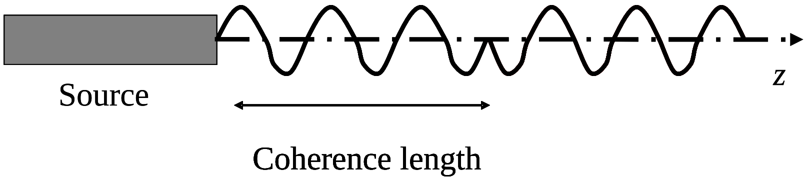
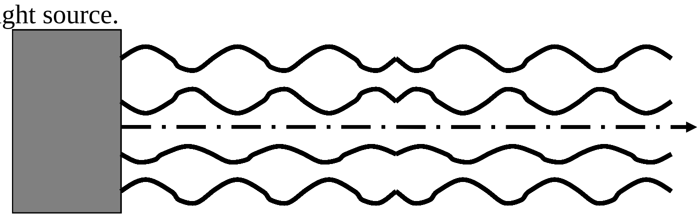
4B20 crib – NB answers longer than would be expected from candidates

Q1 a) [25%] Optical coherence is difficult to define as there are many types and unusual definitions. If we assume that the light has a basic wave property then we are assuming that the light is fully coherent. Unfortunately such light sources are very rare and even the highest quality laser light source will have some degree of unpredictability. The measure of this unpredictability is referred to as its coherence properties and is often expressed in term of a source's coherence length. The problem is that all light sources will have some degree of predictability, hence coherence is often a widely miss-quoted term. As long as the feature size is of the order of or less than the coherence length of the source, then diffraction effects will be seen. Even the most random of light sources will have some degree of predictability in its propagation, even if it is only over a few microns distance.

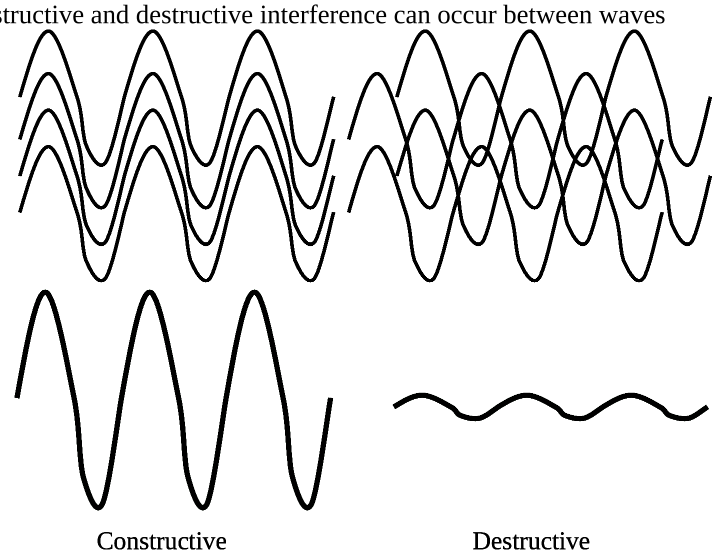
Hence there are two basic definitions that can be applied to coherence, which depend on the relative feature sizes and light source predictability and statistics of emission. The first definition is the degree of *axial coherence* within the source. This means that there is some sort of relationship (not random) between the temporal and spatial emission of optical energy in the same direction as the propagation of the light energy. The coherence length is defined as the distance the light propagates before the ability to predict where you are on the wave is lost.



We can also define the source's *spatial coherence.* This is when a light source has a spatial distribution or physical area and there is some form relationship (not random) between energy emitted from different positions across the area of the light source.

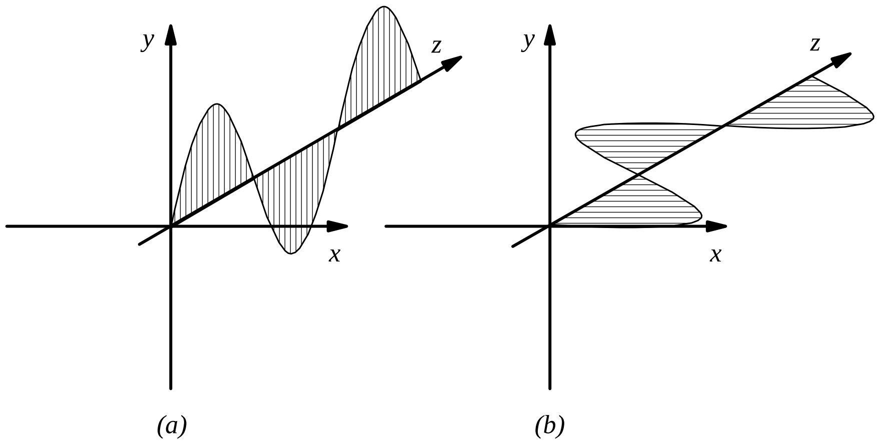


The most important reason for defining coherence properties of the light source is that it dictates which propagation theory and principles can be used to understand its properties within a given application. For instance, coherent sources such as lasers which have a high degree of predictability are subject to the rules of diffraction where constructive and destructive interference can occur between waves



