Q1. Given OD = 0.8m; H/T=0.6, $\phi = 0.5$; $\Psi = 0.4$; $U_{tip} = 60$ m/s; rV_θ = const.
$U_{hub} = 0.6*60 = 36$ m/s; $U_{mid} = 0.8*60 = 48$ m/s; $V_x = V_{x,m} = 0.5*48 = 24$ m/s
$\Delta H = UV_{\theta 2} = \psi \cdot U_m^2 = 0.4 \cdot 48^2 = 921.6\ (\text{m/s})^2$; $V_{\theta,2m} = 921.6 / U_m = 19.2$m/s
$V_{\theta,2t} = V_{\theta,2m} \cdot 0.8 = 15.36$m/s; $V_{\theta,2h} = V_{\theta,2m} / 0.6 = 25.6$m/s;

a). $\dot{m} = \rho A V_x = 1.225 \cdot 24 \cdot \frac{1}{4}\pi[D^2 - (0.6D)^2] = 9.458$kg/s

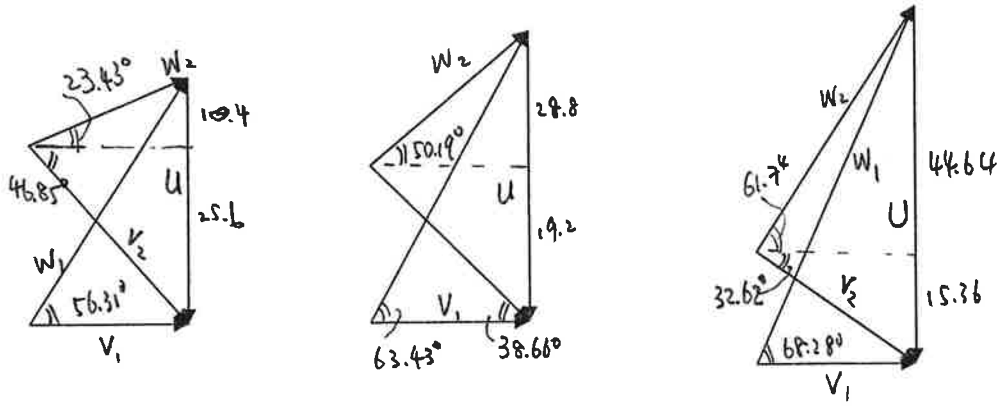
Power = $\dot{m}\Delta H = 9.458 \cdot 921.6 = 8716$w = 8.72kw

b). [20%]

	α_1	β_1	β_2	α_2	W_1	W_2	$V_{\theta 2}$	V_2
Hub	0°	-56.31°	-23.43°	46.85°	43.27	26.16	25.6	35.09
Mid	0°	-63.43°	-50.19°	38.66°	53.67	37.49	19.2	30.73
tip	0°	-68.20°	-61.74°	32.62°	64.62	50.68	15.36	28.49

Velocity Triangles:

Hub Mid Tip



[30%]

c). For $\chi_1 = \beta_1 - i$; $\chi_2 = \beta_2 - \delta$

	Rotor			Stator		
	χ_1	χ_2	$\chi_1 - \chi_2$	χ_1	χ_2	$\chi_1 - \chi_2$
Hub	57.31°	17.43°	39.88°	47.85°	-6.0°	53.85°
Mid	64.43°	45.19°	19.24°	39.66°	-5.0°	44.66°
Tip	69.20°	57.74°	11.46°	33.62°	-4°	37.62°

[15%]

d). $\psi = \Delta H / U^2$; $\Lambda = \phi \frac{1}{2}(\tan\alpha_1^{rel} + \tan\alpha_2^{rel})$

	ϕ	ψ	Λ
Hub	0.666	0.71	0.644
Mid	0.5	0.4	0.800
Tip	0.4	0.256	0.872

[15%]

e). This is a high reaction stage, the loading at the hub is very high, particularly at the stator hub.

From c) and d) above it is clear that the loading at the hub sections is too high: the blade cambers would reach 40° in the rotor and 54° in the stator at the hub. The de Haller numbers are too low. From the loading point of view the rotor hub is over loaded and the turning of the stator hub section is the largest. It would not be possible for the blades to take up such high loading. The problem is resulted from the combination of low hub/tip ratio, high load coefficient and the forced vortex design. A redesign of reducing blade height to increase flow coefficient ϕ, increase blade speed U to reduce loading coefficient ψ, and use a different vortex distribution, say, constant $V_{\theta 2}$ distribution could relieve the problem. [20%]

Q2. calculate properties needed:

