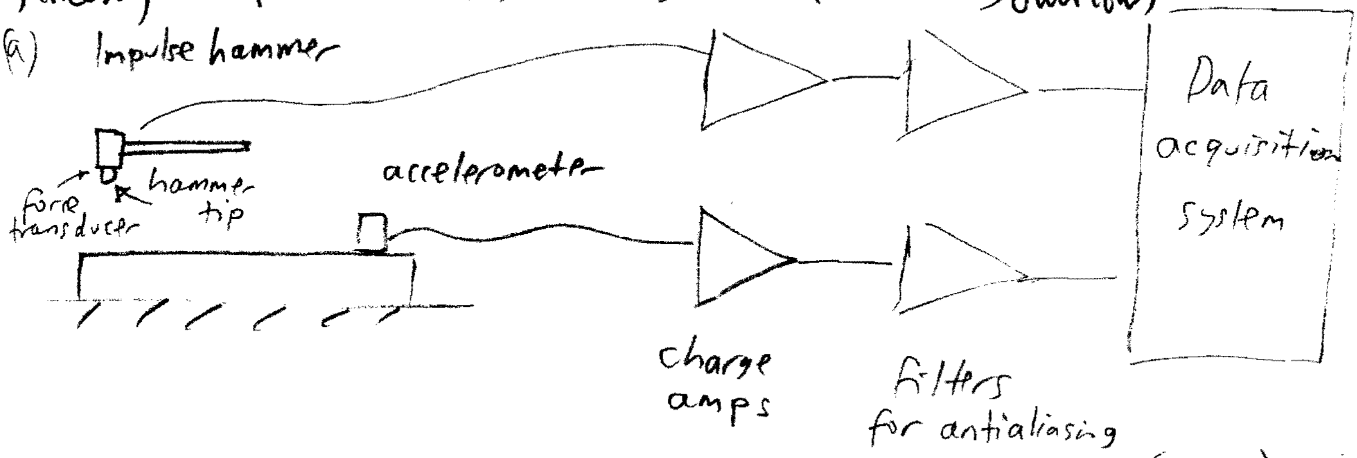
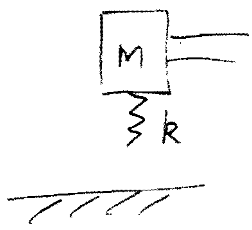
Engineering Tripos Part IIB 2013 4C6 Solutions

1 (a) Impulse hammer



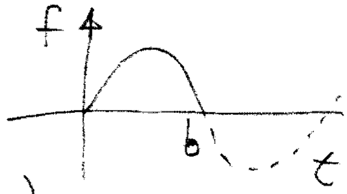
(20%)

(b) (i)



The impulse hammer should be heavy enough to excite the structure but not so heavy as to damage or otherwise affect the structure.

The duration of the impulse $b = \pi\sqrt{\frac{m}{k}}$ because the pulse is a half sine

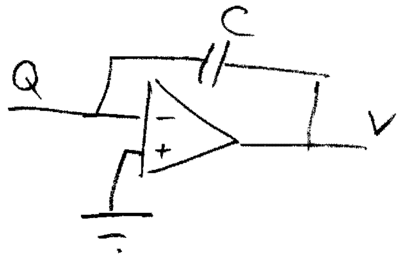


The pulse excites frequencies up to about $\frac{1}{b}$ Hz (rule of thumb)

So select a value of b for the desired frequency range, then mass m to give the right size hammer and k to give the right pulse duration.

1 cont (ii) Charge amplifier converts the pC/N & pC/g output of the piezo force transducer & accelerometer to a voltage ie $Q \rightarrow V$

