4B6 Crib 2013

Q1

(a) If we apply a positive (negative) voltage step to a p-type (n-type) MOS capacitor, which is sufficient to generate an inversion layer at equilibrium, there is a time interval, after the step, when no free electrons (holes) are present at the interface. This is due to the fact that the inversion charge must be thermally generated and this requires a finite time.

During such a time interval, the MOS is said to be in "deep depletion" and the only charge present in the semiconductor is the depletion charge.

$$\rho(x) = -qN_A$$

$$\frac{d\psi}{dx} = \frac{qN_A}{\varepsilon_s}(w - x)$$

$$\psi_s = \frac{qN_A}{2\varepsilon_s}w^2$$

$$w = \left(\frac{2\varepsilon_s}{qN_A}\psi_s\right)^{\frac{1}{2}}$$

$$Q_s = -Q_B = qN_A w$$

which proves the required equation. [30%]

(b) $V = V_i + \psi_s$

$$V_i = -\frac{Q_B}{\varepsilon_i}d$$

$$Q_B = -(2\varepsilon_s qN_A\psi_s)^{1/2}$$

$$V = \frac{(2\varepsilon_s qN_A\psi_s)^{1/2}d}{\varepsilon_i} + \psi_s = 5.06\ \text{V}$$

[20%]

(c)

$$w = \left(\frac{2\varepsilon_s}{qN_A}\psi_s\right)^{\frac{1}{2}} = 2.29 \times 10^{-6}\ \mathrm{m}$$

[10%]

(d)

$$\frac{1}{C_{tot}} = \frac{1}{C_D} + \frac{1}{C_i} = \frac{w}{\varepsilon_s} + \frac{d}{\varepsilon_{ox}}$$

$C_{tot} = 4.05 \times 10^{-5}$ Farad m^{-2} [10%]

(e) (i) In deep depletion the depletion length w increases with the pulse voltage V, so the capacitance decreases until breakdown occurs. [10%]

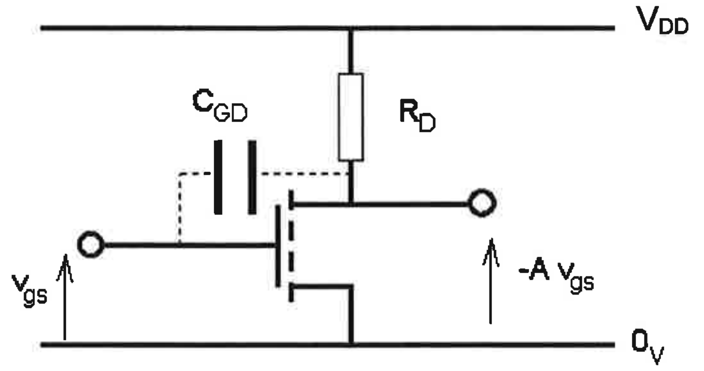
(ii) When the inversion charge is formed, such charge, localized very close to the insulator -semiconductor interface, screens the bulk of the semiconductor, $\psi_S \approx 2\psi_B$ and the depletion region decreases.

At low frequency the inversion charge is modulated by the ac voltage. The total semiconductor capacitance is the parallel between the inversion charge capacitance and the depletion capacitance. The latter is constant, because ψ_S and therefore w are nearly constant, while the former increases with the amount of inversion charge. Eventually the inversion charge capacitance becomes much bigger than the insulator capacitance and $C_{tot}=C_i$.

At high frequencies the inversion charge cannot follow the ac voltage and therefore the semiconductor capacitance is C_D. So $C_{tot}= C_i//C_D=$ constant. [20%]

Q2

(a)



The Miller effect related to a MOSFET inverting voltage amplifier is associated to the increase in the effective input capacitance due to the amplification of the coupling capacitance between the input and output terminals, such as the gate and drain of the MOSFET. [30%]

(b) For an inverting amplifier with gain A and Miller capacitance C_{GD}:

(i) the input current due to the Miller capacitance C_{GD} is:

$$i_{in} = v_{gs}\,(1 + A)\, j\,\omega\, C_{GD}$$ [10%]

