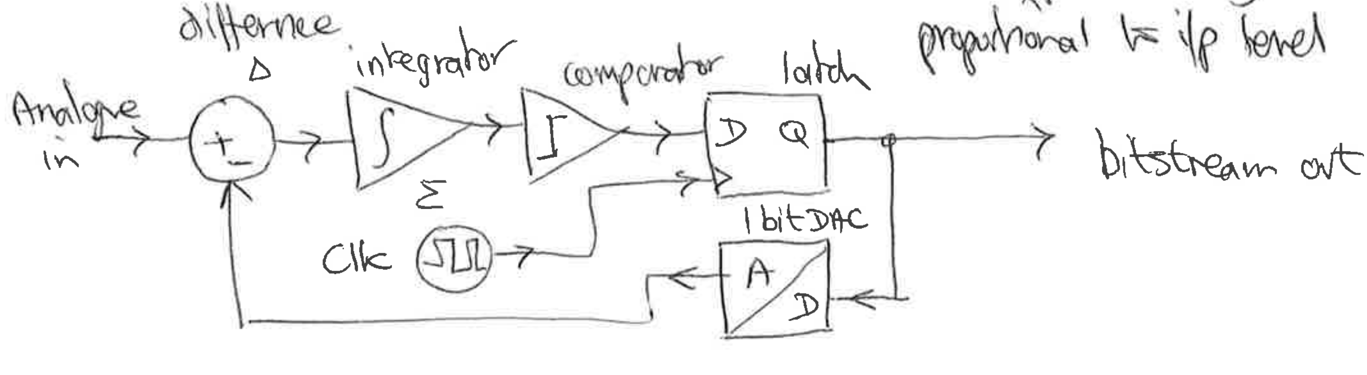
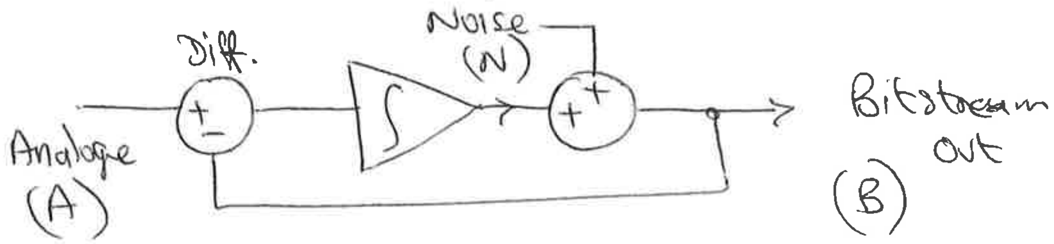
4B13 CRIB 2013

1 (a) ADC types

	rate	bits	principle
Flash	~GS/s	8	• array of comparators – one for each ADC level, operate in parallel, logic to combine into output byte • fast (very) • no sample/hold required
Successive Approximation (common use in μ-controllers)	~MS/s	12	• analogue input compared to internal DAC signal – • MSB to LSB of DAC are switched on in turn & kept at '1' if DAC < input • takes N bits × clk cycles to convert
Delta-Sigma	100 kS/s | 50 S/s	16 | 24	• Single bit DAC output is subtracted from input and the difference integrated & compared to zero to determine next DAC state • DAC pulse density is proportional to i/p level



1(a) contr.



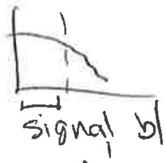
$$B = \int (A-B) + N \quad \text{and } \int \text{ has } 1/f \text{ freq. response}$$

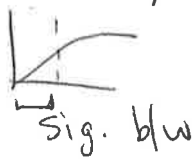
$$\therefore B = \frac{(A-B)}{f} + N \Rightarrow B + \frac{B}{f} = \frac{A}{f} + N$$

$$\therefore B(f+1) = A + fN$$

$$\therefore B = \frac{A}{(f+1)} + N\frac{f}{(f+1)}$$

low pass filter ← high pass

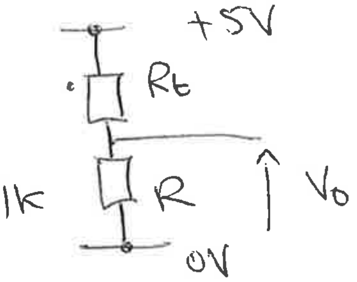




Hence noise is 'shaped' out of signal bandwidth (at low freq.) by converter operation

