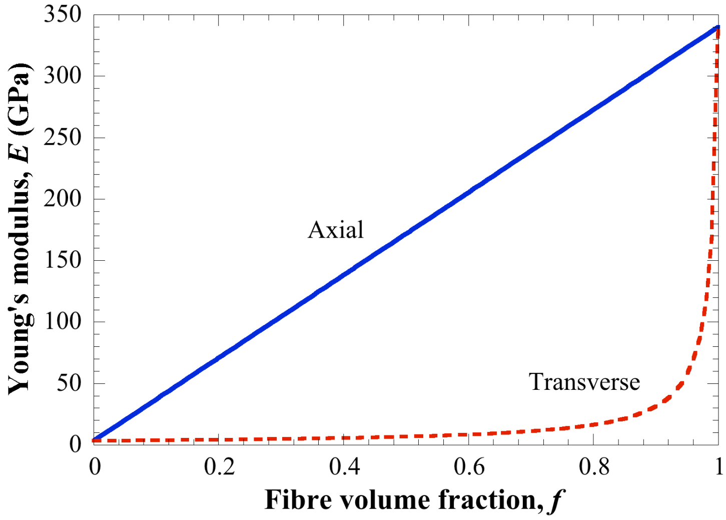
CRIBS
Question 1
(a)



The inverse rule of mixtures gives the accompanying graph for transverse stiffness, the axial stiffness is linear with fibre content.

$$E_1 = fE_\mathrm{f} + (1-f)E_\mathrm{m} = 0.4\times 340 + 0.6\times 3.40 = 138 \text{ GPa}$$

$$E_2 = \left[\frac{f}{E_\mathrm{f}} + \frac{1-f}{E_\mathrm{m}}\right]^{-1} = \left[\frac{0.4}{340} + \frac{0.6}{3.40}\right]^{-1} = 5.63 \text{ GPa}$$

(b) A laminate is made up of a stacked and bonded assembly of unidirectional plies, each having its fibre axis lying at a specified angle to a reference direction. They are used in preference to unidirectional composite material because they are most isotropic with regard to properties of interest.
A balanced laminate is one where the laminate as whole does not exhibit any tensile-shear interactions for any loading angle (A_{16}=A_{26}=0). Tensile-shear interactions are tensile strains arising from applied shear stresses and visa versa and result in in-plane distortion of the laminate.
A symmetric laminate is one possessing a mirror plane lying in the plane of the laminate i.e. the stacking sequence in the top half reflects that in the bottom half. A symmetric laminate does not exhibit bending-stretching coupling (the coupling stiffness [B]=0), i.e. in-plane loading will not generate any out-of-plane distortion and vice versa.

(c)
(i)

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \nu_{21} = 0.0196$$

Substitution of the elastic constants in the equations below provides the components of the lamina stiffness matrix in the principal material axes

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{138}{1-0.3\times 0.0196} = 139\ \text{GPa}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = 9.05\ \text{GPa}$$

$$Q_{12} = \frac{\nu_{12}\,E_2}{1-\nu_{12}\nu_{21}} = 2.72\ \text{GPa}$$

$$Q_{66} = G_{12} = 6.9\ \text{GPa} \qquad Q_{16} = Q_{26} = 0$$

$$[Q] = \begin{bmatrix} 139 & 2.72 & 0 \\ 2.72 & 9.05 & 0 \\ 0 & 0 & 6.9 \end{bmatrix} \text{GPa}$$

The transformed lamina stiffness matrices $[\overline{Q}]$ for the +45° and - 45° plies can be found by substituting the above stiffnesses in the equations below (data sheet)

$$\overline{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2$$
$$\overline{Q}_{12} = \left(Q_{11} + Q_{22} - 4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4 + s^4\right)$$
$$\overline{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2$$
$$\overline{Q}_{16} = \left(Q_{11} - Q_{12} - 2Q_{66}\right)c^3s - \left(Q_{22} - Q_{12} - 2Q_{66}\right)cs^3$$
$$\overline{Q}_{26} = \left(Q_{11} - Q_{12} - 2Q_{66}\right)cs^3 - \left(Q_{22} - Q_{12} - 2Q_{66}\right)c^3s$$
$$\overline{Q}_{66} = \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4 + c^4\right)$$