In the majority of displays, light sources are relatively incoherent, simple geometric rules as used by ray tracing systems apply. However, if coherent light is used, then displays such a projectors will be affected by interference and there will be dependencies between the different optical components that are not desirable when designing the optics of such display systems.

b) [25%] Polarisation is the description of the propagation of light with respect to the orientation of the electric field vector of the propagating wave. All light sources can be represented in terms of an orthogonal set of propagating eigenwaves which are usually aligned to the *x* and *y* axes in a co-ordinate system with the direction of propagation along the *z* axis. These eigenwaves can be used to describe the propagation of light through complex media. The Jones calculus invented by RC. Jones in 1940 allows us to describe these waves and their propagation.



Vertically (a) and Horizontally (b) polarised light

If we have an electromagnetic wave propagating in the *z* direction along the *x* axis then the light is classified as linearly polarised in the *x* direction or <u>horizontally</u> polarised. This wave can be represented as a Jones matrix, assuming an amplitude V_x .

$$V = \begin{pmatrix} V_x \\ 0 \end{pmatrix}$$

If the light is polarised in the direction of the *y* axis, then we have linearly polarised light in the *y* direction of amplitude V_y or <u>vertically</u> polarised light.

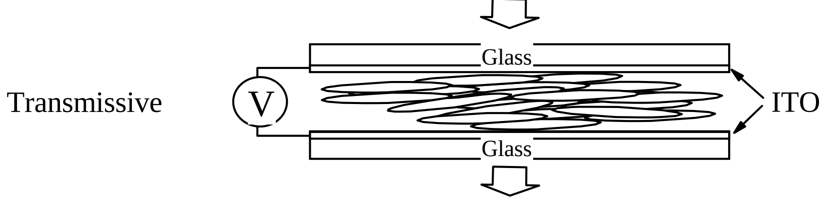
$$V = \begin{pmatrix} 0 \\ V_y \end{pmatrix}$$

We can now combine these two eigenwaves to make any linear state of polarisation we require. We can also represent more complex states of polarisation such as circular states. So far we have assumed that the eigenwaves are in phase (i.e. they start at the same point). We can also introduce a phase difference ϕ between the two eigenwaves, which leads to circularly polarised light. In these examples, the phase difference ϕ is positive in the direction of the z axis and is always measured with reference to the vertically polarised eigenwave (parallel to the *y* axis), hence we can write the Jones matrix.

$$V = \begin{pmatrix} V_x \\ V_y e^{j\phi} \end{pmatrix}$$

We can use Jones calculus to solve the propagation of light through optical systems. There are matrices that describe many different types of optical components such as birefringence, waveplates and polarisors. A combination of optical elements starting from left to right, can be expressed, starting from right to left, as a series of matrix multiplications. The main limit of the Jones matrix technique is that it only describes the forward propagation of the light, more complicated matrices are required for bidirectional propagations (such as Mueller, Berrimann etc).

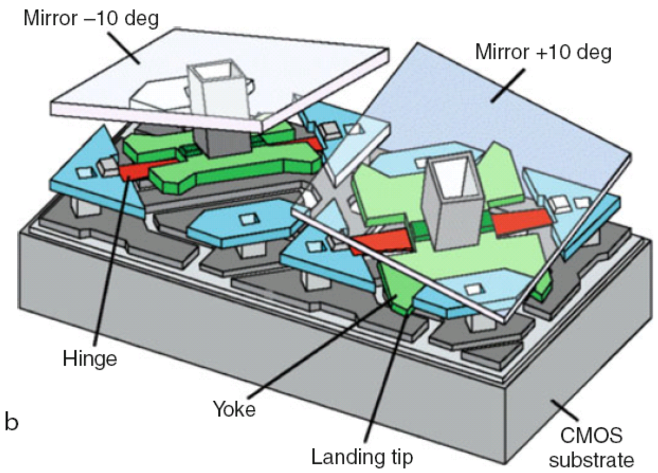
c) [25%] A liquid crystal display would benefit from having a polarised light source as it is a polarisation based modulation technology.