(ii) the effective input capacitance is:

$$C_{eff} = C_{GS} + (1 + A)\, C_{GD}$$ [10%]

(iii) the upper 3dB frequency is $f_{upper} \propto 1/C_{input}$, therefore:

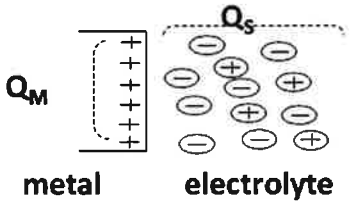
$$\frac{f_{upper,with}}{f_{upper,without}} = \frac{C_{GS}}{C_{GS} + (1 + A)\, C_{GD}}$$ [20%]

(c) Increase of the effective capacitance at the input can lower the bandwidth of the amplifier, reducing its range of operation to lower frequencies.

The physical origin of Miller capacitance here is the overlap of the gate and drain of the MOSFET. Careful design of the device geometry and control of fabrication process will be able to reduce the Miller capacitance significantly. [30%]

Q3

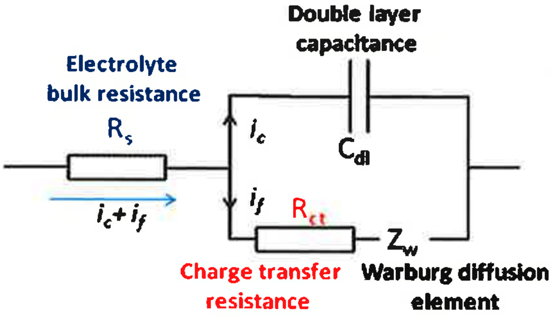
(a) An electrical double layer exists on the interface between and electrode and its surrounding electrolyte. This double layer is formed as ions from the solution adsorbs on the electrode surface. Start from an ideal case: a planar metal electrode in contact with an electrolyte with no chemical reactions occurring between the electrolyte and the metal:

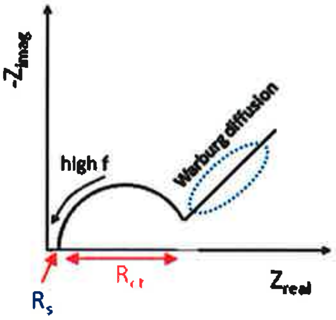


charge neutrality requires:
$Q_M = -Q_S$

The resulting charge distribution – two regions of equal and opposite charge – is known as the electrical double layer and it can be viewed as a capacitor. The double layer capacitance depends on the electrolyte ionic strength and the applied potential if a voltage is applied to the metal. Three models are frequently used to describe the metal/electrolyte interface: Helmholtz model, Gouy-Chapman model and Gouy-Chapman-Stern model. [20%]

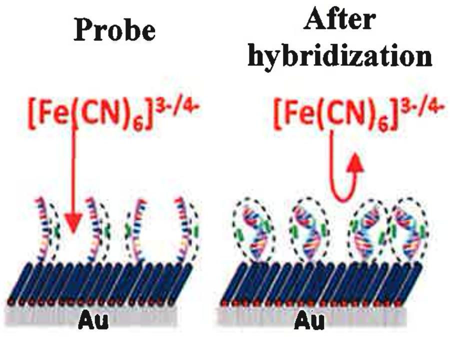
(b) In general, an electrochemical cell can be considered simply an impedance to a small sinusoidal excitation, hence it is able to represent the system's performance by an equivalent circuit that pass current with the same amplitude and phase angle that the real cell does under a given excitation. A frequently used circuit for Figure 2b , called the *Randles equivalent circuit*, is shown below. The parallel elements are introduced because the total current through the working interface is the sum of distinct contributions from the faradaic process, i_f, and double-layer charging, i_c. The faradaic impedance is a series combination of the charge transfer resistance, R_{ct}, and the Warburg diffusion element. The charge transfer resistance represents the resistance for the redox molecules in buffer to exchange electrons with the metal electrode. The Warburg diffusion element represents a resistance to mass transfer/diffusion. The Warburg impedance depends on the frequency of the potential perturbation. At high frequencies, it is small since diffusing reactants (redox molecules in buffer) do not need to move very far. At low frequencies, the reactants have to diffuse further, increasing the Warburg-impedance. It is a constant phase element and appears as a diagonal line with a slop of 45^0 on the Nyquist plot. R_S is the solution resistance between the reference electrode and the working electrode. It depends on the ionic concentration, type of ions, temperature and the geometry of the area in which current is carried. The Nyquist diagram of the system is also plotted below.

