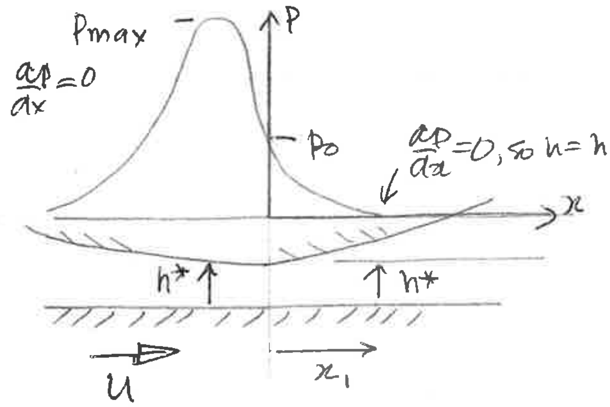
1. Bookwork - but essentially: convergent geometry, viscous lubricant and positive entraining velocity.

(a)



(b) Inlet region $\qquad h = h_0 \exp(-\alpha x) \qquad x \leq 0$

Reynolds' eqn $\qquad \dfrac{dp}{dx} = 6U\eta \dfrac{h - h^*}{h^3}$

$$h = h^* \text{ at } \frac{dp}{dx} = 0$$

So substituting $\qquad \dfrac{dp}{dx} = 6U\eta \left\{ \dfrac{\exp(2\alpha x)}{h_0^2} - \dfrac{h^* \exp(3\alpha x)}{h_0^3} \right\}$

$$\therefore \frac{h_0^3}{6U\eta} \cdot p = \frac{h_0 \exp(2\alpha x)}{2\alpha} - \frac{h^* \exp(3\alpha x)}{3\alpha} + C_1$$

But if $p \to 0$ at $x = -\infty$ then $C_1 = 0$

$$\therefore p = \frac{6U\eta}{\alpha h_0^2} \left\{ \frac{\exp(2\alpha x)}{2} - \frac{h^*}{3h_0} \exp(3\alpha x) \right\}$$

(c) When $x = 0$, $p = p_0$ then

$$p_0 = \frac{6U\eta}{\alpha h_0^2} \left\{ \frac{1}{2} - \frac{h^*}{3h_0} \right\}$$

$$\text{i.e. } p_0 = \frac{U\eta}{\alpha h_0^2} \left\{ 3 - 2\frac{h^*}{h_0} \right\}$$

(d) outlet region $\quad h = h_0 \exp(\alpha x) \quad x > 0$

$$\therefore \frac{dp}{dx} = 6U\eta \left\{ \frac{\exp(-2\alpha x)}{h_0^2} - \frac{h^*}{h_0^3} \exp(-3\alpha x) \right\}$$

$$\therefore \frac{h_0^3}{6U\eta} p = \frac{h_0 \exp(-2\alpha x)}{-2\alpha} + \frac{h^* \exp(-3\alpha x)}{3\alpha} + C_2$$

Now $p = 0$ when $x = x_1$ when also $\frac{dp}{dx} = 0 \therefore h = h^*$

i.e. $\underline{h^* = h_0 \exp(\alpha x_1)}$

$$\therefore \quad 0 = -\frac{h_0}{2\alpha}\left(\frac{h_0}{h^*}\right)^2 + \frac{h^*}{3\alpha}\left(\frac{h_0}{h^*}\right)^3 + C_2$$

$$\therefore \underline{C_2 = \frac{h_0^3}{6\alpha h^{*2}}}$$

So general expression for p is

$$\frac{h_0^3 p}{6U\eta} = \frac{h_0 \exp(-2\alpha x)}{-2\alpha} + \frac{h^* \exp(-3\alpha x)}{3\alpha} + \frac{h_0^3}{6\alpha h^{*2}}$$

when $x = 0,\ p = p_0$ so $\quad \frac{h_0^3 p_0}{6U\eta} = -\frac{h_0}{2\alpha} + \frac{h^*}{3\alpha} + \frac{h_0^3}{6\alpha h^{*2}}$

$$\text{or} \quad \underline{\frac{\alpha p_0 h_0^2}{U\eta} = -3 + 2\frac{h^*}{h_0} + \left(\frac{h_0}{h^*}\right)^2}$$