$\chi_1 = 62.76^{\circ}$, M_b=1.2625; M_∞= 0.65; Mu = 1.42; β_1 =62.76°;

T_1 = 288/1.0845=265.56K; $p_1 = p_{o1}$ 0.7528 =76284.2pa ; $a_1 = \sqrt{\gamma RT} =$ 326.65m/s;

U = $M_b a_1$= 412.4m/s; ρ = P/RT=1.0kg/m^3;

$T_{o1}^{rel} = 265.56 \cdot 1.4033 = 372.66K$; $p_{o1}^{rel} = p_1 (T_{o1}^{rel} / T_1)^{\gamma/(\gamma-1)}$=249712.2pa;

W_1=1.42 · 326.65=463.84m/s; $M_2 / M_1 = 0.5151 \Rightarrow M_2 = 0.7314$

$(p_{o2} / p_{o1})^{rel} = 0.9531 \Rightarrow p_{o2}^{rel} = 238000.7\text{pa}$; $p_2 / p_1 = 2.1858 \Rightarrow p_2 = 166742\text{pa}$

$T_{o2}^{rel} = T_{o1}^{rel} = 372.66\text{K}$; $T_2 = T_1 \cdot (T_2 / T_1) = 265.56 \cdot 1.2676 = 336.62\text{K}$

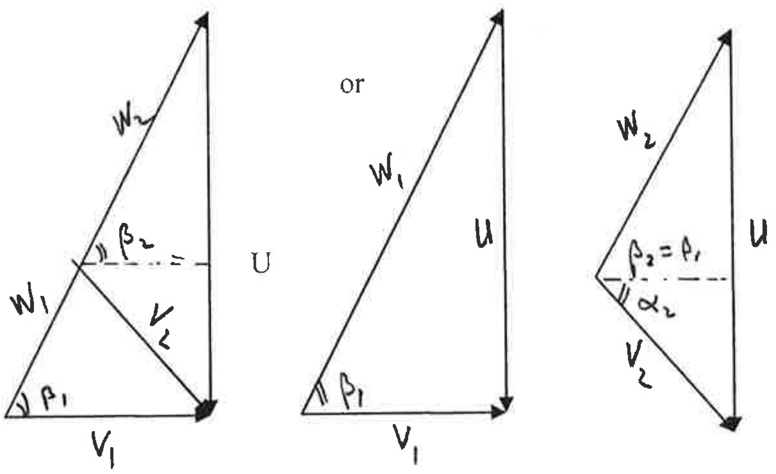
$V_{\theta 2} = U \cdot (1 - \frac{W_2}{W_1}) = U(1 - \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}) = 173.23\text{m/s}$; $U \cdot V_{\theta 2} = 71445.0\,(\text{m/s})^2$

$T_{o2} = T_{o1} + UV_{\theta 2} / Cp = 359.12\text{K}$; $p_{o2} = p_2 (T_{o2} / T_2)^{\gamma/(\gamma-1)} = 209119.2\text{pa}$; $\rho_2 = 1.726\text{kg/m}^3$

$a_2 = \sqrt{\gamma RT_2} = 367.8\text{m/s}$; $W_2 = M_2 \cdot a_2 = 269\text{m/s}$; $\beta_2 = \sin^{-1}(\frac{U\text{-}V_{\theta 2}}{W_2})\ 62.76^{\circ}$;

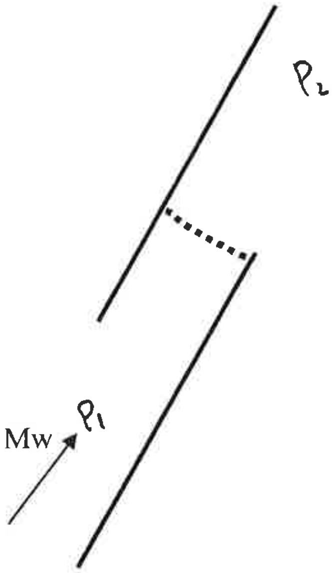
$V_{x2} = \cos\beta_2 \cdot W_2 = 123.13\text{m/s}$; $\alpha_2 = 54.6^{\circ}$; $V_2 = 212.52\text{m/s}$

