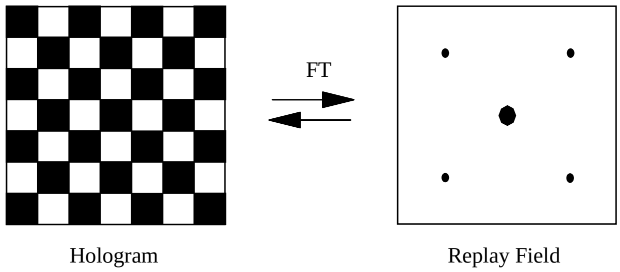
4B11 crib – NB answers longer than would be expected from candidates

Q1 a) [20%] Such a 2-D combination of these pixels in various positions is defined as a Hologram and the pattern generated by the hologram if the far field is the Replay Field. The translation between the two is the Fourier transform.



The far field of a single square pixel is its Fourier transform:

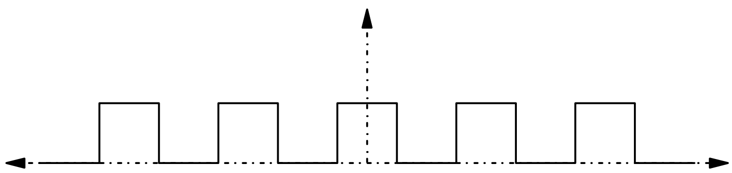
$$F(u,v) = \iint_{\pm\infty} f(x,y) e^{2\pi j(ux+vy)} dxdy = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} A e^{2\pi j(ux+vy)} dxdy$$

$$= \int_{-a/2}^{a/2} e^{2\pi j(ux)} dx \int_{-a/2}^{a/2} A e^{2\pi j(vy)} dy = \frac{A}{2\pi j}\left[\frac{e^{2\pi j(ux)}}{u}\right]_{-a/2}^{a/2} \frac{1}{2\pi j}\left[\frac{e^{2\pi j(vx)}}{v}\right]_{-a/2}^{a/2}$$

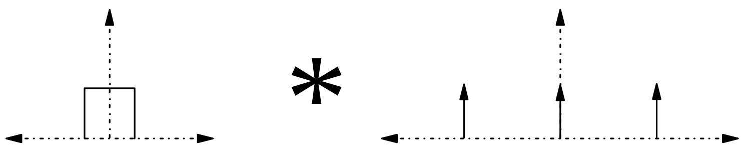
$$= \frac{A}{2\pi j}\left[\frac{e^{\pi jau} - e^{-\pi jau}}{u}\right]\frac{1}{2\pi j}\left[\frac{e^{\pi jav} - e^{-\pi jav}}{v}\right] = Aa^2 \mathrm{sinc}(\pi au)\mathrm{sinc}(\pi av)$$

Assume infinite plane wave illumination, a perfect FT from the lens or free-space, no apodisation

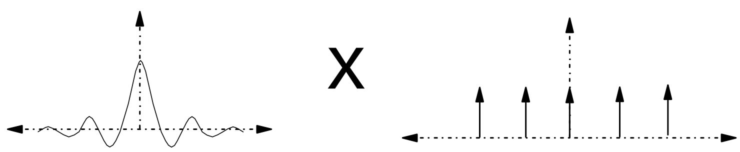
b) [30%] If the aperture is extended vertically to infinity and then the resulting stripe is then repeated horizontally every 2a positions it will form a vertically amplitude grating. If a pixellated pattern such as a grating is viewed from the end it can be modelled as a as a repetitive 1-D function. The repetition rate is defined by the pixel pitch or period.



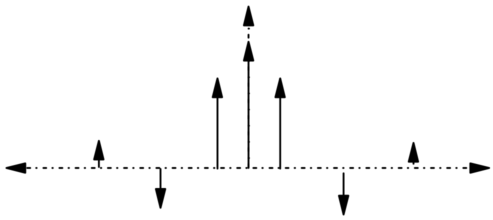
This can be expressed as a convolution of two functions.



Where the delta function train represents the sampling or pixellation function and * represents a convolution. After the Fourier transform we have the replay field by Fourier analysis and the .