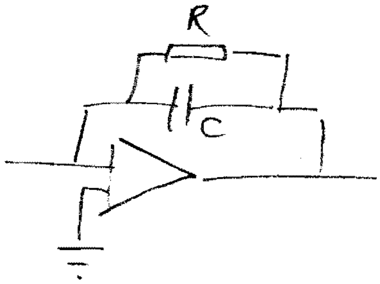
$$V = \frac{-Q}{C}$$



A high pass filter can be included easily

cut-off frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{RC}}$$



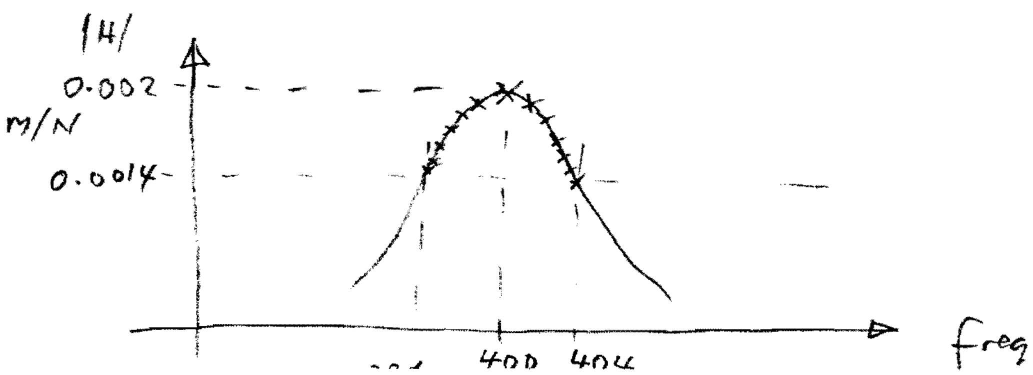
This prevents dc saturation of the charge amplifier output

(30%)

(c)(i) $Q = 50$ $\therefore$ half power bandwidth

$$\frac{\Delta f}{f_n} = \frac{1}{Q} = 0.02$$

$f_n = 400\,Hz$ $\therefore$ $\Delta f = 8\,Hz$



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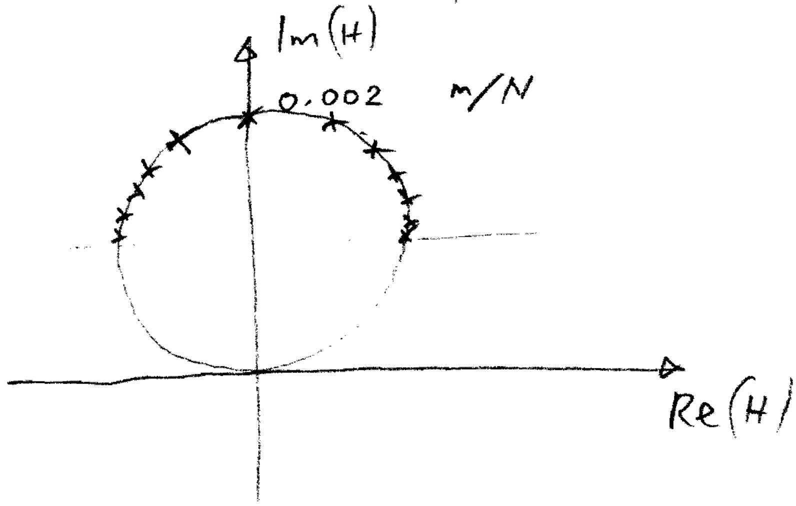
1 cont (c)(ii) For the modal circle need spacing of points $= \frac{5000 \text{ Hz}}{8192 \text{ points}} = 0.61 \text{ Hz/point}$

This means that the half-power peak has about $\frac{8}{0.61} = 13$ points in it



Note the circle could be upside down because we don't know the sign of H.

(30%)

2 (a) Material damping is caused by hysteretic losses during cyclical straining of a material sample. The physical origin is at the atomic/molecular level, for example from dislocation movements or interaction of polymer chains through hydrogen bonds. Often, immaterial damping can be approximated by linear theory based on the correspondence principle of linear viscoelasticity. If measured values are available for the complex moduli of the material, it may be possible to predict the damped mode shapes and natural frequencies. Boundary damping is one name for the various dissipative processes that can occur in a built-up structure. Examples are micro-slipping at joints in bolted or riveted connections, air pumping at lap joints, and losses due to rattling or buzzing at non-rigid connections.
Material damping is likely to dominate either because a single, monolithic piece of material is involved in the vibration, or because the material damping is so high that it dominates over boundary effects. An example of the former would be the vibration of a tuning fork, which has a mode which allows the fork to be held by the stalk without contributing much boundary damping. An example of the latter would be a panel with a damping treatment applied to a level that the loss factor becomes very high, such as a skin panel in a passenger aeroplane with attached internal trim.
Boundary damping tends to dominate in any built-up system made of material with relatively low damping, such as steel or glass. Examples would be a naval ship, or a glass window-pane. (30%)