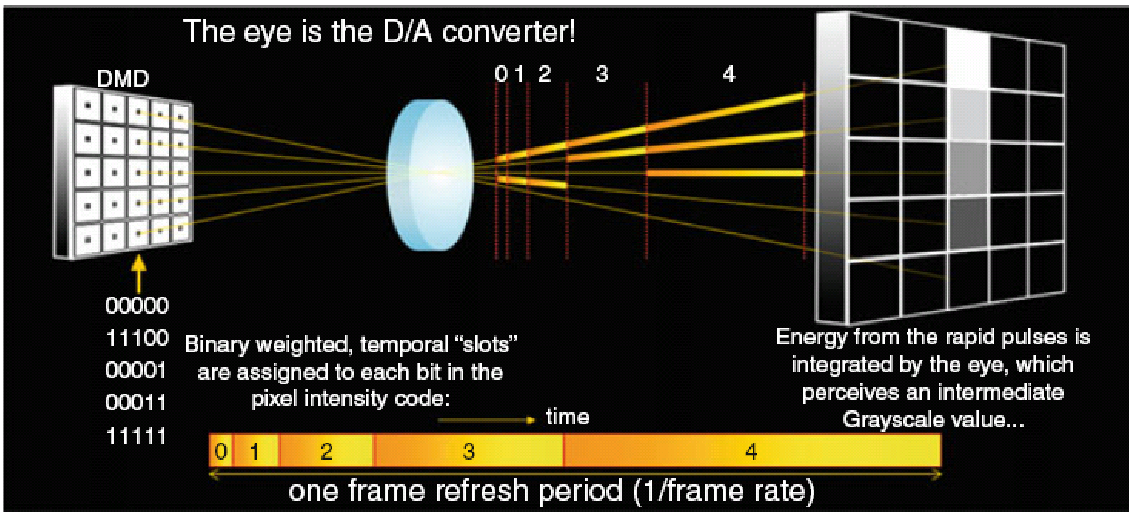
In order for the LC to work it's molecules must be aligned with the incoming light E-field, hence the orientation of the polarisation is critical to the display's performance optically. This is normally achieved using polarisors which filter out all but the desired orientation of polarisation. This suits the modulation, but not the display system as it is in fact very lossy and reduces the efficiency of the display system as photons are being absorbed in the polarisors. In an LCD a polarised backlight would be ideal, however this is very difficult to build in reality as most cheap light sources such as incandescents, fluorescents and LEDs are unpolarised and also incoherent.

Two possible remedies: 1) Polarisation recovery – the unwanted light is reflected rather than absorbed and then optics is used to recover the light back to the desired polarisation. This works using cholesteric (handed) filters but the recovery is not totally efficient and some light is still lost. 2) Use a coherent light sources such as a laser to generate a predictable light wave, however this then allows coherent problems like interference to arise.

d) [25%] The DLP is a good example of a polarisation insensitive modulation technique (along with scattering, e-ink etc). The DLP uses a mirror to deflect light in and out of the aperture of an optical system and is therefore independent of the polarisation state.

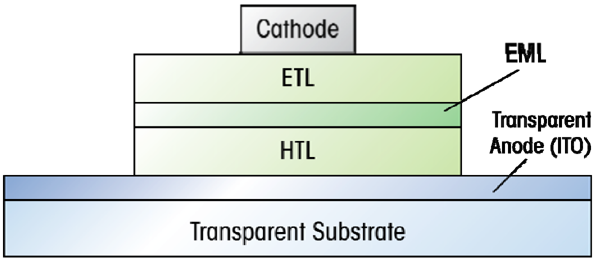


The main problem with the DLP technology is that it is binary, hence it is either on or off only. Hence grayscale and colour have to be done time sequentially (or 3 colour DLPs). This required moving parts such as a colour wheel and also leads to motion artefacts as the eye is a fast detector and can not be easily fooled, especially regarding the speed of colour versus the speed of grayscale response.