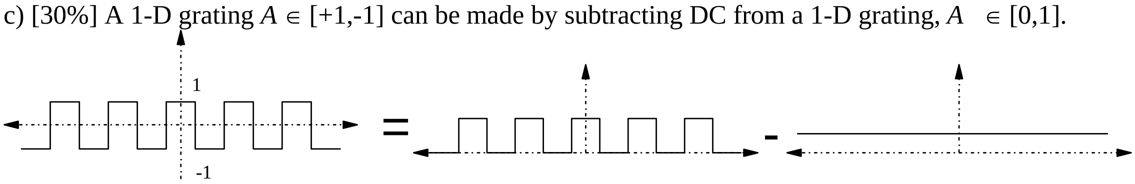


Gives the final result.

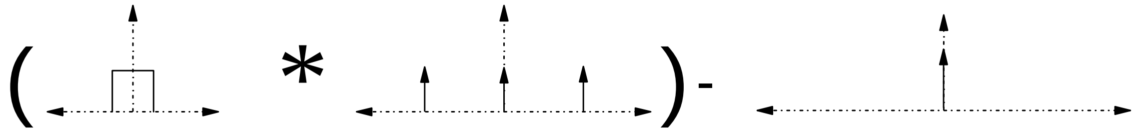


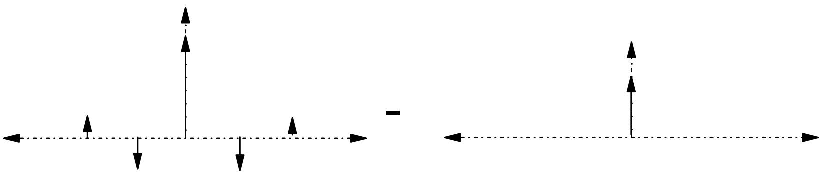
The train of delta functions has a sinc envelope and every second delta function is suppressed by the zeros of the sinc. Each symmetric pair of delta functions above represents a separate order and is repeated every odd harmonic. The envelope of the delta functions above are governed by the sinc function derived in part (a).

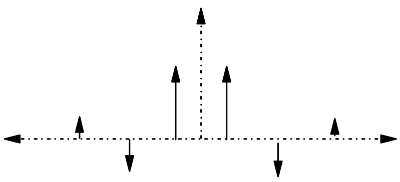
c) [30%] A 1-D grating $A \in [+1,-1]$ can be made by subtracting DC from a 1-D grating, $A \in [0,1]$.



Hence in the Fourier domain we have:







The sinc envelope means that the 41% of the amplitude is in the first order. The amplitude grating will block light, hence the estimated efficiency = 50% (amplitude)*0.41 = 20%. The binary phase grating diffracts all of the light, hence the far field diffraction pattern (no zero order) and the efficiency is now 41%.

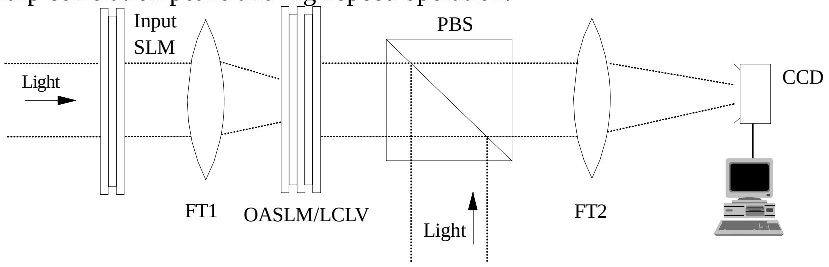
d) [20%] A grating is an ideal candidate for a routing hologram as it is the optimal efficiency that can be achieved with binary phase (41%). Hence the loss in the switch will be minimised. Also, the noise from the grating is very discrete, which means that the crosstalk is very low in positions between the orders and their harmonics. A more subtle advantage of gratings is that they are simple to calculate and can be stored very efficiently. A SLM can be addressed directly with a grating without having to store any data other than periodicity.

