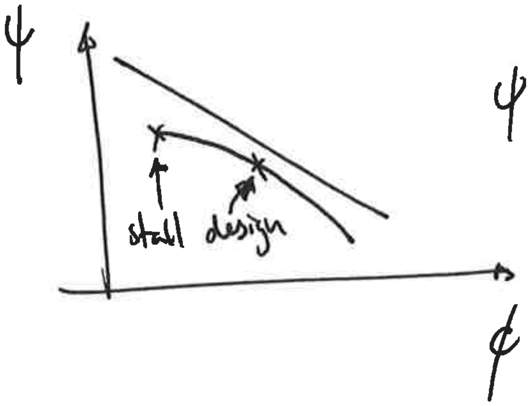
so $\dfrac{\Delta h_0}{U^2} = \dfrac{U}{U^2}(V_{\theta_2} - V_{\theta_1}) = \dfrac{V_x}{U}\tan\alpha_{2,rel} + 1 - \dfrac{V_x}{U}\tan\alpha_1$

$$\psi = \phi(\tan\alpha_{2,rel} - \tan\alpha_1) + 1$$

$$\psi = 1 - \phi(\tan\alpha_1 + \tan\alpha_{2,rel})$$

[15]

b) Pressure rise $\propto \dfrac{\Delta h_0}{U^2} = \psi$
Flow rate $\propto \dfrac{V_x}{U} = \phi$
} at constant U.



$\psi \approx 1 - \phi C$ as $\tan\alpha_1 + \tan\alpha_{2,rel} \approx C$ is approx. constant.

[10]

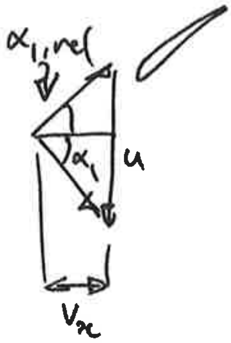
c) i) See diagram for velocity triangles.

$\phi = 0.65$, $\psi = 0.38$ and $\alpha_1 = 18^\circ$ $\Rightarrow \underline{\underline{\alpha_3 = \alpha_1 = 18^\circ}}$ as repeating stages.

Using equation from above: $\alpha_{2,rel} = \tan^{-1}\left\{ \tan\alpha_1 - \frac{(1-\psi)}{\phi} \right\}$

$$= \underline{\underline{-32.17^\circ}}$$

From inlet velocity triangle:

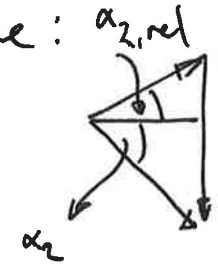


$$\frac{V_x}{u}\underbrace{\tan\alpha_{1,rel}}_{-ve} + \frac{U}{U} = \frac{V_x}{u}\tan\alpha_1$$

$$\alpha_{1,rel} = \tan^{-1}\left\{ \tan\alpha_1 - \frac{1}{\phi} \right\}$$

$$= \underline{\underline{-50.51^\circ}}$$ [15]

From exit velocity triangle:



$$\cancel{V_x}\underbrace{\tan\alpha_{2,rel}}_{-ve} + \frac{U}{V_x} = \cancel{V_x}\tan\alpha_2$$

$$\alpha_2 = \tan^{-1}\left\{ \tan\alpha_{2,rel} + \frac{1}{\phi} \right\}$$

$$= \underline{\underline{42.29^\circ}}$$ [10]

ii) $$\Delta h_{o\,is} = \psi u^2 - 0.5\zeta\left(V_{1,rel}^2 + V_2^2\right)$$

$$= \psi u^2 - 0.025\left(\frac{V_x}{\cos(\alpha_{1,rel})^2} + \frac{V_x}{\cos(\alpha_2)^2} \right)$$

$$\eta = \frac{\Delta h_{o\,is}}{\Delta h_{o\,actual}} = 1 - \frac{0.025\,\cancel{V_x^2}}{\psi \cancel{u^2}}\left\{ \frac{1}{\cos(\alpha_{1,rel})^2} + \frac{1}{\cos(\alpha_2)^2} \right\}\phi$$

$$= 1 - 0.025\,\frac{0.65}{0.38}\left\{ \frac{1}{\cos(\alpha_{1,rel})^2} + \frac{1}{\cos(\alpha_2)^2} \right\} = \underline{\underline{0.882}}$$ [15]

ii) continued

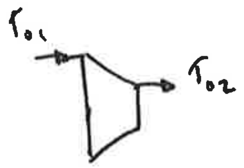
$$\Delta h_0 = \psi U^2 \qquad \Delta h_{0\,is} = \Delta h_0\,\eta$$

$$\Delta T_{0is} = \frac{\Delta h_0 \eta}{c_p} = \frac{\psi U^2 \eta}{c_p} = \frac{0.38 \times 220^2 \times 0.882}{1005}$$

$$= 16.14\,K.$$

$$\text{so } \frac{T_{02is}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{so } \frac{P_{02}}{P_{01}} = \left(\frac{316.14}{300}\right)^{\frac{\gamma}{\gamma-1}} = 1.2$$ [10]