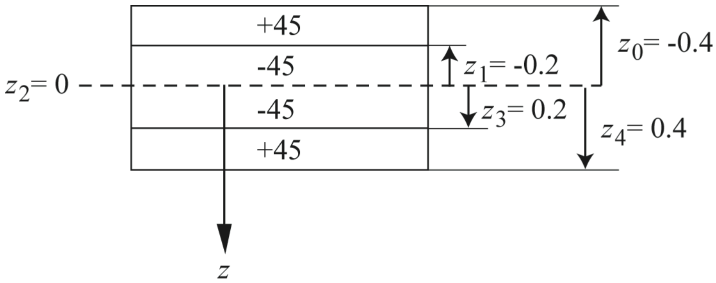
where $c = \cos\theta,\ s = \sin\theta$

$$\left[\overline{Q}\right]_{+45^\circ} = \begin{bmatrix} 45.23 & 31.43 & 32.44 \\ 31.43 & 45.23 & 32.44 \\ 32.44 & 32.44 & 35.61 \end{bmatrix} \text{GPa}$$

$$\left[\overline{Q}\right]_{-45^\circ} = \begin{bmatrix} 45.23 & 31.43 & -32.44 \\ 31.43 & 45.23 & -32.44 \\ -32.44 & -32.44 & 35.61 \end{bmatrix} \text{GPa}$$

The only difference between the stiffness matrices for the two plies is that the shear coupling terms (terms with subscripts 16 and 26) for the -45° ply have the opposite sign from the corresponding terms for the +45° ply.

The laminate extensional stiffness matrix [A] is found by substituting these distances, along with the lamina stiffnesses above into the defining formulae for $[A]$



$$A_{11} = ((z_1 - z_0) + (z_4 - z_3))\left(\overline{Q}_{11}\right)_{+45} + ((z_2 - z_1) + (z_3 - z_2))\left(\overline{Q}_{11}\right)_{-45} =$$

$$= 0.4 \text{ mm} \times \left(45.23 + 45.23\right) \text{ GPa}$$

$$= 36.18 \text{ MNm}^{-1}$$

$$A_{22} = 0.4 \text{ mm} \times \left(45.23 + 45.23\right) \text{GPa}$$

$$= 36.18 \text{ MNm}^{-1}$$

$$A_{12} = 0.4 \text{ mm} \times \left(31.43 + 31.43\right) \text{ GPa}$$

$$= 25.14 \text{ MNm}^{-1}$$

$$A_{66} = 28.49 \text{ MNm}^{-1}$$

$$A_{16} = A_{26} = 0$$

Thus the laminate extensional stiffness matrix [A] is given by

$$\left[A\right] = \begin{bmatrix} 36.18 & 25.14 & 0 \\ 25.14 & 36.18 & 0 \\ 0 & 0 & 28.49 \end{bmatrix} \text{GPa} \cdot \text{mm}$$

It can be seen that A_{16}=A_{26}=0, hence there are no tensile-shear interactions - tensile (normal) strains resulting from applied shear stresses and vice-versa. The laminate is “balanced” apart from “symmetric”.

(ii) The laminate is symmetric, hence [B] = 0 i.e. there is no coupling between bending and stretching. The moment per unit length (units N) is given by

$\{M\}$=$[D]\{\kappa\}$

$$\begin{pmatrix} M_{\text{x}} \\ M_{\text{y}} \\ M_{\text{xy}} \end{pmatrix} = \left[\text{D}\right] \begin{pmatrix} \kappa_{\text{x}} \\ \kappa_{\text{y}} \\ \kappa_{\text{xy}} \end{pmatrix}$$

Beam formula: $M = EI\kappa$ (where M is the bending moment (N m)

$$M = M_{\text{x}} b \text{ (where the width, } b = 10 \text{ mm)}$$

Also $\quad M_{\text{x}} = \text{D}_{11}\kappa_{\text{x}} \quad$ so $\;'EI' = b\text{D}_{11}$