But from (c) $\quad \frac{\alpha p_0 h_0^2}{U\eta} = 3 - 2\frac{h^*}{h_0}$

So equating, & writing $H = h^*/h_0$,

$$3 - 2H = -3 + 2H + \frac{1}{H^2}$$

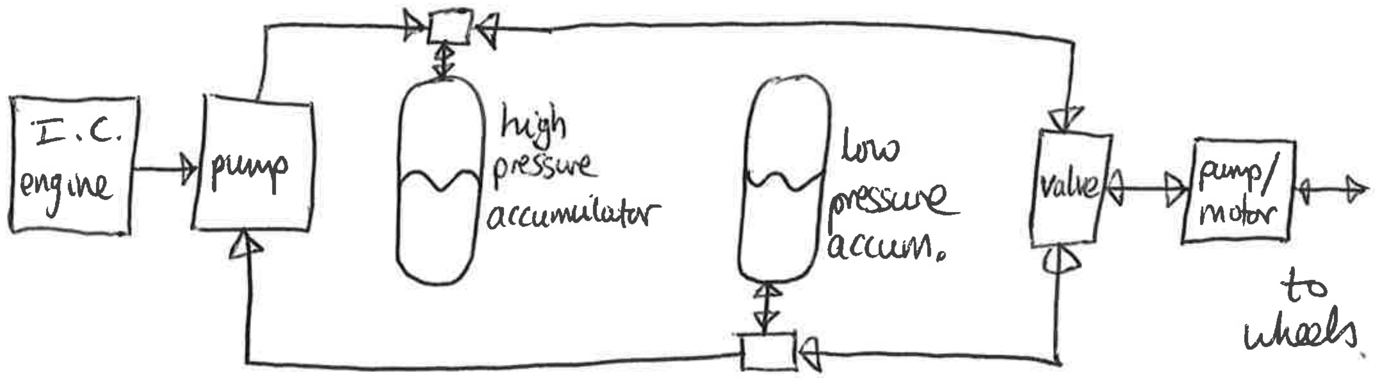
$$\text{i.e.} \quad \underline{6 - 4H = \frac{1}{H^2}}$$

