1) a) 

$$M_{eq} = \frac{mL}{2}$$

$$K_{eq} = \left(\frac{3EI}{(L/4)^3}\right) = 192\frac{EI}{L^3}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \frac{\sqrt{384}}{L^2}\sqrt{\frac{EI}{m}} = \frac{19.6}{L^2}\sqrt{\frac{EI}{m}}$$

b) $\bar{u}(x) = 1 - \cos\left(\frac{2\pi x}{L}\right)$

$$M_{eq} = \int_0^L m\left(1 - \cos\frac{2\pi x}{L}\right)^2 dx = m\int_0^L \left(1 - 2\cos\frac{2\pi x}{L} + \cos^2\frac{2\pi x}{L}\right) dx$$

$$= m\left[L - \left(2\frac{L}{2\pi}\sin\frac{2\pi x}{L}\Big|_0^L + \frac{L}{2}\right)\right] = m\left[L - 0 + \frac{L}{2}\right] = \underline{\underline{\frac{3mL}{2}}}$$

$$K_{eq} = \int_0^L EI\left[\left(\frac{2\pi}{L}\right)^2 \cos\frac{2\pi x}{L}\right]^2 dx$$

$$= EI\left(\frac{2\pi}{L}\right)^4 \int_0^L \cos^2\left(\frac{2\pi x}{L}\right) dx = EI\left(\frac{2\pi}{L}\right)^4\left(\frac{L}{2}\right) = \underline{\underline{EI\frac{8\pi^4}{L^3}}}$$

$$\omega_n = \sqrt{EI\frac{8\pi^4}{L}\frac{2}{3mL}} = \underline{\underline{\sqrt{\frac{16}{3}}\frac{\pi^2}{L^2}\sqrt{\frac{EI}{m}}}} = \frac{22.8}{L^2}\sqrt{\frac{EI}{m}}$$

c) $F_{eq} = 10\left(\bar{u}_1\Big|_{x=\frac{L}{3}}\right) = 10(1+0.5) = 15\text{ N}$

$\omega = \sqrt{\frac{16}{3}}\pi^2 \rightarrow T_n = \frac{2\cancel{\pi}}{\pi^{\cancel{2}}}\sqrt{\frac{3}{16}} \approx 0.28 \rightarrow \frac{t_d}{T_n} = \frac{0.3}{0.28} \approx 1$

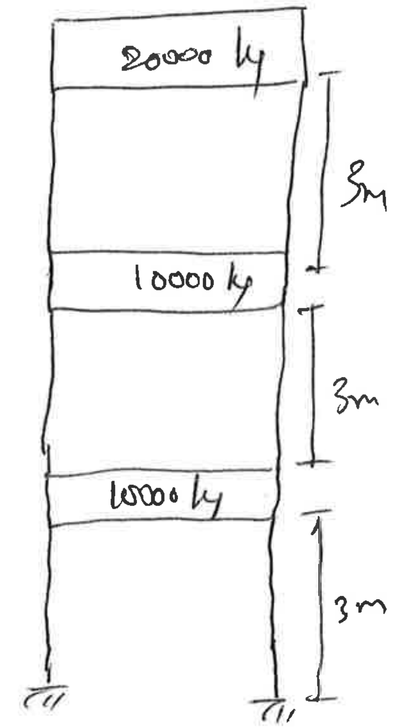
$\rightarrow \text{DAF} \approx 1.55$

$$u_{max} = \frac{F_{eq}}{k_{eq}}\text{DAF}(2) = \frac{15}{8\pi^4}(1.55)(2) = \underline{\underline{6.0\text{ cm}}}$$

d) Present continuous mode shape and identify BC's. Don't need to solve for mode shape.

<u>Or</u> present a lumped mass approach and explain how you would find M_{eq}, K_{eq}.

2 a)



Column Stiffness = $\frac{12EI}{h^3}$

$\therefore$ For each floor = $2 \times \frac{12EI}{h^3} = \frac{24 \times 12 \times 10^6}{3^3}$

$= 1.067 \times 10^7$ N/m.

$m_1 = m_2 = 10000$ kg $\quad m_3 = 20000$ kg

Mode shape = $\{0.4, 0.75, 1.0\}$

$\therefore M_{eq} = 10000 \times 0.4^2 + 10000 \times 0.75^2 + 20000 \times 1^2$
$= 27225$ kg

$K_{eq} = 1.067 \times 10^7 \left[0.4^2 + (0.75-0.4)^2 + (1-0.75)^2\right]$
$= 3.68 \times 10^6$ N/m

$\therefore \omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}} = 11.62$ rad/s $\quad f_n = \frac{\omega_n}{2\pi} = \underline{1.85 \text{ Hz}}$ QED

Modal participation factor:

$$\Gamma = \frac{m_1\bar{U}_1 + m_2\bar{U}_2 + m_3\bar{U}_3}{m_1\bar{U}_1^2 + m_2\bar{U}_2^2 + m_3\bar{U}_3^2} = \frac{10000(0.4+0.75) + 20000\times1}{10000(0.4^2+0.75^2) + 20000\times1}$$

$= \underline{1.16}$ QED

Modal participation factor Γ compares the mass participating in the forcing function with the mass participating in the inertia effects (M_{eq}).