The main drawback with gratings as routing holograms is that they are limited by their periodicity to discrete values. This has two main effects. 1) there are a limited number of locations in the RPF where light can be routed to, hence there are a limited number of fibre channels. 2) The number of channels is also limited by the crosstalk as the chances of picking up an unwanted harmonic or order gets higher with the number of gratings/holograms.

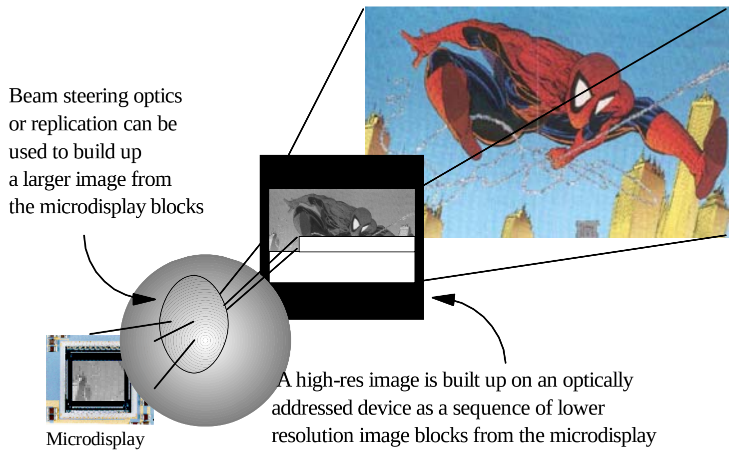
A compromise are Damman gratings which are more complicated than periodic gratings but can be calculated very easily and have discrete noise properties.

Q2 a) [35%] An OASLM takes the intensity of an incoming optical signal and converts it into an image which is displayed on a SLM. It is a way of capturing, processing and storing optical intensity information within an optical system. Two main applications:

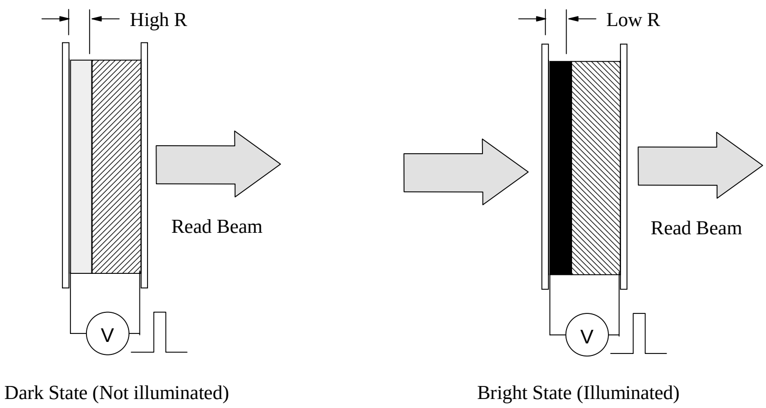
The OASLM can be used as the non-linearity in the Fourier plane of an optical correlator such as a JTC. The OASLM effectively thresholds the intensity of the spectrum from the input plane, which gives good sharp correlation peaks and high speed operation.



A second application is in projection displays, where the OASLM acts as a storage device. The almost continuous resolution can be exploited by storing low res tiles spatially onto the OASLM to make a higher res image or hologram. The OASLM can be used to convert from incoherent to coherent light.

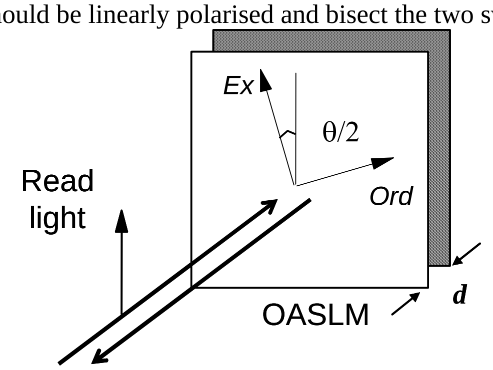


A LC OASLM can be made from a thin layer of LC sandwiched between a conductive glass electrode and a photoconductive layer. There is an external electrical field applied, common to the whole OASLM and is used to time the read and write cycles of the OASLM. The photoconductive layer is usually a thin film such as CdS or amorphous silicon (aH:Si) which has the required photo-active properties. When the external electrical field is positive (write cycle) a voltage appears across the aH:Si and LC layers. If the aH:Si is not illuminated (in a dark state) the its resistivity is very high, and most of the voltage appears across the aH:Si. This means that the voltage across the LC is insufficient to switch the molecules. If the aH:Si is illuminated (bright state), then the resistivity of the aH:Si is low and the voltage appears across the LC, causing it to switch.