H	LHS	RHS
1.3	0.8	0.59
1.4	0.4	0.51
1.37	0.52	0.53
1.36	.56	.54

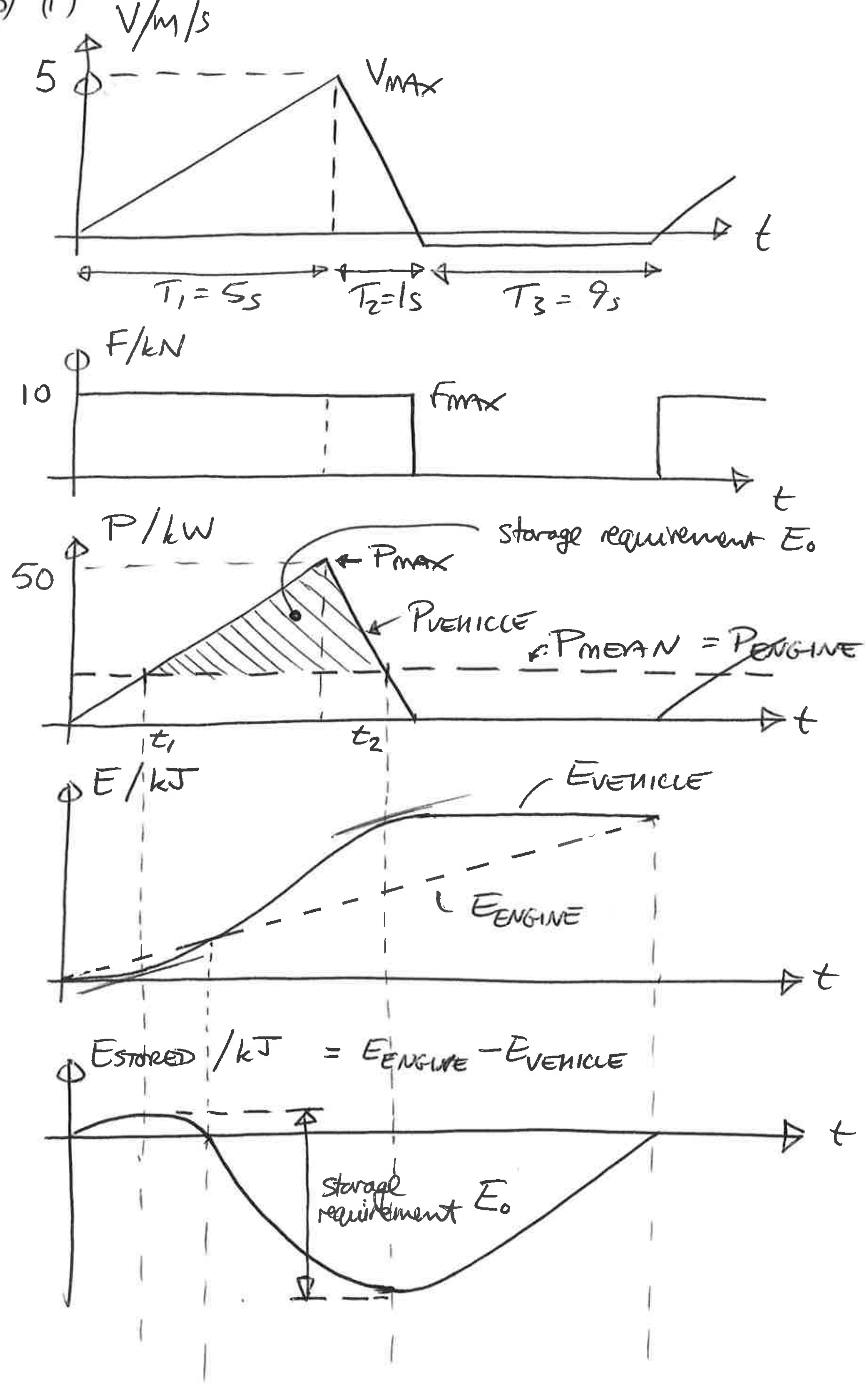
So $\underline{H \simeq 1.37}$

2 (a) (i) In a hybrid arrangement the internal combustion engine operates at its most efficient point, reducing fuel consumption and emissions. The i.c. engine can be much smaller, because it only needs to generate the mean power requirement.

(ii) Earth moving equipment often has hydraulic transmission for the traction drive. Therefore an hydraulic accumulator for energy storage is appropriate



(b) (i)



NOT DRAWN TO SCALE

mean power $P_{MEAN} = \dfrac{\text{energy required during cycle}}{\text{duration of cycle}}$

$$= \frac{\frac{1}{2}(T_1 + T_2)P_{MAX}}{T_1 + T_2 + T_3}$$

$$= \frac{\frac{1}{2}(5+1).50.10^3}{5+1+9} = \underline{\underline{10kW}}$$

peak power of store $P_o = P_{MAX} - P_{MEAN}$

$$= F_{MAX}.V_{MAX} - P_{MEAN}$$

$$= 10.10^3.5 - 10.10^3 = \underline{\underline{40kW}}$$

(ii) from sketch, energy storage requirement

is $E_o = \int_{t_1}^{t_2} P_{VEHICLE}(t) - P_{MEAN}(t)\, dt.$

where $t_1 = T_1 . \dfrac{P_{MEAN}}{P_{MAX}} = 5.\dfrac{10}{50} = 1s$

$t_2 = (T_1 + T_2) - T_2 . \dfrac{P_{MEAN}}{P_{MAX}} = 6 - 1.\dfrac{10}{50} = 5.8s$

$$\therefore E_o = \frac{1}{2}(t_2 - t_1)(P_{MAX} - P_{MEAN})$$

$$= \frac{1}{2} . 4.8.(50.10^3 - 10.10^3) = \underline{\underline{96kJ}}$$

(c) Calculate ratio of $\dfrac{E_o}{P_o} = \dfrac{96kJ}{40kW} = 2.4s$.

Convert to hours $\tau = \dfrac{2.4}{60^2} = 0.67.10^{-3}$ hours

$$\log_{10}\tau = -3.18$$

Select technology B which has $\log_{10}\tau = -3$, the closest to the requirement.