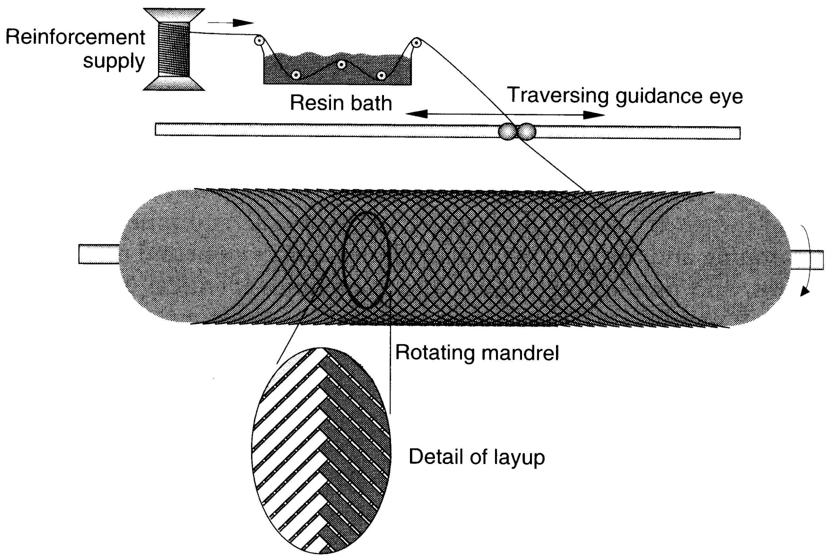
$$\therefore 'EI' = b\text{D}_{11} = 1.93 \times 10 \text{ GPa} \cdot \text{mm}^4 = 1.93 \times 10 \; \frac{10^9 \text{N}}{\text{m}^2} 10^{-12} \text{m}^4 = 1.93 \times 10^{-2} \text{Nm}^2$$

Tip deflection from structures data book is

$$\delta = \frac{FL^3}{3EI} = \frac{FL^3}{3b\mathrm{D}_{11}} = \frac{1\times 0.1^3}{3\times 1.93\times 10^{-2}} = 17.27\ \text{mm}$$

Question 2

(a) Filament winding is a process suited to automation, although limited to certain components shapes (tubes). Fibre tows i.e. bundles of fibres, are drawn through a bath of resin, before wound onto a mandrel or former of the required shape. The equipment comprises of (a) a creel stand, from which the fibre tows are fed under the required tension from a set of reels, (b) a bath of resin, through which the fibre tows pass via a set of guides, (c) a delivery eye, through which the fibres emerge, the position of which is controlled by a mechanical system and (d) a rotating mandrel onto which the fibre tows are drawn. The key parameters are the fibre tension, the resin take-up efficiency and the winding geometry.



(b) The Tsai-Hill failure criterion (the formula for which is given in the Data Sheet) is a basis for prediction of whether a unidirectional composite ply will fail under a given stress state, expressed as normal stresses parallel and transverse to the fibre axis, and the shear stress parallel to the fibre axis, given the critical stresses for these three modes of failure. It effectively takes account of interactions between these modes, particularly transverse tension and shear parallel to the fibre axis, whereas the maximum stress criterion only considers each failure mode in isolation.

(c) Internally pressurised thin cylinder with a radius *r* very small in comparison to the wall thickness *t*.

Assuming that the cylinder radius *r* is very small in comparison to the wall thickness *t*, the relationship between internal pressure *P* and the stresses in the wall can be derived by balancing the forces exerted by the pressure and the stresses.