[25%]

(b) NTC Thermistor



$\beta' = 3100$

$R_T = 500$ @ $0^\circ C$ (273 K)

$$R_T = A e^{\beta'/T}$$

$$500 = A e^{3100/273} \quad \therefore A = 5.85 \times 10^{-3}$$

$$V_o = \frac{1000 \times 5}{(1000 + R_T)} \text{ V}$$

@ $20^\circ C$ $\quad R_T = 230\,\Omega \quad \therefore V_{o_{20}} = 4.065$ V

$40^\circ C$ $\quad R_T = 117\,\Omega \quad \therefore V_{o_{40}} = 4.476$ V

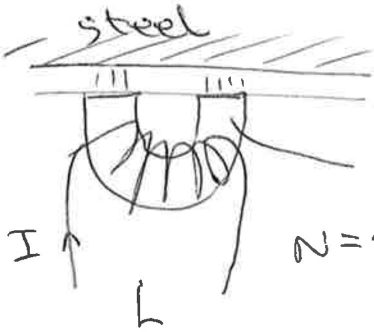
$0^\circ C$ $\quad R_T = 500\,\Omega \quad \therefore V_{o_0} = 3.333$ V

$$\text{Non-linearity} = \left| \frac{\text{actual output} - \text{linear output}}{\text{linear output change}} \right| = \left| \frac{4.476 - [4.065 + (4.065 - 3.333)]}{(4.065 - 3.333)} \right|$$

$$= \frac{0.321}{0.732} \times 100\% = 44\%$$

[25%]

1(c)



$$\oint H \cdot dl = NI = H_m l + H_0 2t$$

$$B_m = B_0 = \mu_0 \mu_r H_m = \mu_0 H_0 = B$$

$$\therefore H_m = H_0 / \mu_r$$

$$\therefore NI = H_0 \left(l/\mu_r + 2t \right) = \frac{B_0}{\mu_0} \left(l/\mu_r + 2t \right)$$

$L = \frac{N\phi}{I}$ where $\phi = BA$ $\therefore L = \frac{N \mu_0 N \cancel{I} A}{\left(l/\mu_r + 2t \right) \cancel{I}}$

$$\therefore L = \frac{N^2 \mu_0 A}{\left(l/\mu_r + 2t \right)}$$

500, $4\pi \times 10^{-7}$, 10^{-5}, 0.025, 1500

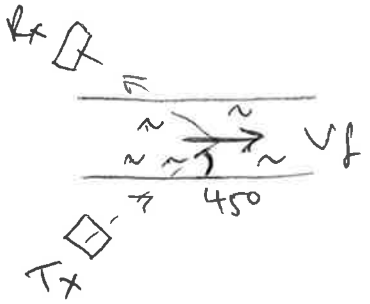
$\Rightarrow$ $t = 10^{-4}$ m $L = 0.01450$ H

$t = 2 \times 10^{-4}$ m $L = 0.00754$ H

$\therefore \Delta L = 7.0$ mH

[25%]

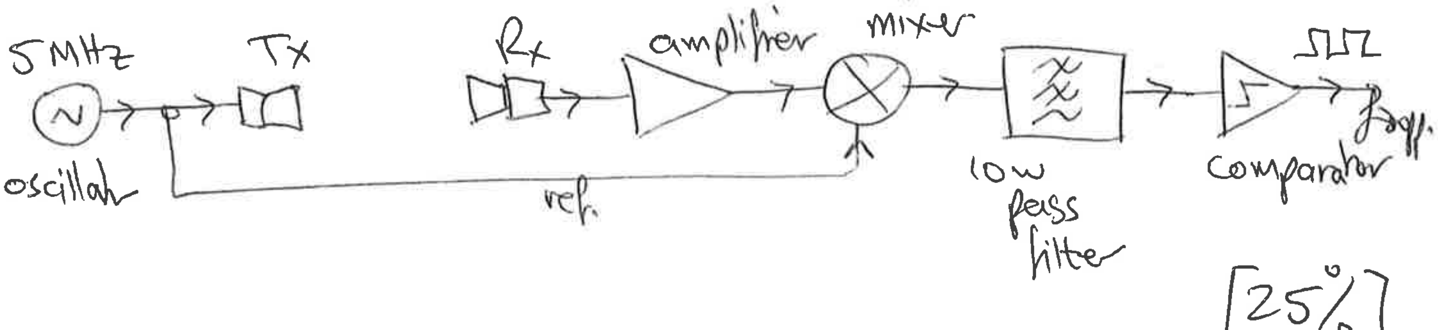
(d)