3. (a)(i) $0 < \theta < 30°$ $y = \frac{L}{2}\left(\frac{\theta}{30}\right)^3$

$\therefore \dot{y} = \frac{3L}{2}\frac{\theta^2}{(30)^3}\dot{\theta}$

ie. $\frac{\dot{y}}{L\dot{\theta}} = \frac{3\theta^2}{2.30^3}$ When $\theta = 30°$. $\frac{\dot{y}}{L\dot{\theta}} = 1/20$

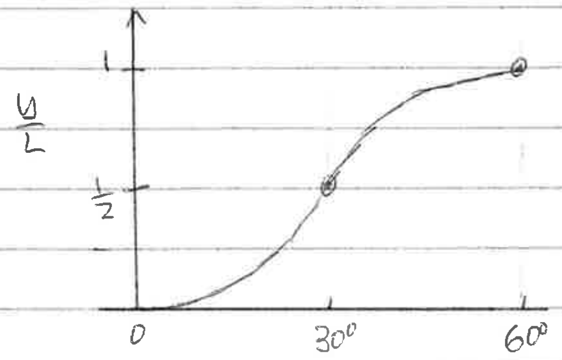
$\frac{\ddot{y}}{L\dot{\theta}} = \frac{3\theta.\dot{\theta}}{30^3}$

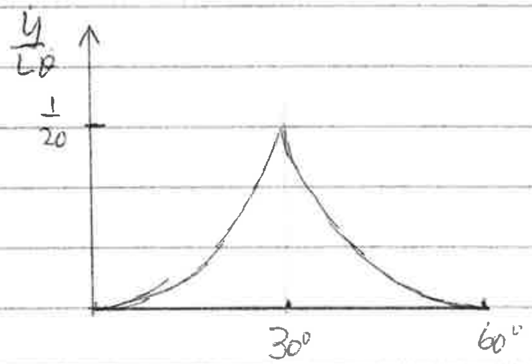
ie. $\frac{\ddot{y}}{L\dot{\theta}^2} = \frac{3\theta}{30^3}$ $\theta = 30°$ $\frac{\ddot{y}}{L\dot{\theta}^2} = \frac{1}{300}$

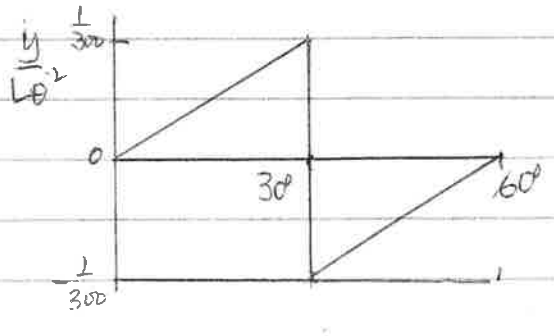
$30° < \theta < 60°$ $\frac{y}{L} = 1 - \frac{1}{2}\frac{(60-\theta)^3}{30^3}$

$\therefore \frac{\dot{y}}{L\dot{\theta}} = \frac{3}{2}\frac{(60-\theta)^2}{30^3}$ $\theta = 30°$ $\frac{\dot{y}}{L\dot{\theta}} = 1/20$

$\frac{\ddot{y}}{L\dot{\theta}^2} = \frac{-6(60-\theta)}{2.30^3}$ $\theta = 30°$ $\frac{\ddot{y}}{L\dot{\theta}^2} = -1/300$