Axial stress

$$\sigma_x = \frac{Pr}{2t}$$

Hoop stress

$$\sigma_y = \sigma_h = \frac{Pr}{t}$$

$$\sigma_{xy} = 0$$

If you treat one of the two plies as if they were present alone, then

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = [T] \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix} \text{ where } [T] = \begin{pmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{pmatrix}$$

where $c = \cos\theta$, and $s = \sin\theta$

$$\sigma_1 = c^2\sigma_x + s^2\sigma_y = c^2\sigma_x + s^2\sigma_h$$
$$\sigma_2 = s^2\sigma_x + c^2\sigma_h$$
$$\sigma_{12} = -sc\sigma_x + sc\sigma_h$$

$$\sigma_x = \frac{\sigma_y}{2}$$

Hence

$$\sigma_1 = c^2\frac{\sigma_h}{2} + s^2\sigma_h$$
$$\sigma_2 = s^2\frac{\sigma_h}{2} + c^2\sigma_h$$
$$\sigma_{12}(=\tau_{12}) = -sc\frac{\sigma_h}{2} + sc\sigma_h = sc\frac{\sigma_h}{2}$$

For $\theta = 45°$, $c = s = 0.707$

$\therefore \sigma_1 = 0.75\sigma_h \qquad \sigma_2 = 0.75\sigma_h \qquad \tau_{12} = 0.25\sigma_h$

The Tsai-Hill criterion

$$\frac{\sigma_1^2}{s_L^2} - \frac{\sigma_1\sigma_2}{s_L^2} + \frac{\sigma_2^2}{s_T^2} + \frac{\tau_{12}^2}{s_{LT}^2} \geq 1$$

$\therefore \sigma_1 = 0.75\sigma_h \qquad \sigma_2 = 0.75\sigma_h \qquad \tau_{12} = 0.25\sigma_h$

$$\frac{0.75^2\sigma_h^2}{400^2} - \frac{0.75^2\sigma_h^2}{400^2} + \frac{0.75^2\sigma_h^2}{20^2} + \frac{0.25^2\sigma_h^2}{25^2} = 1$$

$$\frac{0.75^2\sigma_h^2}{20^2} + 10^{-4}\sigma_h^2 = 1$$

$$0.6025\sigma_h^2 = 20^2$$

$\therefore \sigma_h = 25.76$ MPa

For failure

$$P = \frac{\sigma_h t}{r} = \frac{\sigma_h \cdot t}{r} = \frac{25.74 \cdot 10}{300/2} = 1.71 \text{ MPa}$$

Since the gas rises to 2 MPa, it follows that the pipeline would be damaged.

An alternative process would be to put in the actual pressure and see whether it satisfies the Tsai-Hill criterion. Either method would be acceptable.

$$\sigma_x = \frac{Pr}{2t} = 15 \text{ MPa}$$

$$\sigma_y = \sigma_h = \frac{Pr}{t} = 30 \text{ MPa}$$

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = [T] \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix} \text{ where } \left[T\right] = \begin{pmatrix} 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & -1 \\ -0.5 & 0.5 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} 22.5 \\ 22.5 \\ 7.5 \end{pmatrix} \text{ MPa}$$

$$\frac{\sigma_1^2}{s_L^2} - \frac{\sigma_1 \sigma_2}{s_L^2} + \frac{\sigma_2^2}{s_T^2} + \frac{\tau_{12}^2}{s_{LT}^2} \geq 1$$

$\therefore \sigma_1 = 22.5$ MPa $\qquad \sigma_2 = 22.5$ MPa $\qquad \tau_{12} = 7.5$ MPa

$$\frac{22.5^2}{400^2} - \frac{22.5^2}{400^2} + \frac{22.5^2}{20^2} + \frac{7.5^2}{25^2} > 1$$

Hence the pipeline would be damaged.

(d) Treating one of the two plies alone neglects the changes in stress state induced by the constraint that the +45 ply imposes on the -45 plies and vice versa. The stress transverse to the fibre axis would largely be relieved by the presence of the other ply.

A more rigorous analysis would require to:

Calculate [*Q*] in principal material axes (1, 2), the transformed stiffness matrix $\left[\bar{Q}\right]$ in the global x-y axes, the laminate stiffness matrix [*A*] and the stress resultants [*N*]. The strains would be estimated using

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \left[A\right]^{-1} \begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix}$$

The strains and the stresses in each of the plies in the principal axes would be estimated using the following equations

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = [T]^{-T} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = \left[Q \right] \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix}$$

Once the stresses in each of the plies in the principal axes are calculated, the average stresses can be found.