The commonest form of LC used in OASLMs is FLC. The binary nature and speed are both advantages in the JTC application and the ability to be bistable is very important in applications such as projection displays.

b) [25%] A reflective OASLM uses a layer of FLC twice as it is reflected off the photoconductive layer. In order to obtain binary phase, the SLM must act as a half-wave plate, which in reflection will be a quarter-wave plate. The FLC must have a switching angle of 90 degrees for optimal performance. For binary phase the light should be linearly polarised and bisect the two switched states of the FLC.



For the layout above, the analyser should be horizontally aligned, however if the switching angle of the FLC is 90 degrees and the retardation is pi, then it is not needed as the OASLM will give pure binary phase.

c) [15%] For binary phase the device must act as a half-waveplate with a retardance of pi.

$$\Gamma = \frac{2\pi d \Delta n}{\lambda} = \pi \quad \text{Hence d} = 3.25\text{x}10^{-6}\text{m (3.25um)}$$

However, for a reflective device we need half this thickness hence d = 1.63um.

d) [25%] In order to make a reflective OASLM there must be a reflective layer next to the photoconductive layer sandwiched between the photoconductor and the LC. Most photoconductive layers are not ideal reflectors, although aH:Si is about 60% reflective at 650nm, hence is can be used as a reflective layer. The problem with this is that the aH:Si is also sensitive to 650nm so if care is not taken, the read light will act as a write beam for the OASLM.

Another option is to insert a mirror between the LC and the a-Si:H, however this is not ideal either. A metal mirror will be conductive which is Ok across the layers, but not ideal laterally as the charges from the aH:Si will leak away from the image.

Q3 a) [20%] The term *loss* refers to the amount of optical power which is launched down the optical fibre at the output end of the switch. It is normally the ratio of the optical power launched into each output fibre and the optical power at each input fibre. If the switch is configured to route light to the *k*th fibre in an output array of *n*, then the *crosstalk* is the ratio of light launched down the desired fibre to the light launched down one of the other fibres which are not being routed. Both are normally expressed as power ratios in decibels.

Fan-in loss arises in holographic switch because the only fibre in the output which is on the main optical axis is the one in the centre of the array. When light is steered to the outermost fibres it is at an angle to the central axis which no longer satisfies the perfect launch condition of a single mode fibre. Hence there is a loss which depends on this angle and therefore the position of the output port.

b) [30%] The total input power which appears in the output plane is P_{in}. The total power which is routed into a desired fibre by the CGH is P_{sp} and the remaining power is dissipated into the whole plane as the background or noise power P_{bk}.

$$P_{in} = 2P_{sp} + P_{bk}$$

The factor of 2 is due to the symmetry of the pattern due to binary phase. We can define the CGH efficiency η as the ratio between the power in the spot, P_{sp} and the input power P_{in}.

$$\eta = \frac{P_{sp}}{P_{in}}$$

For *n* fibres in the output array of a 1 to *n* switch, the power into a single fibre will be ηP_{in}. If the CGH has *N*x*N* pixels, then the replay field can also be assumed to contain *N*x*N* 'spatial frequency pixels'. If we assume that the background power is uniformly distributed over the N^2 spatial frequency pixels in the replay field then the background power at each pixel will be.