Q2) (a) [40%] The OLED has three layers, which make up a double hetero-structure and emits light when forward biased, e.g. when the ITO is biased positive wrt the top electrode. The top, electron injecting low work function electrode (cathode) is typically a metal alloy (Mg-Ag or Li-Al) deposited by vacuum evaporation. The bottom, hole-injecting high work function electrode (anode) is typically a metal oxide (e.g. ITO). In regular bottom emitting OLEDs, the anode has to be transparent. The light-emitting layer (EML) can, for example, be

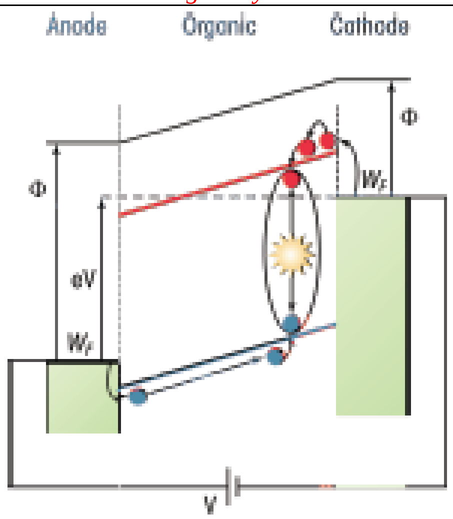
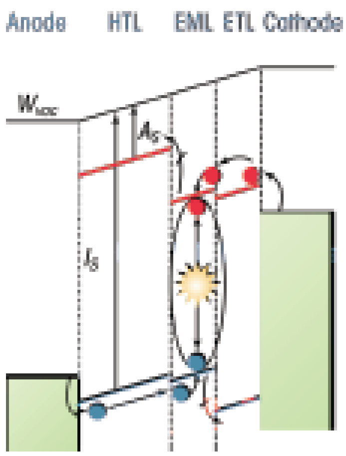
Alq3 in the case of green emission.



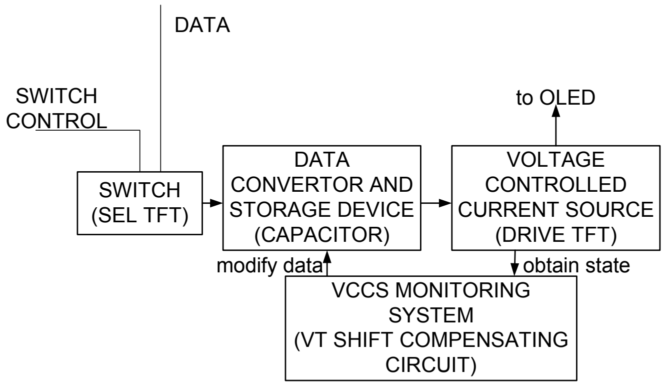
Bottom emitting OLED. ETL and HTL are electron and hole transport layers, respectively.

In the single layer OLED, for equally efficient electron and hole injection and equal carrier mobilities, exciton recombination takes place near the middle of device. Otherwise exciton moves towards electrode of reduced efficiency thereby reducing the charge balance factor (ratio of excitons formed to total electrons through circuit. Figure below depicts the energy levels in single layer OLED; W_{vac} denotes the vacuum level, W_F the Fermi level, and F the work functions.

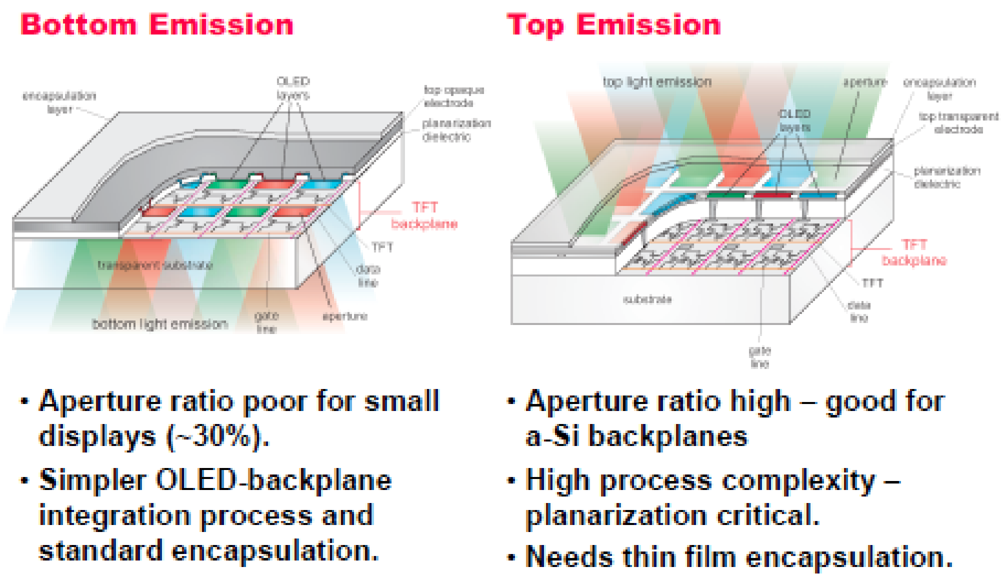
In a double layer OLED, materials are chosen to maximize injection and to pose a barrier for holes to reach the cathode and electrons to reach the anode. Figure below depicts the energy levels in a double heterojunction OLED; I_s denotes ionization energies and A_s the electron affinities.

