4D7 Solutions

1. (a) Characteristic strength $= 1.64\,\sigma$ below the mean (μ) $= 460$ MPa

$$CoV = \frac{\sigma}{\mu} \quad \therefore \sigma = 0.1\mu$$

$$\therefore 460 = \mu(1 - 0.164) \Rightarrow \underline{\mu = 550.2 \text{ MPa}}$$

$=$ ~~[illegible]~~ Mean strength to be provided by the manufacturer

$$\text{Design strength} = \frac{460}{1.15} = \underline{\underline{400 \text{ MPa}}}$$

(b) If mean load $= \bar{S}$ then, since CoV of loads is 25%, $\sigma_S = 0.25\bar{S} = \frac{\bar{S}}{4}$

$$\text{Reliability } \beta = \frac{(550 - \bar{S})}{\sqrt{\left(\frac{550}{10}\right)^2 + \frac{\bar{S}^2}{16}}}$$

(from data sheet formula for β)

$$= 3.8$$

Quadratic in $\bar{S}$ $\Rightarrow$ $\underline{\underline{\bar{S} = 240.5 \text{ MPa}}}$

$$\text{Ratio of mean load / mean strength} = \frac{240.5}{550.2} = \underline{\underline{43.7\%}}$$

(c)(i) If 1 in 1000 bars lies below the characteristic strength then cdf $= 0.001$ (or 0.999 above in tables)

From data sheet this occurs at 3.09 SD below mean

To achieve this with a CoV $= 4\%$

Let Mean strength $= \bar{R}$ $\qquad$ S.D $= \dfrac{\bar{R}}{25}$

$$\therefore \; 460 = \bar{R}\left(1 - \frac{3.09}{25}\right) = 0.876\,\bar{R}$$

$$\Rightarrow \underline{\underline{\bar{R} = 525.1 \text{ MPa}}}$$

[Students might like to note that improved quality control can mean that mean strength can come <u>down</u> even if overall safety goes up (better steel control)]

(ii) New reliability with same load

$$\beta = \frac{(525.1 - 240.5)}{\sqrt{\left(\frac{525.1}{25}\right)^2 + \left(\frac{240.5}{4}\right)^2}} = \frac{284.6}{63.6} = \underline{\underline{4.46}}$$

(iii) New mean load with same reliability

$$3.8 = \frac{(525.1 - \bar{S})}{\sqrt{\left(\frac{525.1}{25}\right)^2 + \left(\frac{\bar{S}}{4}\right)^2}}$$

Quadratic in $\bar{S}$ $\Rightarrow$ $\underline{\underline{\bar{S} = 262.9 \text{ MPa}}}$

(d) Students should note that for the original design ratio of strength to load is $\frac{550}{240} = 2.29$ whereas for second version it would be $\frac{525}{262} = 2.00$.

The reason the ratios are so high is that society expects very high reliabilities for strutures (the "safe as houses" argument) and to provide this with large uncertainty of loads and strengths means that strutures have to be very strong.

Reducing the strength is possible if we have more certainty about the variability, but engineers can only control variability of materials - they have little control over variability of loads (at least for roads and buildings). In effect buildings are made stronger to allow for dients not being able to control loads. They might also comment on the fact that real failures occur more frequently than predicted because of gross errors - things unforeseen by desegnes.

2.

Option 1 – Stainless steel

Will need replacing at 20 and 40 years.

$$\text{Cost now} = 80 + 5 \times 50 = £330\text{K}$$

$$\text{at 20 years} = 80 + (5 \times 50).(1.02)^{20} = £451\text{K}$$

$$\text{at 40 years} = 80 + (5 \times 50)(1.02)^{40} = £632\text{K}$$

$$\therefore \text{WLC} = 330 + \frac{451}{(1.035)^{20}} + \frac{632}{(1.035)^{40}} =$$

$$= 330 + 226.7 + 159.6 = £716.3\text{K}.$$

(of which $80 + \frac{80}{(1.035)^{20}} + \frac{80}{(1.035)^{40}} =$ £140.4 K = materials
£575.9 = traffic delay)