$$P_{bpix} = \frac{(1-2\eta)P_{in}}{N^2}$$

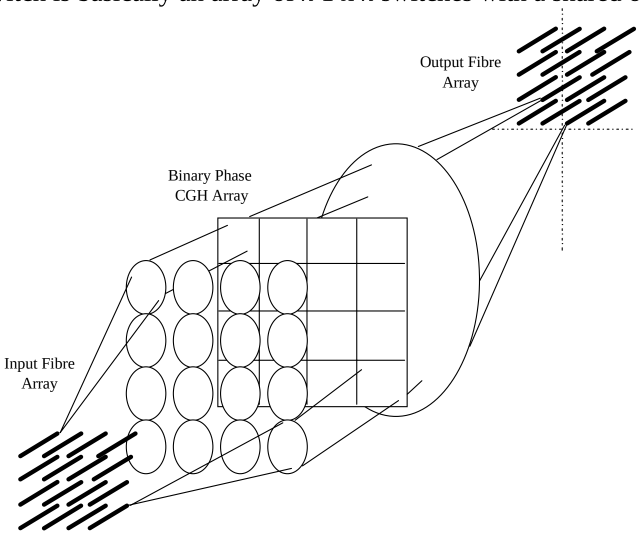
Hence the crosstalk is the ratio of the light routed to a fibre to P_{bpix}.

$$C = \frac{\eta}{1-2\eta}N^2$$

Assumptions:

- The distribution of the background power is uniform.
- The number of CGH pixels is infinite.
- The pixel pitch is effectively zero, hence no sinc envelope.
- The SLM used to display the CGH inevitably has no deadspace.
- The physical alignment of the fibres in the output array is perfect.
- Ferfect optics with no limitations or distortions.
- No fan in loss

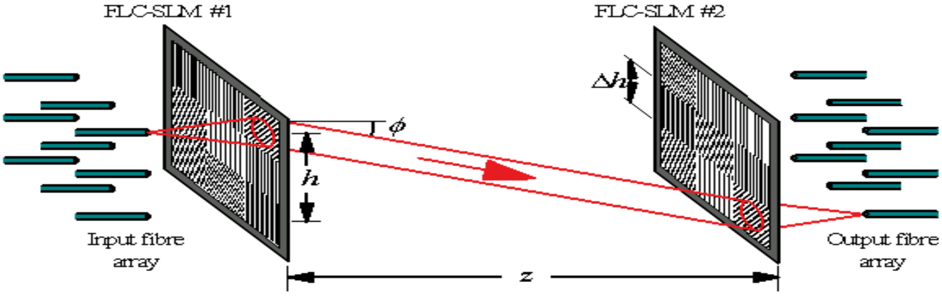
c) [25%] The *n* x *n* switch is basically an array of *n* 1 x *n* switches with a shared output lens



The *n* x *n* analysis for the crosstalk is the same except that we now have the background noise from each of the other *n* - 1 input fibres appearing at the each output fibre along with the ηP_{in} from the routed input. Hence the crosstalk will be.

$$C = \frac{\eta}{1-2\eta}\frac{N^2}{(n-1)}$$

d) [25%] The single hologram *n* x *n* switch is limited in scalability as it can only diffract light over a limited angle. Also the crosstalk boundary becomes probative. We can rectify this by using two holograms to steer the light. The first steer light into the switch, whilst the second steers light out of the switch back onto the output fibre's axis. The most efficient combination for routing is if the second hologram is the complex conjugate of the first routing hologram.

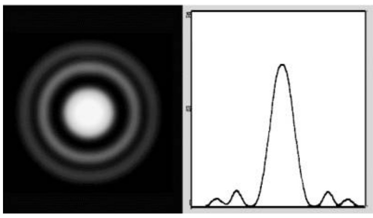


The two hologram switch can be scaled to any size and the loss through the switch does not scale with the number of input and output ports, it does however increase the loss as there are now two holograms routing the same beam. The only parameters which scale with the number of ports are the crosstalk and the physical length *z*. The crosstalk of the two hologram switch is greatly improved as the crosstalk of the first hologram is multiplied by the cross talk of the second hologram.

$$C = \left(\frac{\eta}{1-2\eta} \frac{N^2}{(n-1)} \right)^2$$

Q4 Part (a) [25%] The image (U_i) that is recorded is equal to the convolution of the point spread function (*PSF*) and the image that is predicted by geometrical optics, $U_i = (PSF) \otimes U_g$. Assuming that there are no aberrations, the electric field potential is well-defined at every point of the aperture and the ideal diffraction limited image is just a scaled and inverted version of object. The image is a convolution because diffraction from an aperture or lens pupil convolves the ideal image with the Fraunhofer diffraction pattern of the aperture/lens pupil. The point spread function arises due to diffraction from the aperture/lens pupil and represents the image that an optical system forms of a point source.