$$v_f = \frac{20 \times 10^{-6}}{\frac{\pi}{4} (6 \times 10^{-3})^2} = 0.707 \text{ m/s}$$

$$f_{Dop} = 2 \frac{v_f}{v_s} \cdot f_{u/s} \cdot \cos\theta = 2 \cdot \frac{0.707}{524} \cdot 5 \times 10^6 \cdot 0.707$$

$$= 9.54 \text{ kHz}$$



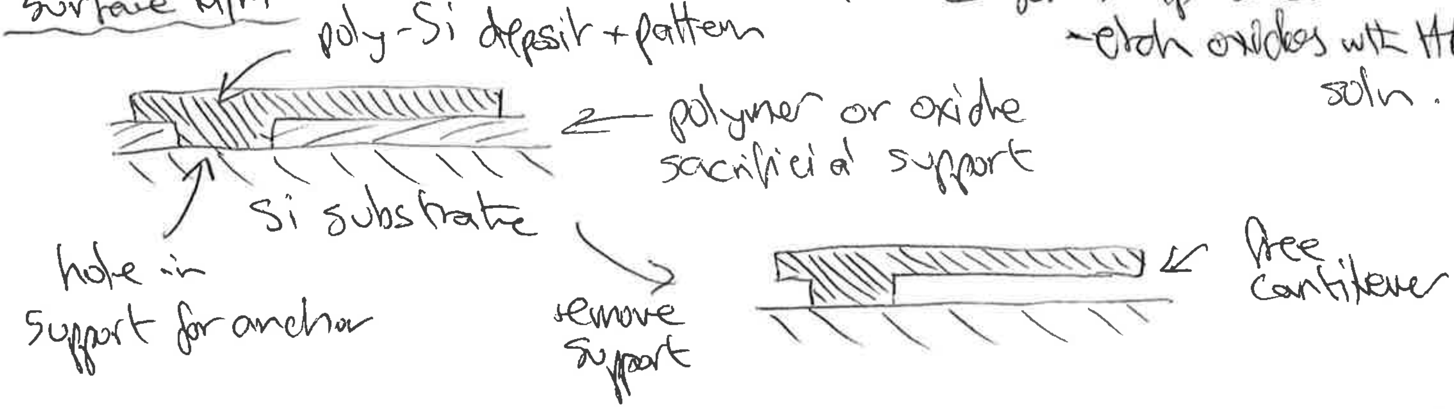
[25%]

2(a) Key processes: photolithography
- spin on photo-sensitive polymer
- expose to light
- develop resist (dissolves exposed sections)
- process eg: etch or deposit onto layer

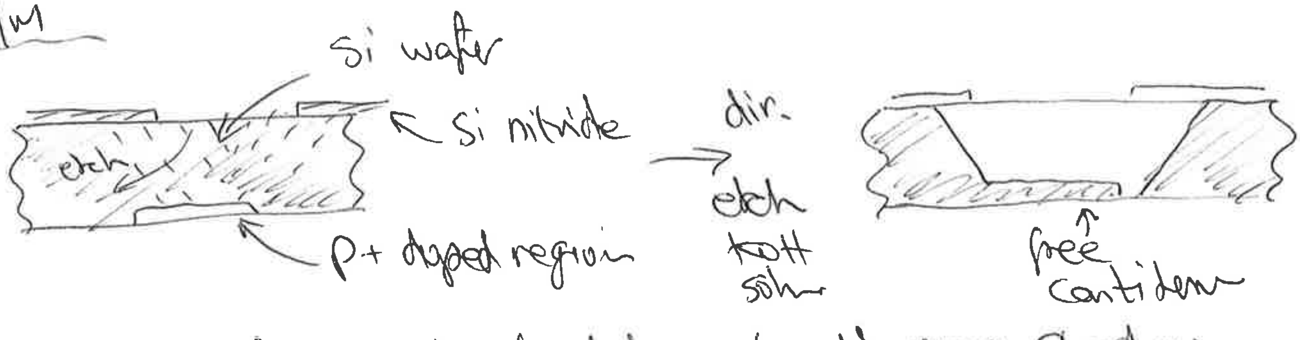
Surface micromachining:
- deposit metals by plating and vacuum processes sputtering or evap.
- etch metals with acids
- deposit poly-Si by plasma process
- deposit oxynitrides from ~~SiH~~ SiH_4 / $SiH_4 + NO_x$ / $+ NH_3$