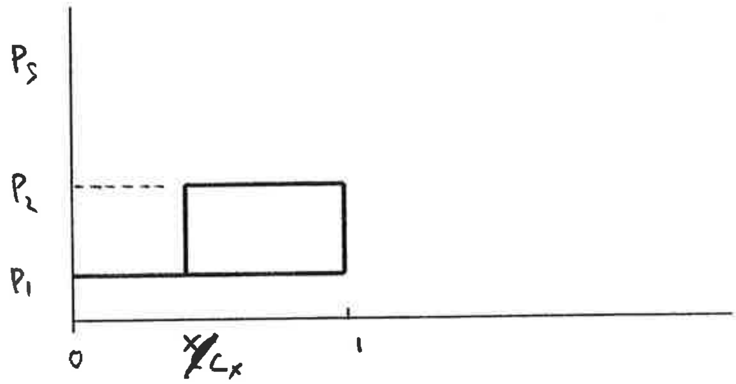
a). Velocity triangles:



Inlet outlet [20%]

b) Shock wave pattern





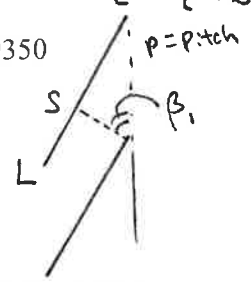
[15%]

c). $p_{2l}/p_1 = 2.1858$; $\quad p_{o2l}/p_{o1} = \dfrac{209119.2}{101330.0} = 2.064$; $\quad T_{o2,isen} = T_{o1}\cdot(p_{o2}/p_{o1})^{(\gamma-1)/\gamma}$=354.25K

$$\eta_{isen} = \frac{T_{o2,isen}-T_{o1}}{T_{o2}-T_{o1}} = \frac{354.25-288}{359.12-288} = 0.9315; \quad \text{or} \quad \Delta s = -R\ln(p_{o2}^{rel}/p_{o1}^{rel}) = 13.786$$

E [25%]

$$T_2\cdot\Delta s/Cp = \frac{336.62\cdot 13.786}{1004.5} = 4.62; \quad \eta_{isen} = \frac{\Delta T_o - T\Delta s/Cp}{\Delta T_o} = \frac{71.12-4.62}{71.12} = 0.9350$$



d). $T\Delta s_{vis} = Cd\rho_\delta V_\delta^3 L$;

geometry: $SE = p\cdot\sin\beta_1 = \dfrac{C}{\sigma}\sin\beta_1 = \dfrac{0.1}{1.4}\sin 62.76^\circ = 0.0635\text{m}$;

$LS = 0.1-0.0635 = 0.0365\text{m}$

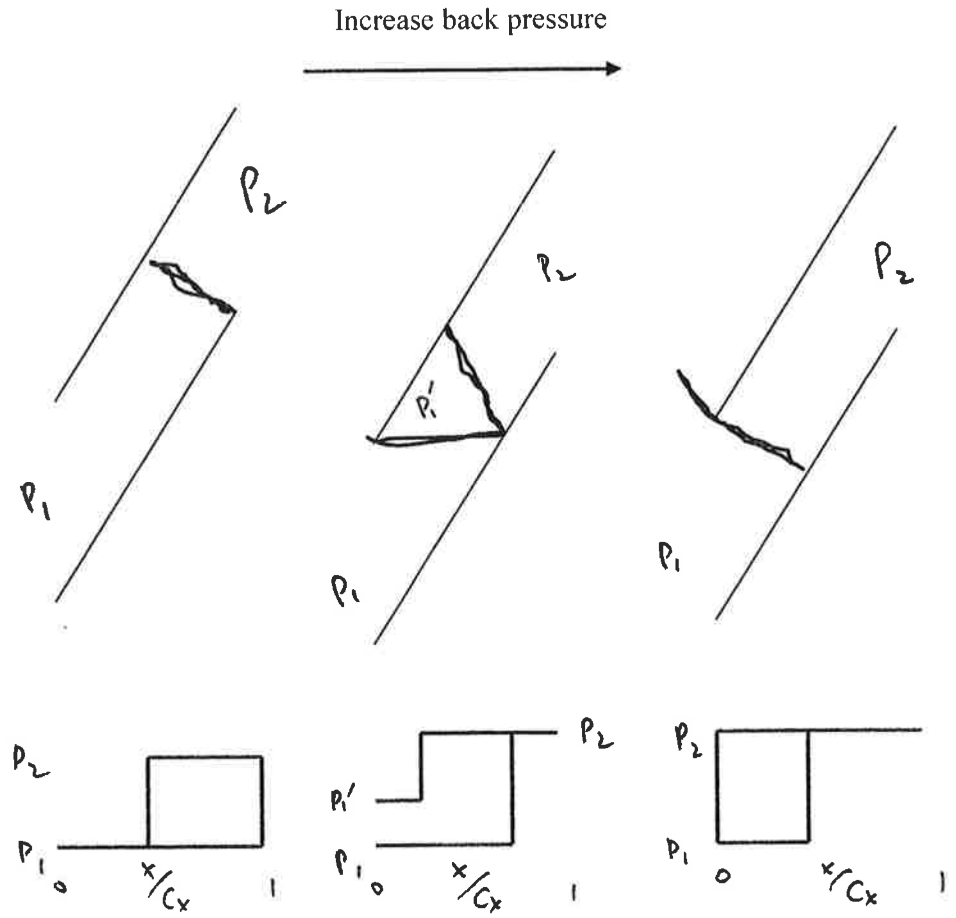
$$\varsigma = \frac{2C}{p\cos\beta_1}Cd\cdot[(1+0.365)\cdot(\frac{W_1}{W_1})^3 + 0.635\cdot(\frac{W_2}{W_1})^3] = 0.01835(1.365+0.635\cdot 0.195) = 0.0273$$