The point spread function of an optical system with a circular aperture is the well-known Airy disc described by:



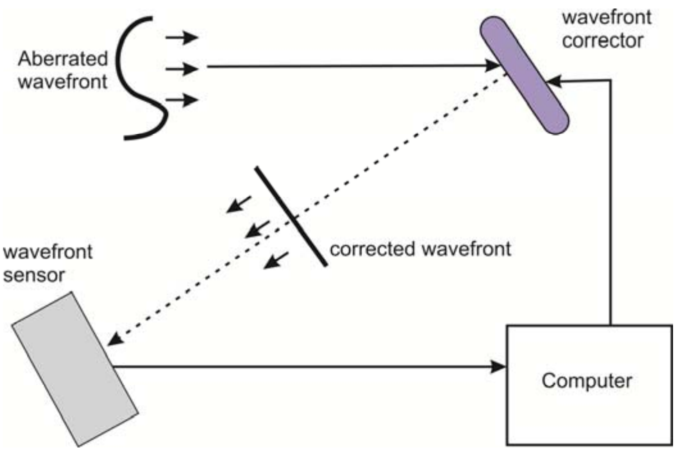
$$I(r) = \left(\frac{A}{\lambda z} \right)^2 \left[2 \frac{J_1(kwr/z)}{kwr/z} \right]^2$$

A convolution represents a superposition integral whereby each point of one function is multiplied by the whole of another function before it is then summed. Equation (1) can be re-written as

$$U(X,Y) = \iint PSF(X-x, Y-y) U_g(x,y) dxdy$$

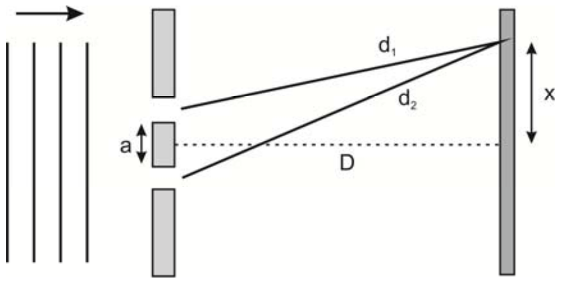
The PSF represents the Fraunhofer diffraction pattern of the aperture/lens pupil of the optical system. This Fraunhofer diffraction pattern is the Fourier transform of the aperture/lens pupil function.

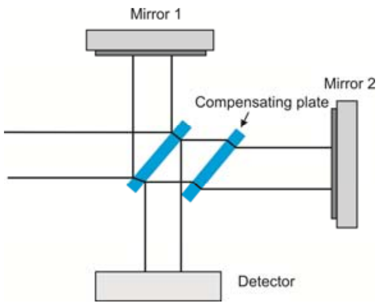
Part (b) **[30%]** A typical layout should look like:



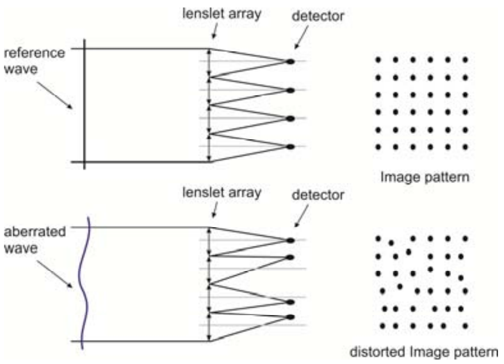
Key points: candidates must include a wavefront corrector and wavefront sensor (WFS) and should that it is a closed loop-system. There should also be a brief description about each component. For example, the wavefront sensor measures the aberrated wavefront. This can be done by encoding the phase delay in terms intensities through interferometric methods, measurements of the slope of the wavefront, or through measurements of the curvature of the wavefront. In the diagram, candidates could also include a Beam splitter to separate the beam into one that will be detected by WFS and one to create image. There should also be a phase correcting element, which is a component to precisely control phase delays. This could be a deformable mirror or spatial light modulator. Finally, the diagram should consist of a computer with which to convert input signals from the WFS to output signals to drive compensator.