Bulk micromachining:
- Si wafer can be etched with hot KOH soln, down xtal planes, etch stop in B doped regions
- directional etching by plasma RIE $CF_4 + O_2$ for steep sides
- etch oxides with HF soln.

Surface M/M

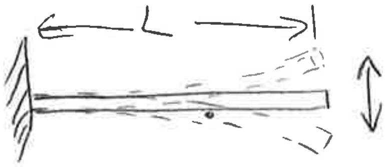


Bulk M/M

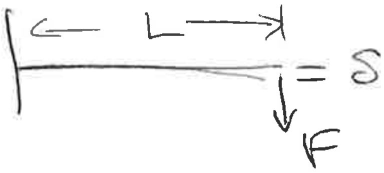


another wafer can be bonded on to add more structures, electrodes etc.

[30%]

2(b) 

Estimate average beam deflection to occur 2/3 along beam (could assume 1/2 way - but it's clearly a bit more).



From databook (structures)

$$\delta = \frac{FL^3}{3EI}$$

$I = \frac{1}{12} bd^3$, $E_{Si} = 150$ GPa, $\rho_{Si} = 2300$ kg m^{-3}

$= 2.08 \times 10^{-21}$ m^4

Taking the stiffness at $2/3 L$: $\frac{F}{\delta} = \frac{3EI}{(\frac{2}{3}L)^3} = \frac{10.1\,EI}{L^3} = S$ $(25.2$ N/m$)$

Resonant freq $= \frac{1}{2\pi}\sqrt{\frac{S}{m}}$

$m = Lwd\rho_{Si} + Lwt\rho_{polymer}$ — 1200 or 1212 kg m^{-3}

$d = 5 \times 10^{-6}$ m
$L = 500 \times 10^{-6}$ m
$w = 200 \times 10^{-6}$ m
$t = 100 \times 10^{-9}$ m

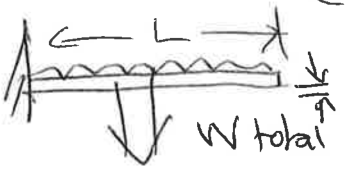
$\therefore m = 1.15 \times 10^{-9} + 1.2 \times 10^{-11}$ kg $= 1.162 \times 10^{-9}$ kg
(+10%) with vapour. (+0.1%)

$$f_{res} = \frac{1}{2\pi}\sqrt{\frac{10.1\,EI}{m}} = \frac{1}{2\pi}\sqrt{\frac{10.1 \cdot 150 \times 10^{9} \cdot 2.08 \times 10^{-21}}{10^{-18} \times \underbrace{1.162 \times 10^{-9}}_{m} \times 500^3}} = 23.4 \text{ kHz}$$

[30%]

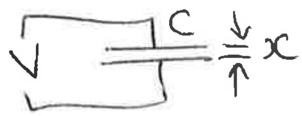
2(c) with $m + 0.1\% \Rightarrow f - 0.05\% \Rightarrow -11.7$ Hz

(or can rework calcs. with new mass) [15%]

2(d) 

$\delta = 100$ nm $\quad \delta = \frac{WL^3}{8EI} = 10^{-7}$ m

$\therefore W = 2 \times 10^{-6}$ N

Force between capacitor plates: 

$W\,dx = dE$

$\therefore W = \frac{dE}{dx}$

$E = \frac{1}{2}CV^2$, $C = \frac{A\epsilon_0}{x}$ $= 0.177$ pF

$\therefore E = \frac{1}{2}\frac{A\epsilon_0 V^2}{x}$

$\therefore \frac{dE}{dx} = W = -\frac{1}{2}\frac{A\epsilon_0 V^2}{x^2} = 2 \times 10^{-6}$ N