[~~10~~ A]
[40%]

2 b) Second mode shape is given as $\{-2.12, -1.55, 1.0\}$.

$$\therefore M_{eq} = 10000\left[(-2.12)^2 + (-1.55)^2\right] + 20000 \times 1^2$$

$$= 88969 \text{ kg}$$

$$K_{eq} = 1.067\times10^7\left[(-2.12)^2 + (-2.12+1.55)^2 + (1+1.55)^2\right]$$

$$= 120.803606 \times 10^8 \text{ N/m}$$

$$\omega_{2n} = 36.848 \text{ rad/s} \Rightarrow f_2 = \underline{\underline{5.865 \text{ Hz}}}$$ [20%]

2c) Consider the first Mode: $f_1 = 1.85$ Hz

$$\therefore T_1 = \frac{1}{f_1} = 0.54 \text{ sec.}$$ [from part (a)]

$$\Gamma = \frac{\Sigma m\bar{u}}{\Sigma m\bar{u}^2} = 1.16$$ [from part (a)]

for $T_1 = 0.54$ $\quad \frac{S_{da}}{a_g} = 2.5$

$$a_g = 0.2g = 0.2\times9.81 = 1.962 \text{ m/s}^2$$

$$S_{da} = 2.5\times1.962 = 4.905 \text{ m/s}^2$$

$$\ddot{y}_{max} = \Gamma\times S_{da} = 1.16\times4.905 = 5.6898 \text{ m/s}^2$$

$$\therefore \ddot{y}_{1max} = 0.4\times5.6898 = 2.2759 \text{ m/s}^2$$

$$\ddot{y}_{2max} = 0.75\times5.6898 = 4.2674 \text{ m/s}^2$$

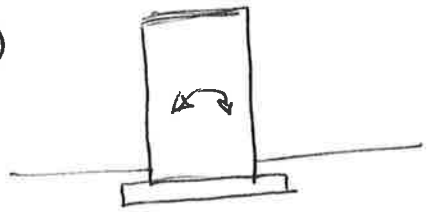
$$\ddot{y}_{3max} = 1\times5.6898 = 5.6898 \text{ m/s}^2$$

$$\text{SF at base} = 10000\times(2.2759 + 4.2674) + 20000\times5.6898$$

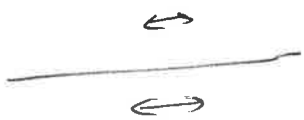
$$= \underline{179.228 \text{ kN}}$$ (shared between ground floor columns) [40%]

[Second mode $T_1 = 0.17$ $\frac{S_{da}}{a_g} \simeq 1.0$ so need not be considered]

3 a)



When the Sh waves arrive they induce vibrations in the structure. The vibrations of the structure can impose additional stresses in the soil surrounding the foundations. This interaction between the soil and the structure under dynamic loading is termed as dynamic soil-structure interaction.

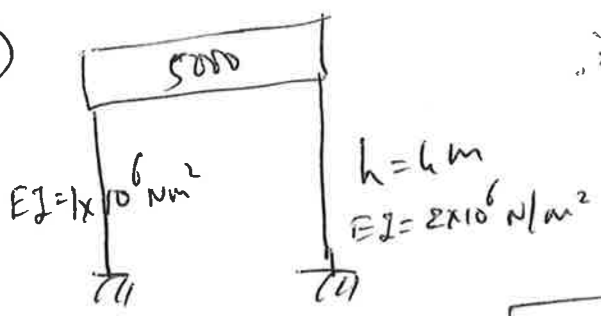


[10%]

3 b) Response spectra methods are predominantly designed for linear elastic systems. However, the soil has a highly non-linear stress-strain history, including the tendency to suffer plastic, non recoverable strains. Hence when attempting to solve soil-structure interaction problems we resort to numerical integration methods, that are used to solve the equations of motions from first principles. It is also possible to have coupled equations to capture solid fluid phases of soil.

[10%]

3 c)



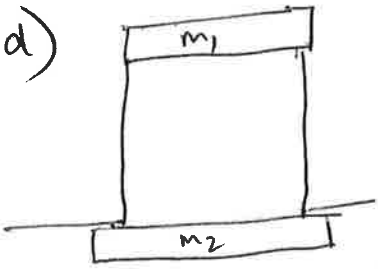
$$\therefore K_1 = \frac{12EI}{h^3} = \frac{12 \times 1 \times 10^6}{4^3} = 187500 \text{ N/m}$$

$$K_2 = \frac{12 \times 2 \times 10^6}{4^3} = 375000 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{K_1 + K_2}{M}} = \sqrt{\frac{187500 + 375000}{5000}} = 10.61 \text{ rad/s}$$

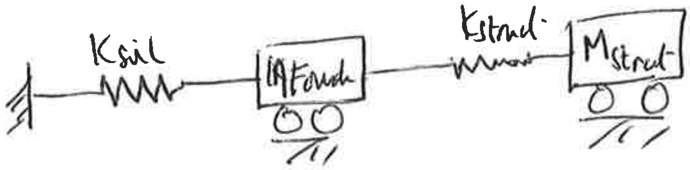
$$f_n = 1.688 \text{ or } \underline{1.7} \text{ Hz}$$

[15%]

3 d) 

Although k_1 & k_2 are two different springs, we can combine them into one structural spring.