The wavefront can be measured using inteferometric techniques such as division of wavefront (Young's slits)or division of amplitude. The latter method includes Michelson interferometers, Mach-Zehnder interferometers, and Twyman-Green interferometers. These methods involve splitting the beam up into separate parts so that they propagate along different paths. When these beams are superimposed, a fringe pattern is observed. By measuring the intensity profile of this fringe pattern at different positions, it is then possible to determine the phase delay of the wavefront.

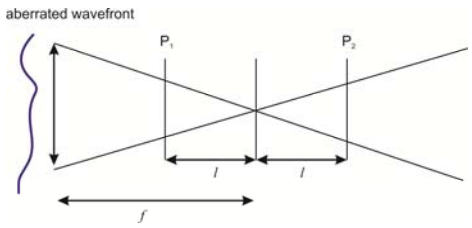




Measurements of the slope of the wavefront can achieved using Shack-Hartmann technique or the pyramid wavefront sensor. Candidates should give a brief description about these methods. For a plane wave incident on a Shack-Hartmann wavefront senor, the spot will be located at the optical axis of the corresponding lenslet. For a distorted wave, on the other hand, the spot is displaced. The displacement of the spot is directly proportional to the slope of the wavefront.

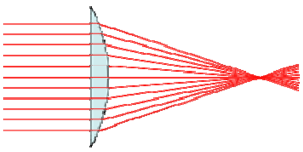


Curvature wavefront sensor measures the second spatial derivative of the wavefront. The wavefront curvature can be deduced from two displaced focal planes. Local curvature of wavefront leads to a convergence closer to one of the two planes this gives rise to intensity variations.

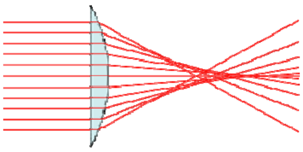


Part (c) [25%]

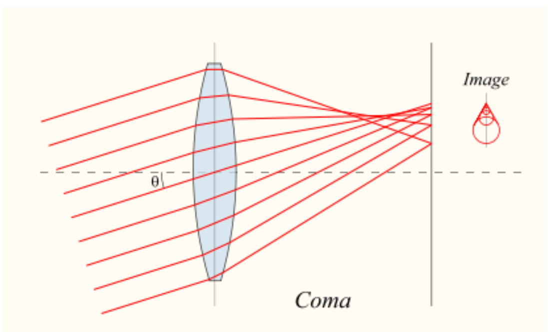
Three sources of aberration are: spherical aberrations, astigimatism and coma.



Spherical aberration A perfect lens focuses all incoming rays to a point on the optic axis. A real lens with spherical surfaces focuses rays more tightly if they enter it far from the optic axis than if they enter closer to the axis. It therefore does not produce a perfect focal point.



Astigmatism occurs when the optical system is not symmetric about the optical axis. This often arises due to manufacturing error in the surfaces of the components or misalignment of the components.



Coma Refers to aberrations due to the imperfection in a lens or other components. This results in off-axis point sources such as stars appearing distorted, appearing to have a tail like a comet. Coma is defined as a variation in magnification over the entrance pupil.

Specifically, aberrations affect the point-spread function. The role of adaptive optics is to make the PSF as close to the diffraction-limited case as possible. $U_i = (PSF) \otimes U_g$ By taking Fourier transforms of the above equation we get,

$$F(U_i) = F(PSF)\,F(U_g)$$

The Fourier transform of the PSF represents the *optical transfer function*, which is a measure of the imaging quality of the system: it represents how each spatial frequency in the object field is transferred to the image. Another quantity is the *Strehl* ratio, which quantifies the degree of aberrations of an optical system. It is defined as the ratio of the peak in the Airy core of corrected image to that of theoretically perfect image. For a perfect image, the *Strehl* ratio is equal to unity.