$x = 5 \times 10^{-6}$ m
$A = 200\,\mu m \times 500\,\mu m$

$\therefore V = 10.6$ V

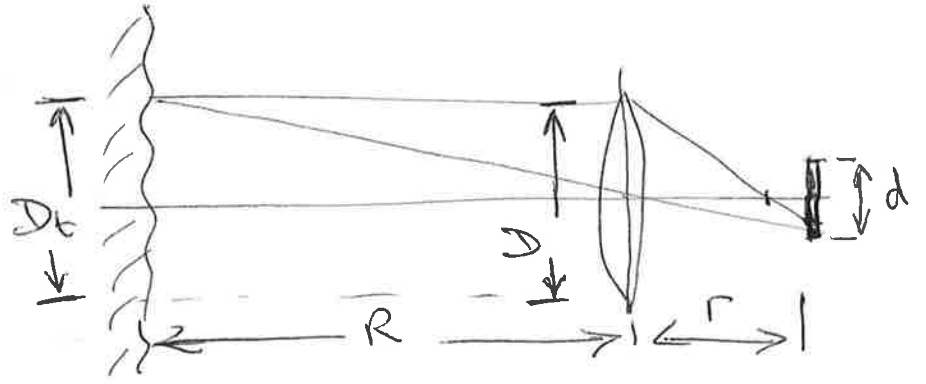
[25%]

3 (a) Stephan's Law $W = \epsilon \sigma_{SB} T^4$

power emitted / unit area
$\epsilon = 0.95$ emissivity
$T =$ temprature K
273 or 293

Lambert's Law $\delta W = \frac{W \cos\theta}{\pi} A \, \delta\omega$

$\delta W =$ power in solid angle $\delta\omega$
$W =$ total power emitted
$A =$ surface area
$\theta =$ angle to surface normal.
(90°)



$$\delta W = \frac{W \cancel{\cos\theta}}{\pi} A \, \delta\omega = \frac{W}{\pi} \cdot \frac{\pi D_t^2}{4} \cdot \frac{\frac{\pi D^2}{4}}{\cancel{4\pi} R^2} \cdot \cancel{4\pi}$$ = power per element

Similar Δ's $\frac{D_t}{R} = \frac{d}{r}$ $\therefore$ $\delta W = \frac{\epsilon \sigma_{SB} T^4}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{4} \frac{d^2}{r^2} \cdot \frac{\pi D^2}{4}$

$$\therefore \delta W = \epsilon \sigma_{SB} T^4 \cdot \frac{\pi}{16} \cdot \frac{d^2 D^2}{r^2} = 2.64 \times 10^{-15} T^4$$

0.95 $\quad 5.67 \times 10^{-8}$ $\quad 273^4$ or 293^4 $\quad 10^{-6}$ $\quad 0.2^2$ $\quad 10^{-2}$

[25%]

$\Delta P = 4.8\ \mu W$
0°C : 1.47×10^{-5} W $\quad \therefore \Delta T = 3.68$ mK
20°C : 1.95×10^{-5} W $\quad \therefore \Delta T = 4.86$ mK
$dT = 1.18$ mK

(b)

$$R_T = A e^{\beta'/T} \qquad \therefore \frac{dR_T}{dT} = -\frac{\beta'}{T^2} A e^{\beta'/T} = -\frac{\beta'}{T^2} R_T$$

3500 $\quad$ 1000 $\quad 293^2$

$$\therefore dR_T = -40.8 \, dT$$

where $dT = 250 \, \Delta P = 1.18$ mK

$$\therefore dR_T = 0.048 \ \Omega$$

3(b) contd. By similar triangles:

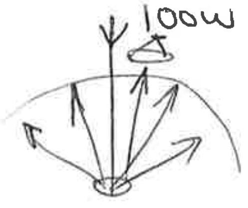
$$\frac{D_t}{R} = \frac{d}{r}$$

$d \leftarrow$ 1 mm, $r \leftarrow$ 200 mm, $R \leftarrow$ 1000 m

$\therefore D_t = 5\,\text{m}$

$\Rightarrow$ square $5\,\text{m} \times 5\,\text{m}$ on ground. [20%]