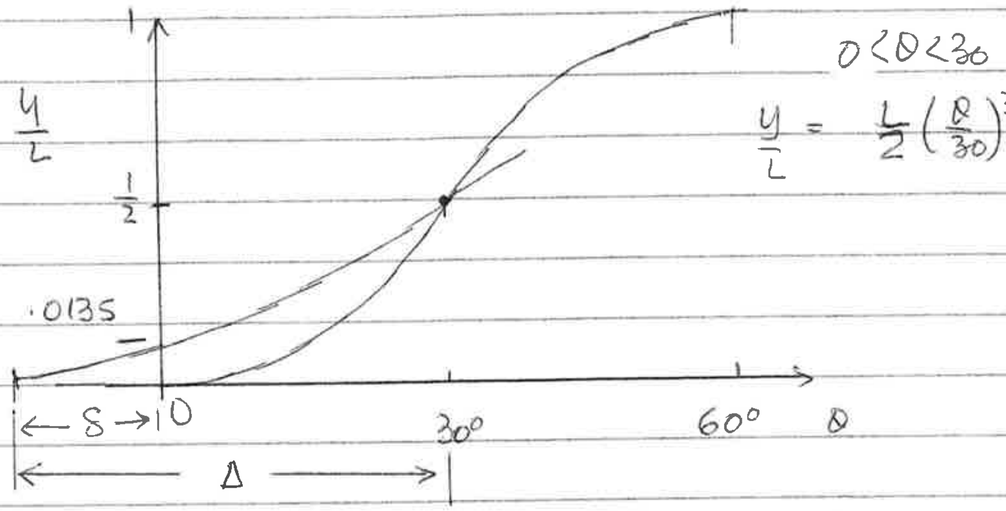






$\dot{\theta}$ in deg/s

(ii)



since $y \propto \theta^3$

$$\frac{.0135}{.5} = \left(\frac{\delta}{\Delta}\right)^3$$

But $\delta = \Delta - 30$ $\therefore$ $\frac{\Delta - 30}{\Delta} = \sqrt[3]{\frac{.0135}{.5}} = 0.3$

$\therefore$ $\Delta = \frac{30}{.7} = 42.9^0$

So raised profile $y = \frac{L}{2}\left(\frac{\theta+\delta}{\Delta}\right)^3$

$$\frac{\dot{y}}{L\dot{\theta}} = \frac{3(\theta+\delta)^2}{2\Delta^3}$$

$$\frac{\ddot{y}}{L\dot{\theta}^2} = \frac{3(\theta+\delta)}{\Delta^3}$$

$\frac{\ddot{y}}{L\dot{\theta}^2}$ max when $(\theta+\delta) = \Delta$ ie $\frac{\ddot{y}}{L\dot{\theta}^2} = \frac{3}{\Delta^2} = \frac{3}{(42.9)^2}$

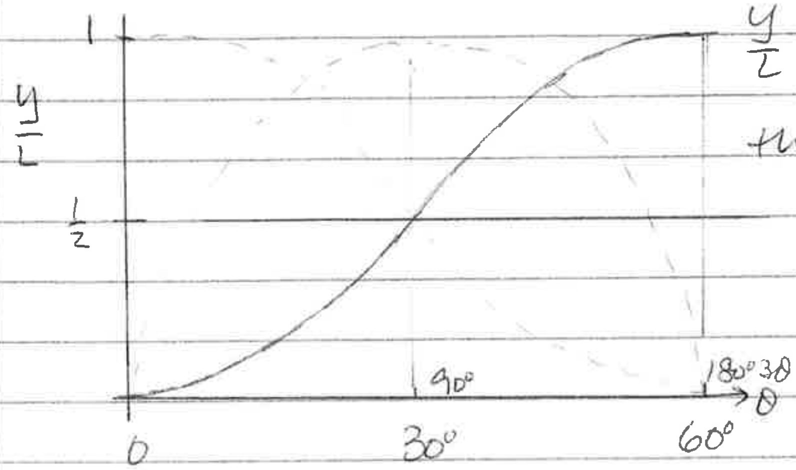
So c/f original design reduced by $\left(\frac{30}{42.9}\right)^2$

$= 0.49$

Original design $\frac{\ddot{y}}{L\omega^2} = \left(\frac{180}{\pi}\right)^2 \times \frac{3}{30^2} = 10.94$

Revised cam $\frac{\ddot{y}}{L\omega^2} = 0.49 \times 10.94 = 5.35$

(iii) Sinusoidal profile



$$\frac{y}{L} = \frac{1}{2} - \frac{1}{2}\cos 3\theta$$

then $$\frac{\dot y}{L\omega} = \frac{3}{2}\sin 3\theta$$

note ω not $\dot\theta$

$$\frac{\ddot y}{L\omega^2} = \frac{9}{2}\cos 3\theta$$

hence

	original	modified	sinusoid
Max $\frac{L\ddot y}{\omega^2}$	10.94	5.35	4.5

(b) Non-conformal contact geometry and significant spring + inertial loads lead to high Hertzian pressures around cam nose – often in excess of 1 GPa. When combined with small magnitude entraining velocities these conditions make severe demands on the lubricant. EHL film thicknesses can fall to very low values so that protection of cam + follower surfaces depends on boundary lubrication often generated by chemical activity of oil additive package – exemplified by ZDDP