Maintenance : Continuous discounting

$$1.035 = \exp(r_c) \Rightarrow r_c = 0.0344$$

$$\therefore A_0 = \int_0^{60} \frac{M\,dt}{\exp(r_c t)} = M\left[\frac{\exp(-r_c t)}{-r_c}\right]_0^{60}$$

$$= \frac{M}{r_c}\left[1 - \exp(-60 r_c)\right]$$

$M = 10\text{K/year}$

$$\therefore \underline{\underline{A_0 = £253\text{K}}}$$

$\therefore$ Total WLC =

Materials	£140.4
Maintenance	£253 K
Traffic	£575.9 K
	£969.3 K

~~2~~ <u>Option 2</u>

$$\text{Cost of Materials} = 130\left[1 + \frac{1}{(1.035)^{15}} + \frac{1}{(1.035)^{30}} + \frac{1}{(1.035)^{45}}\right]$$

$$= £281\text{K}$$

$$\text{Cost of traffic delay} = 2\times 50\left[1 + \left(\frac{1.02}{1.035}\right)^{15} + \left(\frac{1.02}{1.035}\right)^{30} + \left(\frac{1.02}{1.035}\right)^{45}\right]$$

$$= £296.6\text{K}.$$

Cost of maintenance will be $\frac{6000}{10000} \times £253 = £151.8\text{K}$

$\therefore$ Total WLC = <u>£729.4 K.</u>

Thus, option 2 is cheaper overall.

If traffic delay costs are excluded

Option 1 = 393.4 K

Option 2 = 577.6 K.

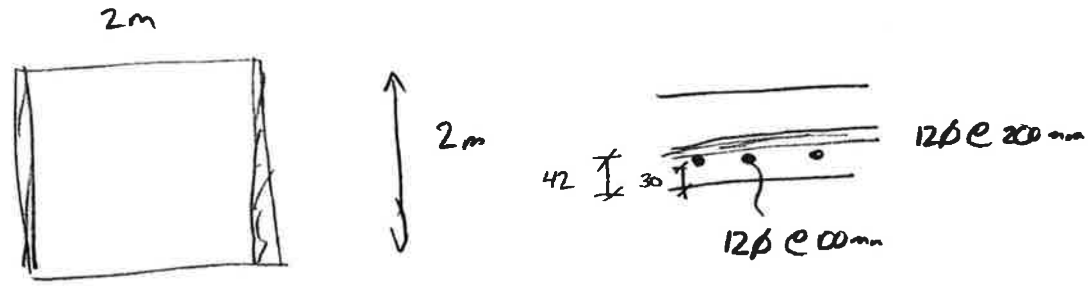
So option 1 is now the cheapest.

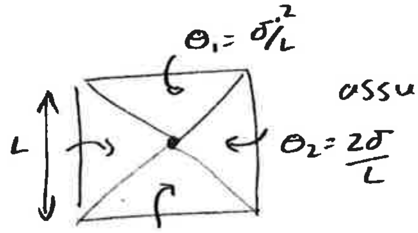
This shows the influence of traffic delay costs.

To include costs of traffic delay during routine maintenance an estimate would be needed of how long the road would need to be closed. Since maintenance should only take a few hours it should be possible to work at night

$\therefore$ No significant difference.

3 a)





assume disp under point load $= \delta$

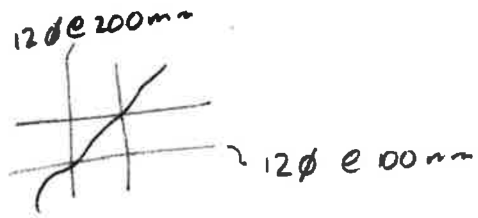
$W.D. = P \cdot \delta$ $\qquad \theta_1 = \theta_2 = \frac{2\delta}{L}$

$E.D = \left(\frac{L}{2} \cdot \theta_1 + \theta_2 \cdot \frac{L}{2}\right) \times 4\, m_p$

$= \frac{2\delta}{\cancel{L}} \cdot \cancel{L} \times 4 m_p = 8\delta m_p$

$W.D = E.D \quad \therefore \quad P = 8\, m_p$

all cracks cross reinforcement at 45^0



Find m_{px} for 12φ at 100 mm $\qquad A_s\,(12\phi) = \frac{\pi\, 12^2}{4} = 113\,mm^2$

$A_s f_y = 0.6 f_{cd} \cdot b \cdot x \quad \rightarrow$ use 1m width

$\frac{A_s f_y}{s} \times 1000 = 0.6 f_{cd} \times 1000 \times x$

$\frac{\pi \cdot 12^2}{4} \times \frac{400}{100} = 0.6 \times \frac{40}{1.5}\, x$