3(c)



Pulse-echo transit time $= \dfrac{2 \times 1000\,\text{m}}{3 \times 10^8\,\text{m/s}} = 6.67\,\mu\text{s}$

$$P_r = \frac{\frac{\pi D^2}{4}}{2\pi R^2} \times 0.4 \times 100\,\text{W} = 50\,\text{nW}$$

$D \leftarrow 0.1$ m, $R \leftarrow 1000$ m

optical power on detector diode.

photon energy $E = \dfrac{hc}{\lambda} = 2.34 \times 10^{-19}$ J/photon

$h \leftarrow 6.626 \times 10^{-34}$, $c \leftarrow 3 \times 10^8$, $\lambda \leftarrow 850 \times 10^{-9}$

$\therefore 50\,\text{nW} \equiv \dfrac{50 \times 10^{-9}}{2.34 \times 10^{-19}} = 2.14 \times 10^{11}$ photons/sec.

$\therefore$ photo current $= \underbrace{2.14 \times 10^{11}}_{\text{flux}} \times \underbrace{1.6 \times 10^{-19}}_{q} \times \underbrace{0.75}_{\eta} = 25.6\,\text{nA}$

[$\Rightarrow$ photo-diode sensitivity ~ 0.5 A/W ✓]

[35%]

3(d) Photo-current = 25.6 nA @ 1000 m; I is inverse square law with height

$\therefore$ @ 5000 m, photo-current on clear day $= \dfrac{25.6}{25} = 1.02\,\text{nA}$

Detection limit = 10 pA $\therefore$ factor of 102 in power margin.

$\hookrightarrow \equiv 10\log_{10}(102)\,\text{dB} = 20.1\,\text{dB}$

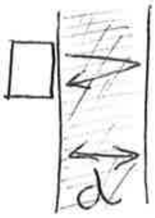
Total path length $= 2 \times 5000\,\text{m} = 10\,\text{km}$

$\therefore$ allowable attenuation $= \dfrac{20.1\,\text{dB}}{10\,\text{km}}$

$\Rightarrow$ coeff. atten. $= 2\,\text{dB km}^{-1}$

[20%]

4 (a)



$$t_{echo} = \frac{2d}{v_s} = \frac{10 \times 10^{-3}}{2850} = 3.51 \mu s$$

$$Z_{comp} = \rho v_s = 1750 \times 2850 = 5 \times 10^6 \ kg\,m^{-2}s^{-1}$$

$$Z_{PZT} = 7500 \times 4000 = 30 \times 10^6 \ kg\,m^{-2}s^{-1}$$

$$\text{Ref. coeff.} = \frac{(Z_{comp} - Z_{PZT})^2}{(Z_{comp} + Z_{PZT})^2} = 0.51$$

$\therefore$ Trans. coeff $= 0.49$

[20%]

(b) $\text{PZT Pulse power} = \frac{V^2}{R} \cdot \eta = \frac{120^2}{5000} \cdot 0.15 = 0.432 \ W$

Power coupled to composite $= 0.432 \times 0.49 = 0.211 \ W$

Power @ back face, reflected to front $= 0.211 \cdot 10^{-\frac{32}{10} \cdot 0.01} = 0.196 \ W$
(assume 100% refl. @ air interface)

Power coupled back to PZT $= 0.196 \times 0.49 = 0.096 \ W$

Power converted back to electrical $= 0.096 \times 0.15 = 0.0144 \ W$

$$P = \frac{V_r^2}{R} \Rightarrow 0.0144 = \frac{V_r^2}{5000} \quad \therefore V_r = 8.5 \ V$$

[35%]

(c) $$Z_m = \sqrt{Z_{PZT} Z_{comp}} = 12.2 \times 10^6 \ kg\,m^{-2}s^{-1}$$

with $v_s = 2500$ m/s $\Rightarrow \rho = 4880 \ kg/m^3$

Thickness $= \lambda/4$ where $\lambda = \frac{v_s}{f} = \frac{2500}{8 \times 10^6} = 313 \ \mu m$

$\therefore$ matching layer $= 78 \ \mu m$

[15%]