(b) For the undamped problem, $K\underline{u} - \omega_n^2 M\underline{u} = 0$ (1)

For the damped problem, try $\underline{x}\, e^{i\omega t}$:

$$-\omega^2 M\underline{x} + i\omega C\underline{x} + K\underline{x} = 0$$

If $C = \alpha M + \beta K$:

$$(-\omega^2 + i\omega\alpha) M\underline{x} + (1 + i\omega\beta) K\underline{x} = 0$$

This has the same form as (1), so $\underline{x} = \underline{u}$ will work as a mode vector, then need $-\omega_n^2 = \dfrac{-\omega^2 + i\omega\alpha}{1 + i\omega\beta}$

$$\therefore -\omega^2 + i\omega\alpha + i\omega\omega_n^2\beta + \omega_n^2 = 0$$

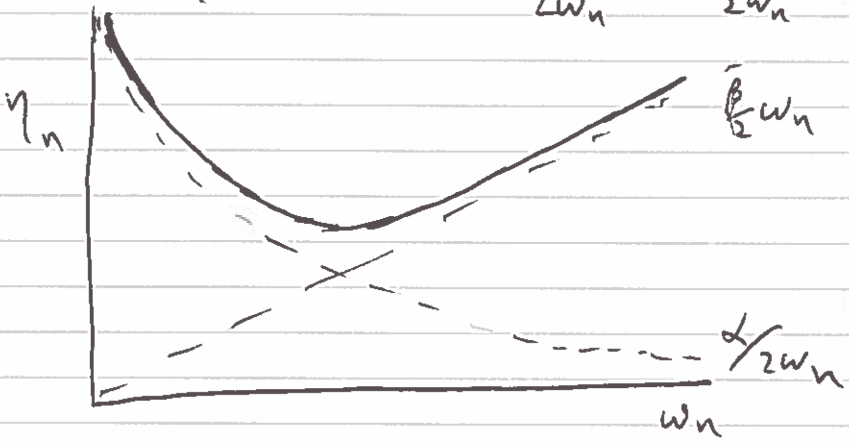
This is a quadratic equation for $i\omega$, so

$$i\omega = \frac{1}{2}\left[-\alpha - \beta\omega_n^2 \pm \sqrt{(\alpha + \beta\omega_n^2)^2 - 4\omega_n^4}\right]$$

(20%)

(c) If α, β small: $i\omega \simeq \frac{1}{2}\left(-\alpha - \beta\omega_n^2 \pm 2i\omega_n\right)$

So if $\omega = \omega_d(1 + i\eta_n)$, $\eta_n \simeq \dfrac{\alpha + \beta\omega_n^2}{2\omega_n} = \dfrac{\alpha}{2\omega_n} + \dfrac{\beta}{2}\omega_n$



(d) $M = m\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $K = \begin{bmatrix}k+s & -s\\ -s & k+s\end{bmatrix}$, $C = \begin{bmatrix}c_1+c_2 & -c_2\\ -c_2 & c_2+c_3\end{bmatrix}$

Any combination $\alpha M + \beta K$ has elements 11, 22 equal.
So need $c_1 = c_3$ for consistency.
Then any combination of values c_1, c_2 satisfies the condition: need $\beta = \frac{c_2}{s}$ to get off-diagonal elements right.

Then need $\alpha m + \beta(k+s) = c_1 + c_2$

$$\therefore \alpha = \frac{1}{m}\left[c_1 + c_2 - \frac{c_2}{s}(k+s)\right]$$

Modes are always $\begin{bmatrix}1\\1\end{bmatrix}$, $\begin{bmatrix}1\\-1\end{bmatrix}$ by symmetry.

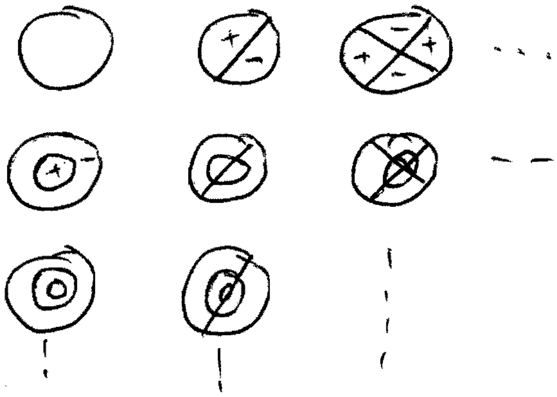
Undamped frequencies $\omega_n = \sqrt{\frac{k}{m}}, \sqrt{\frac{k+2s}{m}}$. Complex ω from (b).