$x = 28.3\ mm \qquad d = 120\ mm$

$m_{px} = \frac{A_s f_y}{s} \times 1000 \times (120 - 28.3/2) =$

$= \frac{113 \times 400 \times 1000}{100} (120 - 28.3/2) = 47.88 \times 10^6\ Nmm$

or $47885\ \frac{Nmm}{mm}$

3 a) (contd) for 12Ø at 200mm

$$\frac{\pi\,12^2}{4} \times \frac{400}{200} = 0.6 \times \frac{40}{1.5} \times x$$

$d = 150 - 42 = 108\text{ mm}$

$x = 14.1\text{ mm}$

$$m_{py} = \frac{113 \times 400 \times 1000}{200}\,(108 - 14.1/2)$$

$= 22.811 \times 10^6\text{ Nmm}$

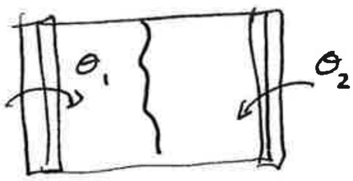
or $22811\ \frac{\text{Nmm}}{\text{mm}}$

$m_p = m_x \cos^2\alpha + m_y \cos^2\alpha$

$= 47885 \cdot 0.5 + 22811 \times 0.5 = 35348\ \frac{\text{Nmm}}{\text{mm}}$

$P = 8 \times 35348 = 282784\text{ N} = 283\text{ kN}$

3 b)



$\theta_1 = \theta_2 = \frac{2\delta}{L}$

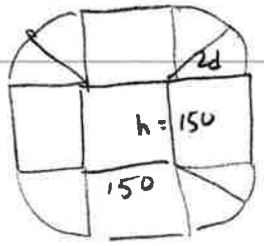
W.D. $= P.\delta$

E.D. $= L \cdot \left(\frac{2\delta}{L} + \frac{2\delta}{L}\right) m_{px} = 4\delta\, m_{px}$

W.D. = E.D $\therefore$ $P = 4 m_{px} = 4 \times 47885 = 191.5\text{ kN}$

c)

use ave d for slab : 150 - 36 = 114 mm



$\rho_1 = \frac{A_{sy}}{bd}$

$= \frac{113 \text{ mm}^2}{1000 \times 114}$

$= 0.00099$

$\rho_L = \sqrt{\rho_1 \rho_2}$

$= \sqrt{0.00099^2/2} = 0.0007$

perimeter $= 4h + 2\pi(2d)$

$= 4 \times 150 + 2\pi(2 \times 114) = 2033$ mm

$v_{Rd,c} = \frac{0.18}{\gamma_c} k (100 \rho_1 f_{cc})^{1/3} \geq v_{min}$

$k = 1 + \sqrt{200/d} \leq 2.0$

$= 2$

$= \frac{0.18}{1.5} \times 2 (100 \times 0.0007 \times 40)^{1/3}$

$= 1.57$ MPa

$v_{min} = 0.035 k^{3/2} f_{cc}^{1/2}$

$= 0.626$

$P_{max} = 1.57 \times 2033 \times 114 = 364$ kN > 191.5 kN

flexural collapse occurs first

3d) bookwork

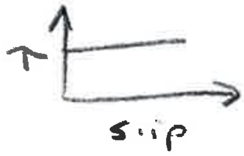
An axial force is beneficial for both failure modes. Membrane forces increase the capacity of thin slabs. An axial force also acts to reduce the crack widths in shear at a given load thereby increasing the shear capacity. However you would need to be sure you can rely on the axial force throughout the lifetime of the structure

Membrane forces associated with failure mode in Fig. 1

4 a) bookwork

The bond stress between the steel and the concrete causes tension to build up in the concrete. From equilibrium for the reinforcement $\frac{d\sigma_s}{dx} = \tau . 2\pi r$ so a high bond stress causes a more rapid build up of stress in the concrete. However once the fully developed crack pattern has formed, there is insufficient length over which sufficient tensile forces can be transferred to the concrete to cause cracking.