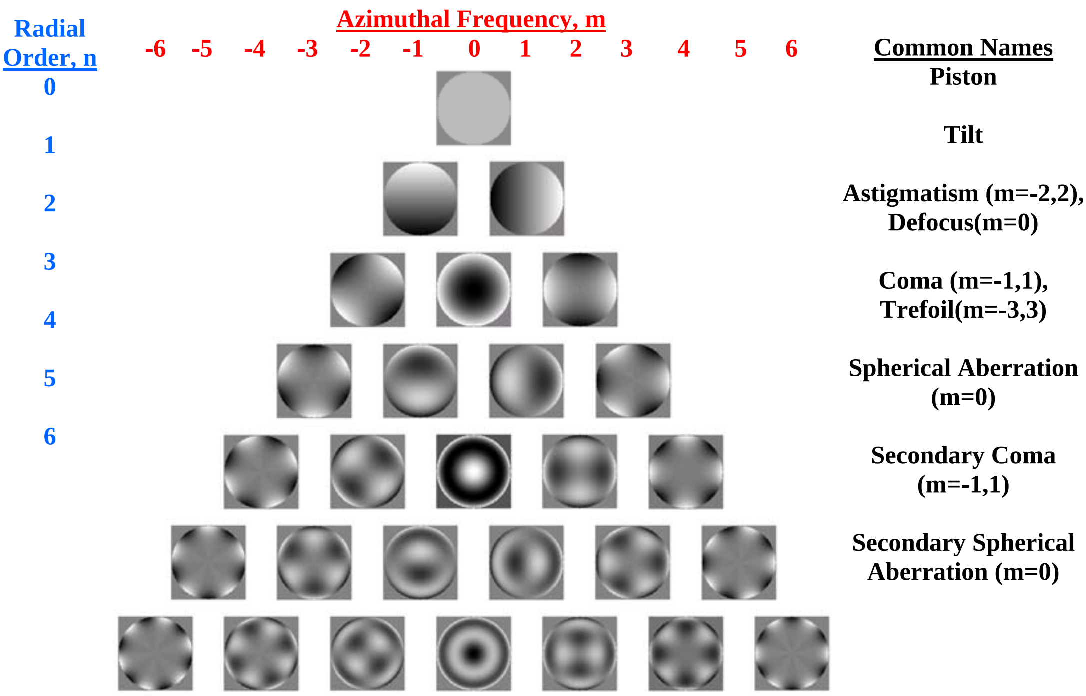
Part (d) [20%] Zernike polynomials are used to describe aberrations in an optical system as they form a complete set of functions or modes that are orthogonal over a circle of unit radius. Other power series descriptions are not orthogonal. Wave aberrations in an optical system with a circular pupil can be accurately described by a weighted sum of Zernike polynomials. Zernike polynomials have nice mathematical properties. They are orthogonal over the continuous unit circle. All their derivatives are continuous. They efficiently represent common errors (e.g. coma, spherical aberration) seen in optics. They form a complete set, meaning that they can represent arbitrarily complex continuous surfaces given enough terms.

The Zernike polynomials (even and odd) are defined as:

$$Z_n^m(\rho,\theta) = N_n^m R_n^{|m|}(\rho)\cos(m\theta) \quad \text{for } m \ge 0,\ 0 \le \rho \le 1,\ 0 \le \theta \le 2\pi$$
$$= -N_n^m R_n^{|m|}(\rho)\sin(m\theta) \quad \text{for } m < 0,\ 0 \le \rho \le 1,\ 0 \le \theta \le 2\pi$$

for a given n: m can only take on values of $-n, -n+2, -n+4, \ldots, n$

mode j	order n	frequency m	$Z_n^m(\rho,\theta)$
0	0	0	1
1	1	-1	$2\rho\sin(\theta)$
2	1	1	$2\rho\cos(\theta)$
3	2	-2	$\sqrt{6}\rho^2\sin(2\theta)$
4	2	0	$\sqrt{3}(2\rho^2-1)$
5	2	2	$\sqrt{6}\rho^2\cos(2\theta)$



Candidates should describe, with diagrams if possible, the first few low order Zernike polynomials.