Question 3

(i) The failure strain of the epoxy matrix much exceeds that of the carbon fibres, thus the carbon fibres **fail** first. A shear lag zone is developed along the broken fibres and this blunts out the local stress concentration so that a macroscopic transverse crack does not develop from the broken fibre. Instead, the load from the broken fibre is transferred back into the remaining composite in a diffuse manner. With increasing remote load, other fibre breaks occur, in accordance with a Weibull statistical distribution in strength. A peak load is achieved such that the remaining fibres are no longer able to bear the applied load.

In contrast, the axial compressive strength is dictated by plastic microbuckling from a region of fibre waviness. The matrix shears between fibres and the compressive strength is given by $\sigma = \tau_Y / \phi$ where τ_Y is the shear strength and ϕ is the fibre misalignment angle (on the order of a few degrees). The microbuckle does propagate across the composite in a crack-like fashion, and so the compressive strength is dictated by the region of largest waviness.

(ii) The pull-out toughness is a result of the linear drop in traction T with crack opening displacement u. Consider the shear-lag problem. The stress on the broken end of the fibre equals zero. At the other end of the shear lag zone, write the tensile strength of the composite σ_1, the composite modulus as E_1 and the fibre modulus as E_f. The matrix modulus is negligible compared to the fibre modulus and so the composite modulus scales with the fibre volume fraction V_f according to

$$E_1 = V_f \, E_f$$

At this end of the shear lag zone, there is no-slip and so the axial stress within the fibre is $\sigma_f = \sigma_1 / V_f$.

Axial equilibrium for the fibre within the shear lag zone dictates that

$$\frac{\pi d^2}{4} \frac{d\sigma_f}{dx} = \pi d \tau_f$$

where τ_f is the shear stress on the fibre from the matrix. Solution of the above differential equation gives

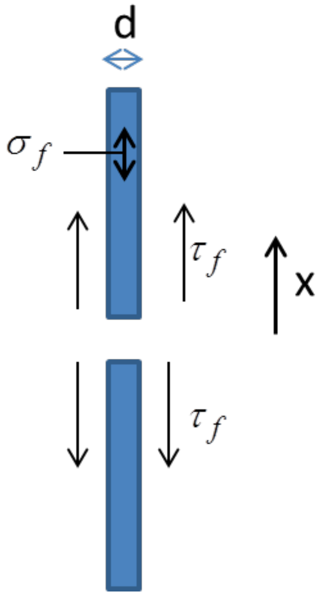
$$\ell = \frac{d\sigma_1}{4 V_f \tau_f}$$

where ℓ is the length of the shear lag zone.

The traction from the fibre drops from the peak value $T = \sigma_1$ to zero as the crack opens from zero to a value of $u = \ell$. Thus, the pull-out toughness is

$$G_p = \frac{1}{2} \sigma_1 \ell = \frac{\sigma_1{}^2 d}{8 V_f \tau_f}$$

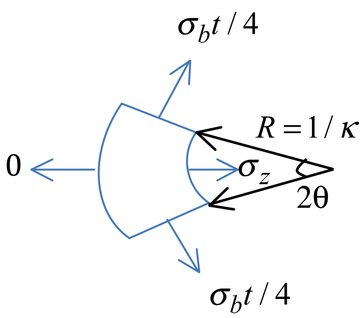
Note that the toughness increases with diminishing shear strength τ_f.



(iii) A through thickness tensile stress develops due to the imposed moment M on a beam of depth t and finite initial curvature κ. The applied bending moment M gives rise to a bending stress field of amplitude

$$\sigma_b = \frac{M(t/2)}{I} = \frac{Mt}{2I}$$

where I is the 2nd moment of area.



Now consider radial force equilibrium on a segment of the beam.

$$2\left(\frac{\sigma_b}{2}\cdot\frac{t}{2}\sin\theta\right) = \sigma_z R 2\theta$$

For a small θ, this gives

$$\sigma_z = \frac{t\sigma_b}{4R} = \frac{t\kappa\sigma_b}{4} = \frac{t^2\kappa M}{8I}$$

This tensile stress can lead to splitting of the beam at mid-depth.