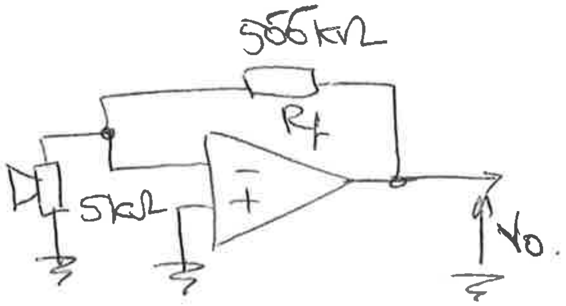
(d) Drive amplitude reduced by factor of 100 $\therefore$ Rx amplitude also $\div 100$ plus additional attenuation $\times 10^{-\frac{32}{10} \times 0.03} = 0.80$ for power $= \times\sqrt{0.80} = \times 0.895$ on voltage.

4(d) contd.

Hence signal is $\frac{8.5}{100} \times 0.895 = 76\,mV$

Noise sources:

for x100 op-amp



Feedback resistor : $v_n = \sqrt{4kTRB} = 0.26\,mV\,rms$ ($T = 300$, $k = 1.38\times10^{-23}$, $R = 500\times10^3$, $B = 8\times10^6$)

Transducer resistance : $\frac{v_n}{10} \times 100 = 2.60\,mV\,rms$

$i_n \times R_f = 2\times10^{-12} \times \sqrt{8\times10^6} \times 500\times10^3 = 2.83\,mV\,rms$

$e_i \times gain = 8\times10^{-9} \times \sqrt{8\times10^6} \times 100 = 2.26\,mV\,rms$

$v_n\ total = \sqrt{\Sigma v_n^2} = \sqrt{(0.26^2 + 2.83^2 + 2.26^2 + 2.6^2)}$

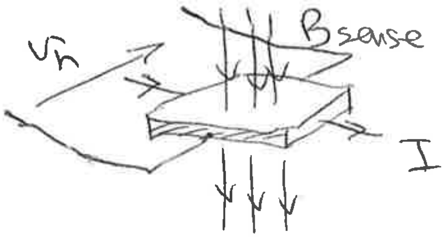
$= 4.47\,mV\,rms$

$\therefore$ Sig./noise $= \frac{76}{4.47} = 17:1$ pk/rms

or $\sim 3:1$ pk/pk.

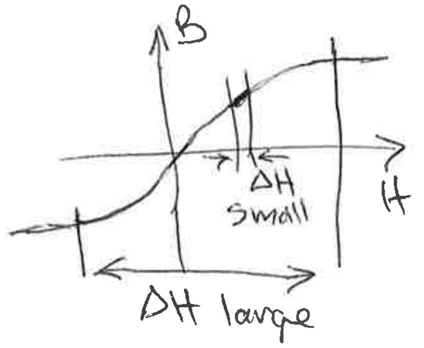
[peak-peak noise signal ~ 6x Vrms] [30%]

5 (a) Hall effect :



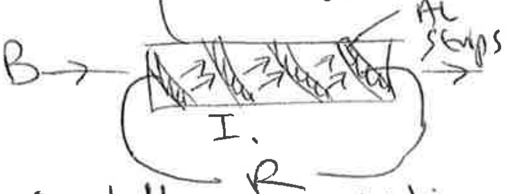
slice of semiconductor material with carrier flow along one axis (current flow). A magnetic field coupling through the face of the slice causes charge carriers to deflect along orthogonal axis. An electric field builds up to oppose Lorentz force on carriers ⇒ hence output voltage, V_h

Inductive sensor : a core of high permeability magnetic material forms an inductor wound with a coil. The inductor is excited with an alternating current – an external field biases the B-H loop to one side causing a change in μ_r : a small excitation gives a linear local ΔH drive and changing Bsense changes the inductance. For a large drive, this becomes a fluxgate magnetometer, where induced voltage transients in a pick-up coil become assymmetric on the time axis (equivalent to even harmonics of the drive excitation).