Single Layer OLED	*Double Layer OLED*
	

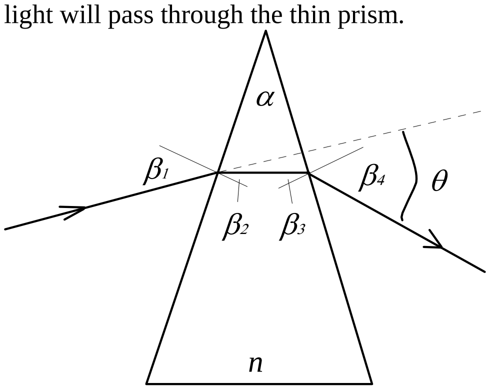
(b) [30%] A display consists of a matrix of picture elements or pixels. In the active matrix OLED (AMOLED) display, each pixel requires a stable current source while the active matrix liquid crystal display (AMLCD) is driven by a voltage source. Although the addressing is the same for both, in addition to the switching transistor used in the AMLCD pixel to uniquely turn on and off the pixel, the AMOLED pixel requires a current driving stage (since the OLED is a current-driven device) and a stage to compensate for V_T-shift in the TFT. This is depicted in the following illustration where the three additional boxes (besides SWITCH) are needed to generate the stable current source for the OLED.



(c) [30%] With the aid of architectural schematics briefly elaborate on the benefits and trade-offs of bottom versus top emission AMOLED displays.



Q3 a) [20%] The principle of Snell's law can be used to solve the optical problem of light propagation through a thin wedge shaped prism such as the one shown below. The refraction of the rays at each surface dictate the how light will pass through the thin prism.



From Snell's law we have:

$$n\sin\beta_2 = \sin\beta_1 \qquad n\sin\beta_3 = \sin\beta_4$$

And the total deflection of the ray through the prism is θ such that.

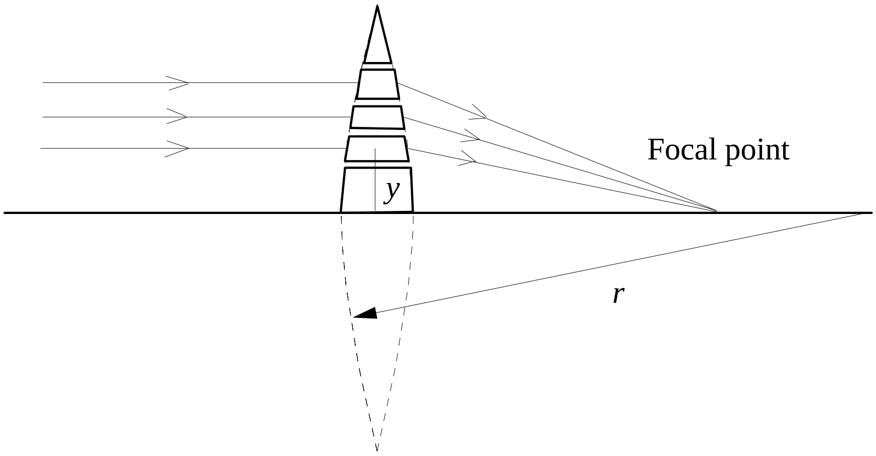
$$\beta_2 + \beta_3 = \alpha$$

$$\theta = \beta_1 - \beta_2 - \beta_3 + \beta_4 = \beta_1 + \beta_4 - \alpha$$

If we have a thin prism such that α is small and a small angle of incidence such that $\sin\beta \approx \beta$ then the total deflected angle can be approximated to:

$$\theta = (n-1)\alpha$$

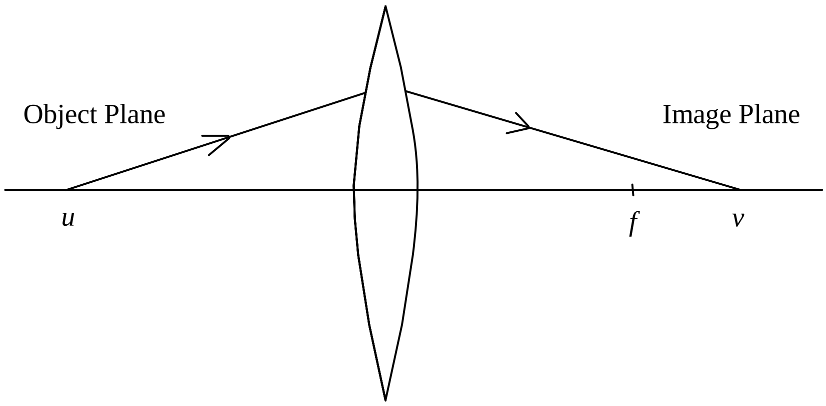
b) [40%] This is the basic principle used in all most geometric ray problems. Deviation from small values of α and β leads to aberration in the optical system, hence these values form a solid basis for good lens design and minimisation or potential aberrations. They do, however limit what can be done in an optical system, especially if size is a constraint. A good example of how this property can be used is shown below, where a thin lens is made from a series of thin prism sections.



Each prism section is at a height *y* from the optical axis of the thin lens. Hence as they are thin lenses, the apex angle can be expressed as $\alpha = 2y/r$ where *r* is the radius of curvature of both surfaces, hence the deflected angle of a ray passing through each prism section will be:

$$\theta = (n-1)\frac{2y}{r}$$

This deviation of each ray means that if parallel rays are incident on the lens, then they will all converge to the same point called the focal point or focal length of the lens. We can represent this as $f = y/\theta = r/2(n-1)$. Moreover, parallel rays incident at an angle to the lens will converge to a different point on the *optical axis*. From this we can define the classical optical system, assuming that the thin lens is circular symmetric about the optical axis.

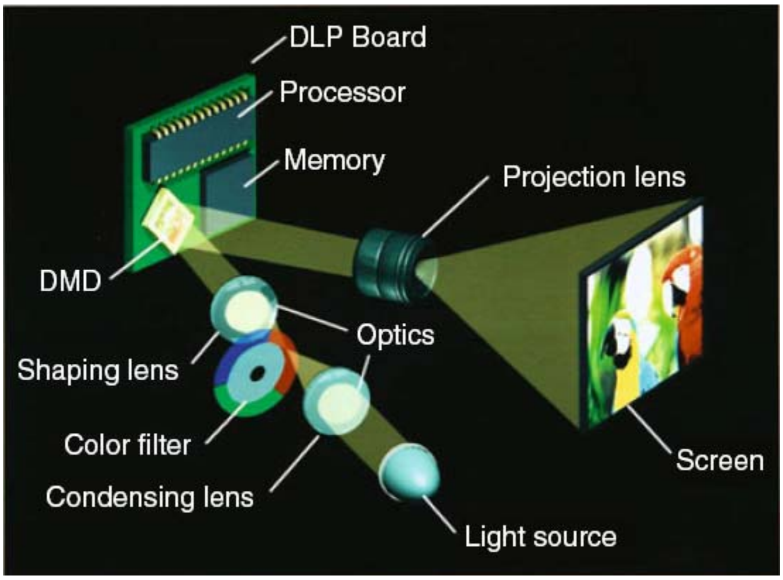


From the dimensions shown, we define object distance as *u* and the image distance as *v* and we can then use classical geometrical optic theory to give the relationship:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This relationship forms the basis of most geometrical optical systems and is one of the fundamental relationships which are exploited on a regular basis in optical design procedures. It is important, however to have a convention for signs when describing optical systems, as some image planes will be real and others will be virtual, depending on the lens system described.

c) [40%] Single chip DLP based projector. Uses an arc lamp as its light source. This is then focussed onto the DLP by the condensing lens which may also include a hologiniser to create a uniform light illumination for the DLP. The colour is filtered by the colour wheel to form the sequence optimal for the design of the projector (not just RGB). The reflected light is then captured by the projection lens and magnified onto the projection screen.



The projection lens is optimised using a ray tracing package to minimise aberrations and remain as paraxial as possible. Hence it will be a multi-element lens to satisfy the needs of the projector and also allow focus and variable zoom.

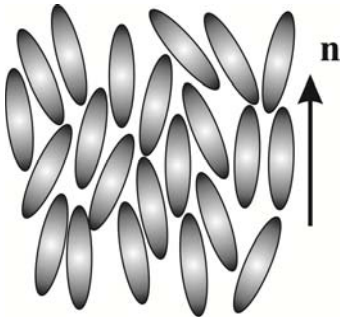
Two main considerations of this lens are: 1) Achromatic performance. The lens must perform the same across the RGB spectrum, hence the elements are designed to compensate each other at different wavelengths. This is usually combined with different glasses to match dispersion with the optical design. 2) Off axis projection. The screen is never centred on the optical axis of the lens, hence the lens must project off axis which pushes the paraxial approximation to its limit. The larger the image and the more off axis, then the longer the throw and the more elements needed (expensive).