(iv) Splitting of the panel occurs from the edge of the hole, due to the low mode II toughness of the CFRP. These splits lead a reduction in the stress concentration associated with the hole, and consequently the panel is almost notch insensitive.

Question 4

(a) The force resultant at the built-in end has the maximum value of

$$N_x = \sigma t = \frac{Myt}{I} = \frac{FLRt}{\pi R^3 t} = \frac{FL}{\pi R^2} = \frac{1\times10^3 \cdot 3\times10^3}{\pi\cdot(40/2)^2} = 2.38\times10^3 \text{ Nmm}^{-1} = 2.38\times10^6 \text{ Nm}^{-1}.$$

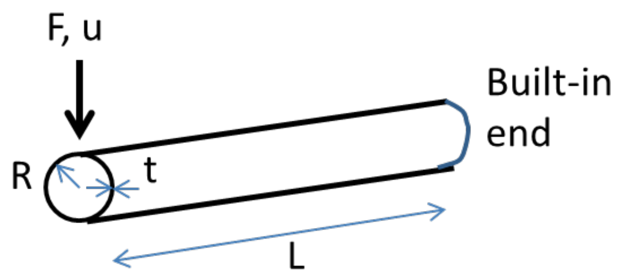
For the CFRP, write t_0 as the thickness of the 0 layers, and E_1 138 GPa as the axial modulus of the 0 plies (from table 2 of the databook). The tensile strain allowable is taken to be $e = 0.4\%$ from table 1 of the databook. Then,

$$t_0 = \frac{N_x}{E_1 e} = \frac{2.38\times10^6}{138\times10^9 \cdot 0.4\times10^{-2}} = 4.31\times10^{-3} \text{ m} = 4.31 \text{ mm}$$

And the total thickness is $t = t_0 / 0.8 = 5.4$ mm.

For the aluminium, the Young's modulus is 70 GPa and tensile strain allowable is $e = 0.3\%$ giving

$$\boxed{t = \frac{N_x}{Ee} = \frac{2.38\times10^6}{70\times10^9 \cdot 0.3\times10^{-2}}} = 11.3 \text{ mm}$$



(b) The end deflection is $u = \dfrac{FL^3}{3E_1 I}$ where $I = \pi R^3 t_0$

For CFRP, this gives $t_0 = \dfrac{FL^3}{3\pi E_1 R^3 u} = \dfrac{1\times10^3 \cdot 3^3 \times 10^3}{3\cdot\pi\cdot138\times10^3 \cdot 20^3 \cdot 0.5}$ =5.18 mm, and a total wall thickness of $t = 1.25\cdot t_0 = 6.5$ mm.

For the aluminium we assume $u = \dfrac{FL^3}{3EI}$ where $I = \pi R^3 t$.

This gives a thickness of t = 10.2 mm

So take t = 6.5 mm for the CFRP and t = 11.3 mm for the aluminium.

(c) The lost profit is (Premium + Cost/mass) × Mass

Mass $= 2\pi RtL\rho = 2\cdot\pi\cdot20\times10^{-3}\cdot6.5\times10^{-3}\times3\cdot1500 = 3.67$ kg

For the CFRP, mass = 3.67 kg and cost is £100/kg

For aluminium, Mass $= 2\pi RtL\rho = 2\cdot\pi\cdot20\times10^{-3}\cdot11.3\times10^{-3}\times3\cdot2700 = 11.50$ kg and cost is £2/kg

Thus, lost profit for CFRP is (£100 + £100) × 3.67 = £734
For Aluminium, the lost profit is (£100 + £2) × 11.5 = £1173
So choose CFRP.

(d) Use hand lay-up of pre-preg on a mandrel for a small batch run, followed by an autoclave cure. Filament winding or pultrusion would be suitable for large batch runs.
In filament winding, on any curved surface, the fibres will tend to follow a geodesic path – i.e. the shortest one. This can cause problems with some shapes, since it may be difficult to ensure that fibres cover some parts of the surface or lie in certain orientations (0 or 90º).