(NB over 90% of the viscous loss upstream of the shock wave!)

$$T\Delta S = \varsigma\cdot\Delta h_1 = \varsigma\cdot Cp\cdot(T_{o1}^{rel}-T_1) = 2937.0; \quad \Delta\eta \approx \frac{T\Delta S}{Cp\cdot\Delta T_o} = \frac{2937}{1004.5\cdot 71.12} = 0.0411$$

[15%]

e).



At low p_2, choking flow condition; aft-loaded; strong shockwave and high losses.

Mid loaded; low shock loss, lower V cubed, high efficiency.

Front loaded, high loss due to strong shock; low V cubed; close to instability

[25%]

Q3

a) (i) 4 equations of motion (MASS, MOMx, MOMy, Energy) + 1 equation of state ($p=\rho RT$ for perfect gas) [10%]

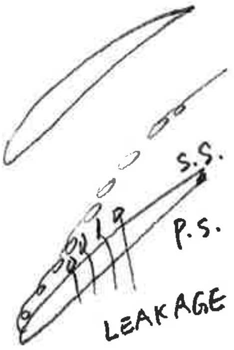
(ii) 4 equations => 4 characteristic wave:

- S Entropy convection @ V_x "
- ω Vorticity convection @ V_x downstream for subsonic V_x
- p+ Downstream pressure @ V_x+a "
- p- upstream pressure @V_x-a upstream for subsonic V_x [15%]

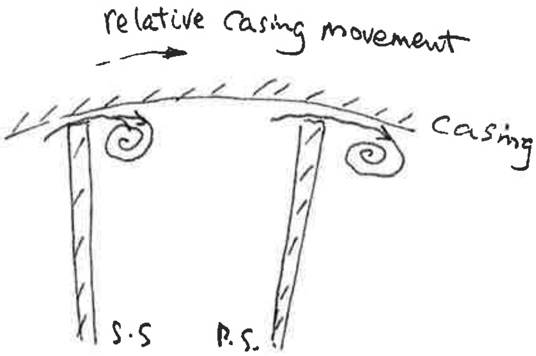
(iii) Need to specify a boundary condition for each wave that enters the computational domain. For subsonic axial flow (usual turbomachinery case) the S, ω & p+ waves travel downstream so need three boundary conditions at the domain inlet; p- is the only up-going wave so only need one boundary condition at exit. The commonest boundary conditions for subsonic flow are P_o, T_o and α at inlet (3 BCs), and p at exit (1BC). [15%]

(iv) $\text{CFL} = \dfrac{(a+|u|)\Delta t}{\Delta x} < 1$; No wave is allowed to move more than a cell spacing in a single time-step (explicit scheme). The fastest moving wave is p+ with speed $a+|u|$, hence $(a + |u|)\,\Delta t < \Delta x$. [10%]

b). (i)



Tip Leakage Trajetory rolls up into vortex like structure



Leakage flow rolls up into vortex, get high loss region

Tip leakage is primarily a pressure driven flow (is, inviscid). However, cavity wall boundary layers add to leakage and vortex mixing is a viscous effect. So involves both inviscid and viscous mechanisms. [20%]

(ii). Euler solve double mesh in all 3-directions => $2^3 = 8$ times the mesh, However $\Delta t \propto \Delta x$ => half time step length, hence run time x $2^4 = 16$ for fine mesh.

Improved resolution by $\Delta x/2$ for a second order scheme ought to improve the capturing of any inviscid effects. Hence leakage flow ought to be better captured as it is primarily pressure driven.

However, would still rely on numerical dissipation to generate the mixing process and loss. [15%]

(iii) Using a Navier-Stokes solver would also be slower as there are more calculations (equations) per iteration. However, it would be able to capture viscous effects such as casing end-wall boundary layer growth and viscous mixing within the vortex. However, results may be very dependent on turbulence model and mesh quality. [15%]