∴ The simple discrete model can be;



[20%]

3 e) unit weight of soil = $\gamma_d = 16 \text{ kN/m}^3$ (take $g = 10 \text{ m/s}^2$)

$v_s = 120 \text{ m/s} = \sqrt{\frac{G}{\rho}}$

∴ Shear modulus of soil $G = \rho v_s^2 = 1600 \times 120^2$

$= 23.04 \text{ MPa}$

Use Wolf's formulae for K_h - horizontal soil stiffness

$2l = 3 \text{ m}$ $\quad e = 0.5$

$2b = 3$

$l/b = 1 \quad e/b = \frac{0.5}{1.5} = 1/3$

$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\,(l/b)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1 + l/b}\right)\left(\frac{e}{b}\right)^{0.8}\right]$

$K_{soil} = \frac{23.04 \times 10^6}{2 - 0.3}\,[9.2]\,[1.4152] = 2.6469 \times 10^8 \text{ N/m}$.

$M_{foundation} = 3 \times 3 \times 0.5 \times 2400 \text{ kg/m}^3 = 10800 \text{ kg}$.

Soil + participation $= 108000 \text{ kg}$

$K_{struct} = 562500$ N/m.

$M_{struct} = 5000$ kg.

Solving the 2DOF' system with above

$\omega_1 = 10.59$ rad/s $\qquad f_1 = 1.6863$ Hz

$\omega_2 = \cancel{15.6\ldots}$ 47.25 rad/s $\qquad f_2 = \cancel{24.9\ldots}$ 7.52 Hz (7.52 Hz)

The first mode is almost same as for fixed structure and the second mode ~~has~~ quite high nat. freq (~~20~~ Hz)

[30%]

3 f) If the soil below the foundation liquefies the structure can sink/Rotate. The settlements in themselves may not be a problem but rotations can make the structure unserviceable.

Liquefaction resistant measures such as in-situ densification of foundation soil can reduce the risks to the structure.

[15%]

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4 a) Marks will be obtained for a clear description, preferably with diagrams, of the convection cells for hurricanes (with winds spiralling **inwards** towards the centre and rising up alongside the eye wall), contrasted with the downwards-and-**outwards** "first gust" phenomenon in thunderstorms. Additional marks will be obtained for further descriptions of secondary flows and causative processes in the two cases.

b) D'Alembert's paradox arises from the inviscid flow theory prediction that there this no drag on any solid body immersed in a flowing fluid. Drag however arises from two sources not covered by inviscid theory – form drag and friction drag. Form drag is the most important in wind engineering, and arises due to boundary layer separation. Marks will be awarded for a clear explanation of this, preferably with explanatory diagrams.

c) Galloping is an aeroelastic instability which arises as a sharp-cornered structure oscillates across-wind, its motions changing the apparent angle of attack of the incident wind leading to the switching of separation points and consequent dynamic alterations to the pressures felt. The resulting feedback between cross-wind forces and motion can result in an oscillatory divergent cross-wind response. Structures susceptible to galloping include those with hexagonal and octagonal sections, roughly-hemispherical radio telescopes, cables consisting of bundles of wires and cables whose cylindrical cross-sections are augmented by ice accumulation or by rivulets of rain running along their underside.

d) Full marks will be obtained for a clear description of the sharp positive pressure spike followed by a longer negative suction for explosive blasts, contrasted with the longer duration positive pressures of a gas blast.

e) Full marks will be obtained for a clear description of how aerodynamic admittance, a concept in the statistical theory of buffeting, essentially acts as an "areal-averaging" reduction factor as local pressures are integrated to obtain total (possibly mode-generalised) forces. Wind pressure fluctuations across the face of a structure are de-correlated, and thus do not act in dynamic synchrony. The aerodynamic admittance is thus a form of reduction factor that takes account of this decorrelation, and the frequency-dependent admittance depends on the size of the structure compared to the size of gusts at a particular frequency.

f) Marks will be obtained for recognising that a suspension bridge is a tension structure which gains much of its stiffness from tension stiffening due to dead loads. Finite element analysis to obtain mode shapes and frequencies should thus proceed in stages. First a static model should be created with dead loads applied to determine the static tensions in cables. This should be a nonlinear model, with some form of path-following to determine the nonlinear equilibrium. Only then should eigenvalues and eigenvectors be extracted from a linearization (which includes geometric stiffness effects due to tension stiffening) around the previously computed equilibrium.