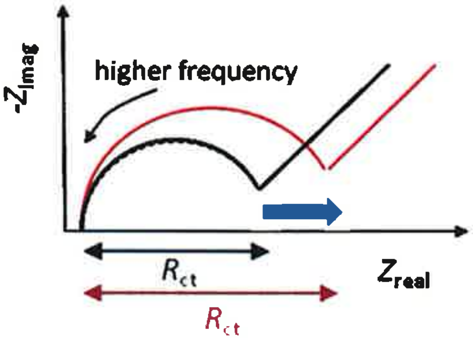




[40%]

(c) DNA biosensors with EIS measurement is based upon detection of the intrinsic negative charge of the target DNA. Hybridization with the immobilized probe single-stranded DNA causes an increased in the electrostastic barrier for the negatively charged redox molecules Ferricyanide $[Fe(CN)_6]^{3-}$ and Ferrocyanide $[Fe(CN)_6]^{4-}$ to exchange electrons with the Au electrode, resulting in an increase in charge transfer resistance R_{ct}.



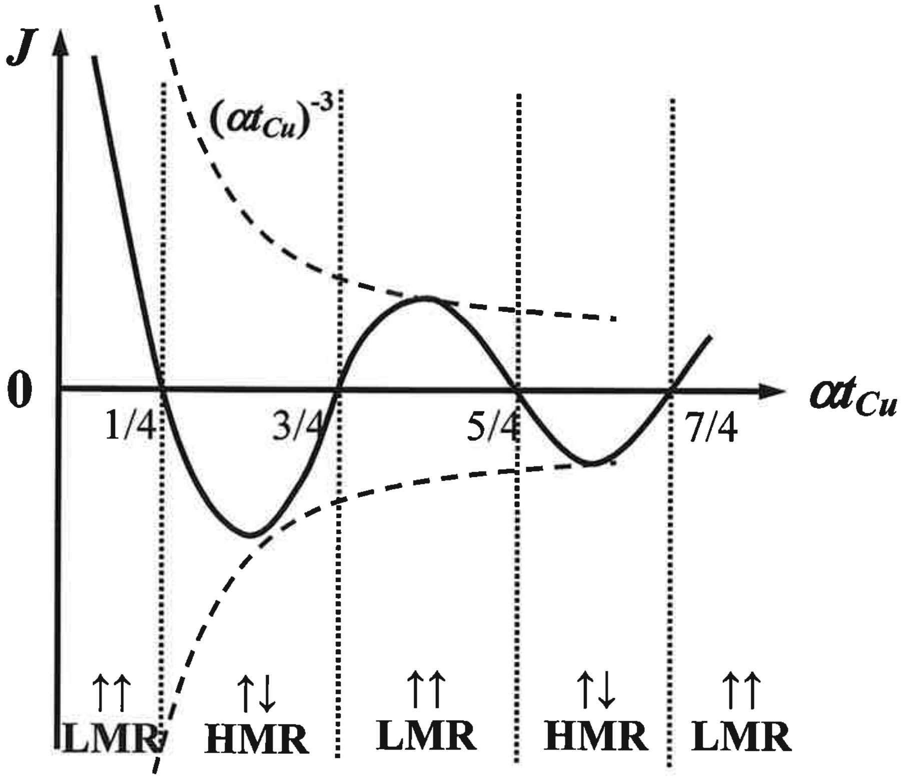


[30%]

(d) To improve the sensitivity, one can: (i) optimize the mixing ratio of the DNA probe and Mercaptohexanol; (ii) optimize the measurement buffer ionic strength; (iii) use peptide nucleic acid probe (neutral) rather than DNA probe (negatively charged). [10%]