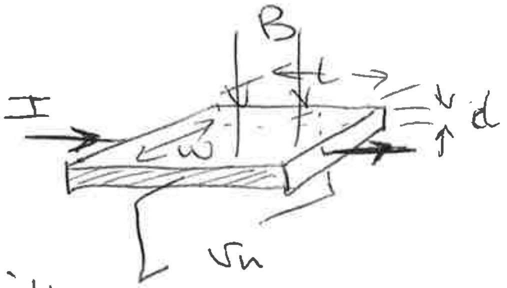
Magneto-resistive : (spin valve GMR or 'barber pole' MR)

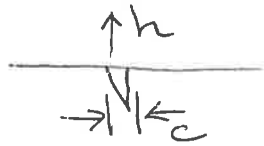
A stack of Co-Cu-Co layers in thin films to form a resistor. NiFe layers each side align with external magnetic fields – one of which has its direction pinned by a Fe/Mn poled layer. A ~10+% change in resistance is seen as the magnetisation angles between the layers change. Or 1% ΔR for 'barber pole' permalloy strip with Al electrodes. [30%]



Current flow & magnetisation angle

5(b)





$B_0 = 10\,mT$

$B = B_0 \left(\frac{c}{h}\right)^2$

carrier drift velocity, $v_d = \frac{V}{L}\mu = \frac{5}{200\times10^{-6}} \cdot 0.14 = 3500$ m/s

$= 2.5\,mT$

$Bq\,v_d = \frac{v_h}{w} q \quad \therefore\ v_h = w B \cancel{\frac{q}{q}} v_d$ where $B = 10^{-2}\left(\frac{1}{2}\right)^2$

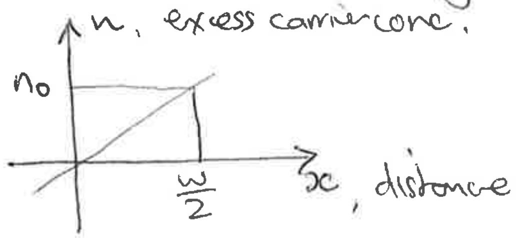
$\therefore\ \underline{v_h = 1.75\ mV}$ ($\equiv 0.7\ V/_T$ responsivity)

(c) $R = \frac{\rho\, l}{wd} = \frac{0.045}{10^{-5}} = 4500\,\Omega$

$\boxed{30\%}$

$v_n = \sqrt{4kTRB}$ noise voltage rms.

For response bandwidth consider carrier diffusion back across slice centre line – assuming linear excess carrier conc. for simplicity:



$n = \frac{2n_0}{w}x$

Fick's Law

$F = -D\frac{dn}{dx}$, $D = \frac{\mu kT}{q}$

$\frac{dn}{dx} = \frac{2n_0}{w}$

$= 3.62\times10^{-3}$ @ 300K

N = total excess carriers one side $= Ld\int_0^{w/2} \frac{2n_0}{w}x\,dx = \frac{n_0 wLd}{4}$

Consider 1 side: $\frac{dN}{dt} = FLd = -D\frac{2n_0}{w}Ld$ where $n_0 = \frac{4N}{wLd}$

$\therefore\ \frac{dN}{dt} = -D\frac{2}{w}\frac{4N}{wLd}Ld = -\frac{8DN}{w^2}$

Soln. of form $N = N_0 e^{-t/\tau}$ where $\tau = \frac{w^2}{8D}$

$t_{rise} \simeq 2.2\tau = \frac{(200\times10^{-6})^2\, 2.2}{8\times3.62\times10^{-3}} = \underline{3.04\ \mu s}$ $\quad f_{-3dB} = \frac{1}{2\pi\tau} = \underline{115\ kHz}$

5(c) cont'd.

$v_n = \sqrt{4 . kTRB}$

$B = 115\,kHz$
$R = 4800\,\Omega$

$\therefore v_n = 2.93\ \mu V\ rms.$

For a signal-to-noise ratio of say 5:1, the minimum signal ≈ 15 μV. This gives a magnetic flux density limit of $15/0.7 \approx 20\,\mu T$.

$$20 \times 10^{-6} = 0.01 \left(\frac{c}{1}\right)^2$$ where c is crack size in mm.

$\therefore$ $c = 0.045$ mm for sensivity limit.

However, checking the rise-time @ 10 m/s with $t_{rise} = 3.04\,\mu s$ ⇒ min. crack dimension = 30 μm ≡ 0.03 mm. So, this looks fine :)

[40%]