iii) 6 stages. $T_{02} = T_{01} + \dfrac{6U^2\psi}{c_p}$



$$\& \ \frac{T_{02}}{T_{01}} = \left\{\frac{P_{02}}{P_{01}}\right\}^{\frac{\gamma-1}{\gamma\eta_p}} \quad \text{so} \quad \frac{P_{02}}{P_{01}} = \left\{\frac{T_{02}}{T_{01}}\right\}^{\frac{\gamma\eta_p}{\gamma-1}}$$

$$= \left\{\frac{\cancel{T_{01}}\left(1+\frac{6U^2\psi}{T_{01}c_p}\right)}{\cancel{T_{01}}}\right\}^{\frac{\gamma\eta_p}{\gamma-1}}$$

$$= 2.62$$ [15]

$$\Delta T_{0is} = T_{01}\left(2.62^{\frac{\gamma-1}{\gamma}} - 1\right) = 94.99\,K$$

$$\Delta T_0 = \frac{6U^2\psi}{c_p} = 109.8\,K$$

$$\eta_{is} = \frac{94.99}{109.8} = 0.865$$ [10]

Q3

a) POWER TURBINE EXIT

$$P_{06} = P\left[1 + \frac{\gamma-1}{2}M_6^2\right]^{\frac{\gamma-1}{\gamma}}$$

$$= P\left[1 + 0.2 \times 0.3^2\right]^{3.5}$$

$$= 1.064 \text{ bar}$$

[15]

b) $C_{pa}(T_{03} - T_{02}) = C_{p\,ex}(T_{04} - T_{05})$

COMPRESSOR WORK = TURBINE WORK

$$\frac{T_{03}}{T_{02}} = 12^{\frac{\gamma-1}{\gamma R_p}} \Rightarrow T_{03} = 2.2 \times T_{02}$$

$$T_{03} = 660\text{K}$$

$$T_{05} = T_{04} - \frac{C_{pa}(T_{03} - T_{02})}{C_{pex}} = 998.5$$

[10]

$\frac{P_{05}}{P_{04}} = 0.281$

$$P_{05} = P_{04} \times \left(\frac{T_{05}}{T_{04}}\right)^{\gamma/\gamma-1 R_p} = \cancel{\text{bar}}\ 3.36\text{bar}$$

$$T_{06} = T_{05} \times \left(\frac{P_{06}}{P_{05}}\right)^{\frac{(\gamma-1)R_p}{\gamma}} = \cancel{\ }\ 998.5 \times \left(\frac{1.064}{3.\cancel{\ }}\right)^{\frac{0.3\times0.9}{1.3}}$$

$\frac{T_{06}}{T_{05}} = 0.788$

$$T_{06} = 786.4\text{K}$$

[10]

$$\text{POWER} = \dot{m}\, C_{p\,ex}(T_{05} - T_{06}) = 100 \times 1.2 \times (912.3 - 736.1)$$

$$\text{POWER} = 25.45\text{kW}$$

[10]

c) $\frac{\dot{m}\sqrt{C_p T_{04}}}{A_4 P_{04}} = \text{CONST}$ $\quad \frac{\dot{m}\sqrt{C_p T_{05}}}{A_5 P_{05}} = \text{CONST}$ (SAME CONST)

$$\therefore \frac{\sqrt{T_{05}}}{P_{05} A_5} = \frac{\sqrt{T_{04}}}{P_{04} A_4}$$

NB VANES ON BOTH TURBINES ARE CHOKED

[10]

Q3

c CONT)

$$\left(\frac{T_{05}}{T_{04}}\right) = \left(\frac{P_{05}}{P_{04}}\right)^{\frac{\eta_p(\gamma-1)}{\gamma}}$$

$$\sqrt{\frac{T_{05}}{T_{04}}}\,\frac{P_{04}}{P_{05}} = \frac{A_5}{A_4}$$

$$\sqrt{\left(\frac{P_{05}}{P_{04}}\right)^{\frac{\eta_p(\gamma-1)}{\gamma}}}\,\frac{P_{04}}{P_{05}} = \frac{A_5}{A_4}$$ SO $\frac{P_{04}}{P_{05}}$ IS FIXED

$\frac{T_{04}}{T_{05}}$ IS ALSO FIXED

$$\Rightarrow \quad T_{04} - T_{05} = T_{04}(1-c) = k(T_{04})$$

$$C_{pA}(T_{03}-T_{02}) = C_{p\,ex}(T_{04}-T_{05})$$

[10]

$$\therefore \quad \underline{\underline{T_{03}-T_{02} = k' T_{04}}}$$ WHERE k', k, c ARE CONSTANTS

d) T_{02} DROPS FROM 300K TO 245K

$$T_{04} = 1300 \times \frac{245}{300} = \underline{\underline{1061.7\text{K}}}$$ [10]

PRESSURE RATIO'S MAINTAINED SO TEMPERATURES SCALE.

$$\text{POWER} = \dot{m}\, C_{p\,ex}(T_{05}-T_{06})$$

$$\frac{\dot{m}\sqrt{C_p T_0}}{A P_0} = \text{CONST} \quad \Rightarrow \quad \dot{m} \propto \frac{1}{\sqrt{T}}$$ [10]

$$\text{POWER} = 25.45 \times \frac{245}{300} \times \sqrt{\frac{300}{245}} =$$

$\underline{\underline{23.0\text{ kW}}}$

[15]