[30%]

3(a) $u(r,\theta) = \cos n\theta \, J_n(kr)\, e^{i\omega t}$

so $u(a,\theta) = 0 \rightarrow J_n(ka) = 0$

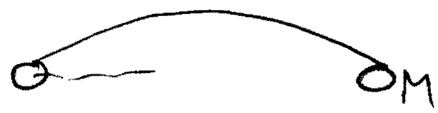
Given a, allowed values of k are given by the zeros of $J_n(ka)$. Modes in a 2D family:



Radial section along antinodal radius follows J_0, J_1, J_2 etc.

(25%)

(b)



Total force ↑ on boundary is $-2\pi a T \left.\frac{\partial u}{\partial r}\right|_{r=a}$

So for mass: $M \frac{\partial^2 u}{\partial t^2} = -2\pi a T k J_0'(ka) e^{i\omega t}$

$\therefore M\omega^2 J_0(ka) = 2\pi a T k J_0'(ka)$

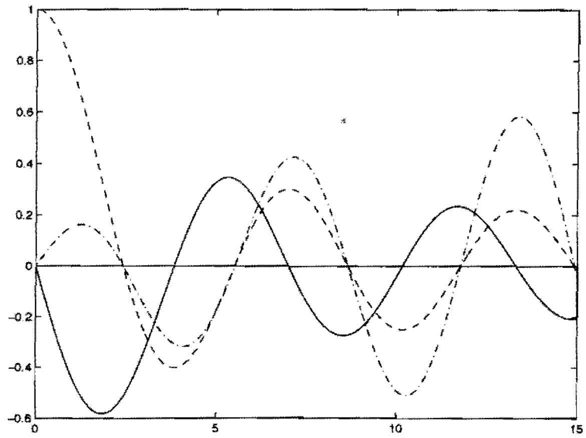
But $\omega^2 = k^2 T/m$, so $M k^2 \frac{T}{m} J_0 = 2\pi a T k J_0'$

ie $ka J_0(ka) = 2\pi \frac{a^2 m}{M} J_0'(ka)$

If the rigid ring is fixed and prevented from moving, this corresponds to adding a single constraint to the problem so interlacing should be seen, Fixing the ring means removing one degree of freedom, so the frequencies from part (a) would be expected to interlace between each pair of frequencies with the ring allowed to move.

So sketch the solution to the frequency equation just found. The result is shown below: dashed line shows $J_0(z)$, solid line shows $J_0'(z)$ which has zeros and max/min points interchanged with dashed line.
Now superimpose a multiple of $zJ_0(z)$ (dash-dot line) and see where it crosses the solid line: these intersections give the values of ka corresponding to the natural frequencies, the details depending on the value of the multiplying factor $\frac{2\pi a^2 m}{M}$.

3 cont



The result shows an intersection at $z = 0$ corresponding to the expected rigid-body mode at zero frequency. The next intersection lies *above* the first zero of $J_0(z)$, and *below* the first zero of $J_0'(z)$. The second intersection lies between the second zeros of these two functions, and so on. But the zeros of $J_0(z)$ correspond to the natural frequencies of the fixed-edge membrane from part (a), so we see interlacing behaviour exactly as expected.

(50%)

(c) The membrane frequency of a given mode can only be changed by the effect of movement of the rigid ring if the ring is capable of motion with the same variation $\cos/\sin n\theta$. For $n = 0$ we have just seen that this corresponds to rigid displacement of the ring. For $n = 1$, this variation corresponds to rigid rotation of the ring: the rotation has two degrees of freedom depending on the axis of rotation, and those correspond to the two choices $\cos\theta, \sin\theta$. So the $n = 1$ modes of the membrane are affecting by ring motion in a very similar way to the $n = 0$ modes investigated in part (b), with a pair of rigid body modes at zero frequency, and the remaining natural frequencies interlacing with the results from part (a).
For $n > 1$, the ring has no possible motion corresponding to $\cos/\sin n\theta$. The result is that the membrane behaves exactly the same as in part (a), with a fixed boundary, and the frequencies are unchanged.