Q4

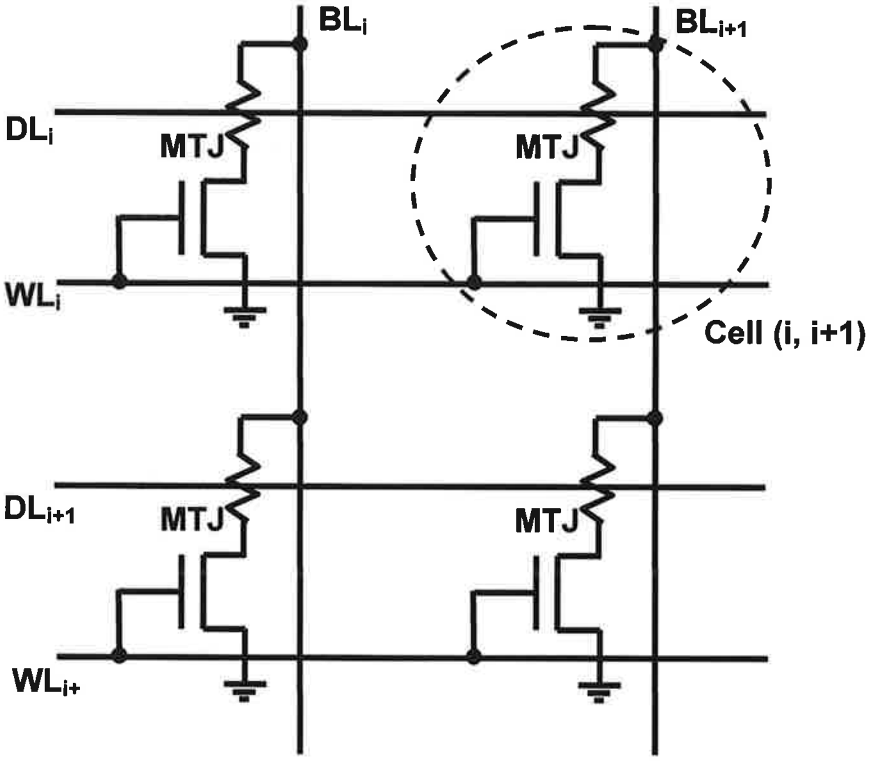
(a) Since the free energy $E = -J\overrightarrow{M}_{Co,1} \cdot \overrightarrow{M}_{Co,2}$ and $J \sim cos(2\pi\alpha t_{Cu})/(2\pi\alpha t_{Cu})^3$, the in-plane magnetisations in the two Co layers, $\overrightarrow{M}_{Co,1}$ and ,$\overrightarrow{M}_{Co,2}$ are in parallel and anti-parallel directions when $J > 0$ and $J < 0$, respectively. The zero field magneto-resistance in the CPP configuration is low LMR (high HMR) when the magnetisations are parallel (anti-parallel).



[50%]

(b) (i) Memory matrix as shown here:

BL / DL / WL ↔ Bit Line / Digit Line / Word Line



[25%]

(ii) Assume the position of the chosen cell is (i, i+1) as shown in the above sketch.

Write: Supply suitable current in DL_i and BL_{i+1}, so that each of them separately can only half-switch (tilt) the magnetisation of the free layer of the MTJ in the chosen cell but the combined magnetic field will be able to switch it to the desired parallel or anti-parallel configuration, which represents the bit of information to be stored there.

Read: Set WL_i to High and all the rest WLs to Low;

Set BL_{i+1} to High and all the rest BLs to Low;

Sense the current level on BL_{i+1} to find out that Cell (i, i+1) is in a high/low magneto-resistance state, in order to determine the stored bit of information.

(NB: the Read operation is non-destructive.) [25%]

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ENGINEERING TRIPOS PART IIB 2013
NOTES, MODULE 4B6: SOLID STATE DEVICES

1. Final crib (v. 5.1) is attached.

2. List of numerical answers:

   Q1(b) 5.06 V
   Q1(c) $2.29\text{x}10^{-6}$ m
   Q1(d) $4.05\text{x}10^{-5}$ Farad m^{-2}

3. Revised examination paper (v 5.1) is also attached, in which the missing "-" sign from $E = - J\ M_{Co,1}.M_{Co,2}$ in Question 4 was added.

D. P. Chu (Principal Assessor)
May 2013 (ver.1)