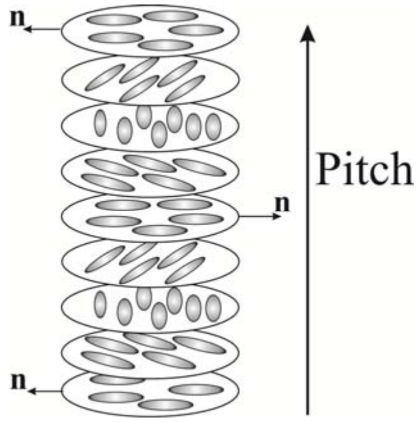
Q4 Part (a) [25%] Answers should include any four examples from the following liquid crystal phases described in the lectures: nematic, chiral nematic, smectic A, smectic C, chiral smectic C, and blue phase.

Nematic phase. For the nematic phase, illustrated in the figure, answers should include a discussion about some of the characteristics of the phase and should mention that the phase has long range orientational order but no positional order. In the diagram, the unit vector known as the director, which describes the average point direction of the molecules should also be defined. The nematic phase is identified by the two- and four-point brushes that are observed in the Schlieren texture when viewed on a polarizing optical microscope. It is the most commonly used LC phase in display technology. Because of the non-polar nature of the phase the properties along the director in the positive direction are equivalent to this in the opposite direction. This is know as the n = -n invariance. For thermotropic nematic liquid crystals, the phase is formed as droplets on cooling from the isotropic phase. The transition from the nematic to isotropic liquid phase is called the clearing point. A uniformly aligned nematic phase can be treated as a unixially positive anisotropic slab whereby the optic axis is aligned parallel to the director.

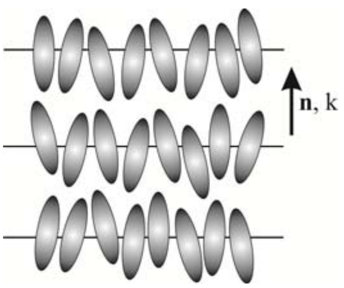
The chiral nematic phase is not thermodynamically distinct from the nematic phase, the difference is that macroscopically it exhibits a helical structure. This phase can be formed using naturally chiral molecules (molecules which cannot be superimposed onto their mirror image) or chiral additives. The important length scale is the pitch; the distance over which the director rotates by 360°. This helical structure can take two possible configurations: left and right handed structures.

Due to the **n** = -**n** invariance, the periodicity is only half the value of the pitch. Unlike the nematic phase, the optic axis is aligned with the helix axis rather than along the director. This means that the structure is uniaxially negative. The pitch can be of the order of the wavelength of light leading to a selective reflection band.





Nematic phase **Chiral nematic phase**

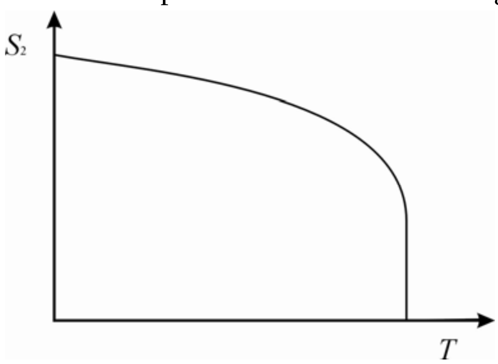


Smectic A phase The smectic A phase is the least ordered smectic phase. Unlike the nematic phase, this phase possesses both positional and orientational order. It is identified by 2-point brushes using optical polarizing microscopy. However, within each layer, molecules orient similar to the nematic phase. For the smectic A phase, the layer spacing is of the order of the molecular length. For polymorphic compounds, cooling of a nematic phase to an underlying smectic phase results in a significant increase in viscosity and results in a slowing of the display device. The rotational symmetry of the phase prevents the formation of a helix with chiral additives.

Birefringence. For a planar aligned nematic liquid crystal, the optical anisotropy is defined by the birefringence which is given by $\Delta n = n_{||} - n_{\perp}$ where $n_{||}$ and $n_{\perp}$ correspond to the refractive indices parallel and perpendicular to the director, respectively.

Dielectric anisotropy The dielectric anisotropy represents the difference between the relative dielectric permittivity parallel and perpendicular to the director. $\Delta\varepsilon = \varepsilon_{||} - \varepsilon_{\perp}$ Dielectric anisotropy can be *positive* or *negative* leading to different orientations in the presence of an electric field.