(25%)

4(a) Air flow near the neck region is much faster than in the main volume or outside. The kinetic energy of this air motion can be idealised as a rigid mass ρLS moving at the mean speed of flow in the neck. Provided the frequency is very low the wavelength of sound is much longer than the neck size and the dimensions of the cavity. The pressure can then be regarded as uniform throughout the volume, acting on the real piston and the invisible neck piston. (15%)

From Data Sheet the Helmholtz resonance frequency with the piston immobilised by $K \to \infty$ is $\omega = c\sqrt{\frac{S}{VL}}$.

(b) New volume $= V + Ax + Su$, ie. change $V' = Ax + Su$
Same mass of air, so density changes to $\rho + \rho'$ such that

$$V\rho = (V+v')(\rho+\rho'), \text{ ie } V\rho' + \rho V' \simeq 0$$

$$\therefore \rho' \simeq -\rho\frac{V'}{V} = -\frac{\rho}{V}(Ax+Su)$$

$$\therefore \text{pressure change } p' = -\frac{\rho c^2}{V}(Ax+Su)$$

So equations of motion for the masses are:

$$\begin{cases} M\ddot{x} = p'A - Kx + Fe^{i\omega t} = -Kx - \frac{\rho c^2 A}{V}(Ax+Su) + Fe^{i\omega t} & (1) \\ \rho SL\ddot{u} = p'S = -\frac{\rho c^2 S}{V}(Ax+Su) & (2) \end{cases}$$

These match the given equations. (35%)

(c) For harmonic response, $x = Xe^{i\omega t}$, $u = Ue^{i\omega t}$

and we wish to calculate $V' = (AX + SU)e^{i\omega t}$

From (2), $-\cancel{\rho S}L\omega^2 U = -\frac{\cancel{\rho}c^2\cancel{S}}{V}(AX+SU)$

$$\therefore U\left(L\omega^2 - \frac{c^2S}{V}\right) = \frac{c^2A}{V}X$$

$$\therefore U = \left(\frac{c^2A}{L\omega^2V - c^2S}\right)X$$

$$\therefore SU + AX = \left[\frac{c^2AS}{L\omega^2V - c^2S} + A\right]X \qquad (3)$$

4 cont. From (1), $F = -M\omega^2 X + KX + \frac{\rho c^2 A}{V}(AX + SU)$

$$= (K - M\omega^2)X + \frac{\rho c^2 A}{V}(AX + SU)$$

$$= (AX + SU)\left[\frac{\rho c^2 A}{V} + \frac{(K - M\omega^2)}{A + \frac{c^2 AS}{L\omega^2 V - c^2 S}}\right] \quad \text{using (3)}$$

$$= (AX + SU)\left[\frac{\rho c^2 A}{V} + \frac{(K - M\omega^2)(L\omega^2 V - c^2 S)}{L\omega^2 A V}\right]$$

So TF is $\dfrac{AX + SU}{F} = \dfrac{V}{\rho c^2 A + \frac{(K - M\omega^2)(L\omega^2 V - c^2 S)}{L\omega^2 A}}$

$$= \frac{LAV\omega^2}{\rho c^2 A^2 L\omega^2 + (K - M\omega^2)(L\omega^2 V - c^2 S)}$$

So as $\omega \to 0$, $TF \to 0$

The reason is that at very low frequency, if the piston M is pushed out of V, the same volume of air is drawn in through the vent. The air behaves incompressibly, so no net volume change occurs and virtually no sound is radiated.

(50%)

Answers: 4C6 2013

2 (b) Complex frequency given by $i\omega = \frac{1}{2}\left[-\alpha - \beta\omega_n^2 \pm \sqrt{\left(\alpha + \beta\omega_n^2\right)^2 - 4\omega_n^4}\right]$

(c) $\eta_n \approx \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2}$

(d) Require $c_1 = c_3$.

Modes $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, undamped natural frequencies $\sqrt{\frac{k}{m}}$, $\sqrt{\frac{k+2s}{m}}$,

damped natural frequencies from (b).

3 (b) Boundary condition $M\frac{\partial^2 u}{\partial t^2} = -2\pi a T \left.\frac{\partial u}{\partial r}\right|_{r=a}$

giving $kaJ_0(ka) = \frac{2\pi a^2 m}{M} J_0'(ka)$