The nematic liquid crystal phase has the lowest degree of order. A sketch of the orientational order parameter as a function of the temperature is shown in the figure



b) [25%] In this case, the threshold voltage is related to the dielectric anisotropy by

$$V_{th} = \pi \left(\frac{K_{11}}{\varepsilon_0 \Delta\varepsilon} \right)^{\frac{1}{2}}$$

Therefore, the threshold voltage is inversely proportional to the square-root of the dielectric anisotropy. Larger values of the dielectric anisotropy result in lower threshold voltages assuming that the splay elastic constant remains the same.

If the dielectric anisotropy is increased by a factor of four then the threshold voltage must be reduced by a factor of 2.

If the dielectric anisotropy is negative the liquid crystal will not alter its alignment under application of an applied electric field.

c) [25%] Consider the nematic as a uniaxial birefringent slab with the fast axis aligned in the vertical direction. The retardation can then be expressed by the Jones matrix:

$$W_0 = \begin{pmatrix} e^{-j\frac{\Gamma}{2}} & 0 \\ 0 & e^{j\frac{\Gamma}{2}} \end{pmatrix}$$

For an expression of the retardation whereby the fast axis is aligned at some arbitrary angle ϕ to the vertical direction we use the rotation matrices,

$W = R(-\phi)W_0 R(\phi)$ where $R(\phi) = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$ This gives

$$W = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} e^{j\frac{\Gamma}{2}}\cos\phi & e^{-j\frac{\Gamma}{2}}\sin\phi \\ -e^{j\frac{\Gamma}{2}}\sin\phi & e^{j\frac{\Gamma}{2}}\cos\phi \end{pmatrix} = \begin{pmatrix} e^{-j\frac{\Gamma}{2}}\cos^2\phi + e^{j\frac{\Gamma}{2}}\sin^2\phi & e^{-j\frac{\Gamma}{2}}\sin\phi\cos\phi - e^{j\frac{\Gamma}{2}}\sin\phi\cos\phi \\ e^{-j\frac{\Gamma}{2}}\sin\phi\cos\phi - e^{j\frac{\Gamma}{2}}\sin\phi\cos\phi & e^{j\frac{\Gamma}{2}}\cos^2\phi + e^{-j\frac{\Gamma}{2}}\sin^2\phi \end{pmatrix}$$

$$W = \begin{pmatrix} e^{-j\frac{\Gamma}{2}}\cos^2\phi + e^{j\frac{\Gamma}{2}}\sin^2\phi & -j\sin\frac{\Gamma}{2}\sin 2\phi \\ -j\sin\frac{\Gamma}{2}\sin 2\phi & e^{j\frac{\Gamma}{2}}\cos^2\phi + e^{-j\frac{\Gamma}{2}}\sin^2\phi \end{pmatrix}$$

Including crossed polarisers we get,

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{-j\frac{\Gamma}{2}}\cos^2\phi + e^{j\frac{\Gamma}{2}}\sin^2\phi & -j\sin\frac{\Gamma}{2}\sin 2\phi \\ -j\sin\frac{\Gamma}{2}\sin 2\phi & e^{j\frac{\Gamma}{2}}\cos^2\phi + e^{-j\frac{\Gamma}{2}}\sin^2\phi \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ E_y \end{pmatrix} \qquad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -E_y j\sin\frac{\Gamma}{2}\sin 2\phi \\ 0 \end{pmatrix}$$

The intensity is this multiplied by its complex conjugate, so we get $I = I_0 \sin^2(2\phi)\sin^2\left(\frac{\Gamma}{2}\right)$

but $\Gamma = \frac{2\pi d\Delta n}{\lambda}$ Therefore $I = I_0 \sin^2(2\phi)\sin^2\left(\frac{\pi d\Delta n}{\lambda}\right)$

d) [25%] For maximum transmission the optic axis of the nematic liquid crystal must be aligned at $\phi = \frac{\pi}{4}$ to the polarizers. Therefore, the equation in part (c) becomes, $I = I_0 \sin^2\left(\frac{\pi d\Delta n}{\lambda}\right)$

To ensure maximum transmission, the term in brackets must equal to $\frac{\pi d\Delta n}{\lambda} = \frac{(2m+1)\pi}{2}$ where *m* = 0, 1, 2, …. For the lowest order *m* = 1 so $d = \frac{\lambda}{2\Delta n}$ .Substituting in values we get d = 2.75 μm