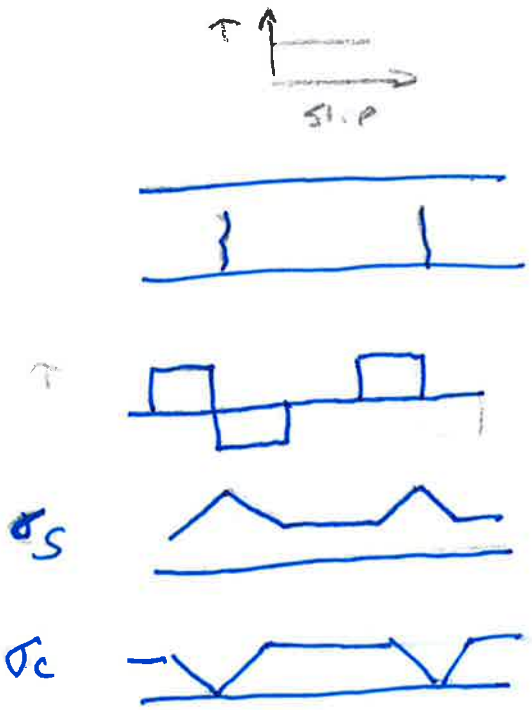
An example of a constant bond stress-slip relationship:



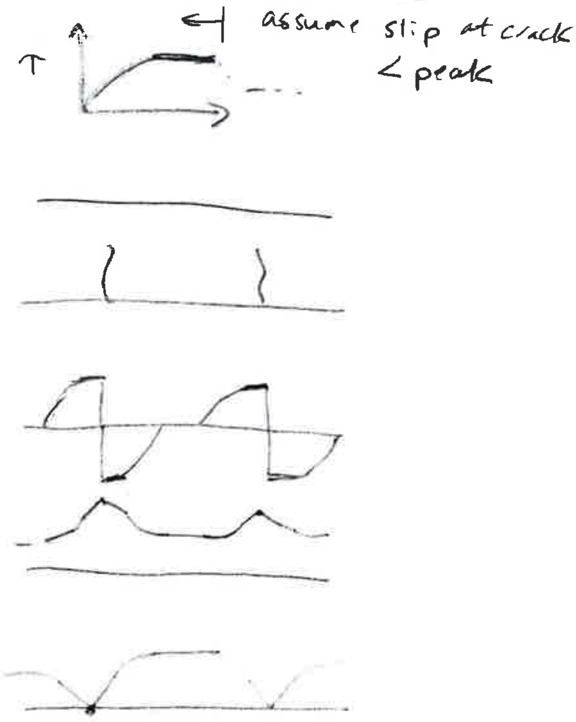
An example of a non-linear stress-slip relationship

The integration of the slip over the crack spacing leads to the crack width.

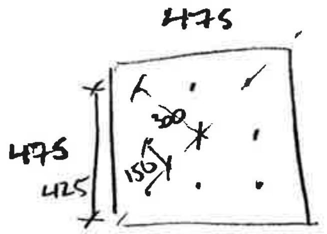
i)

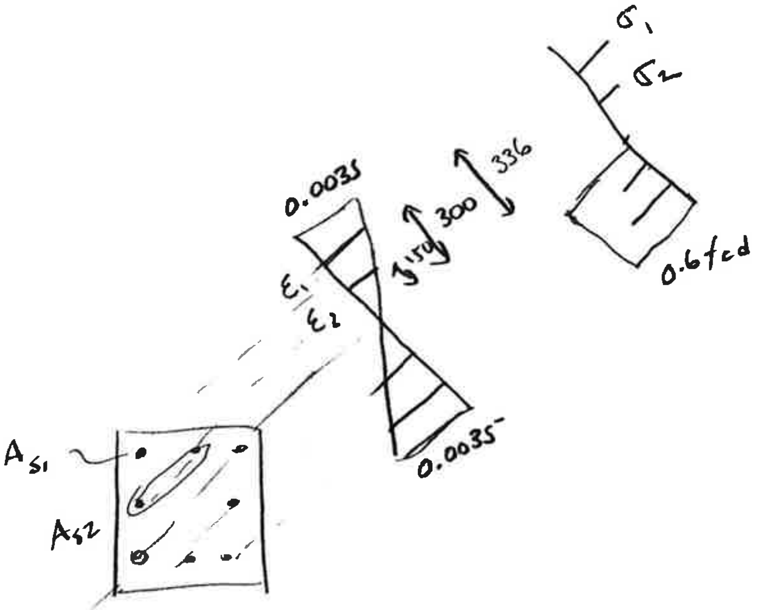


ii)



4b) i)





$$A_s = \frac{\pi \times 32^2}{4} = 804.2\ mm^2$$

$$\varepsilon_1 = \frac{0.0035}{336} \times 300 = 0.00313$$ ∴ steel has yielded

$$\varepsilon_2 = \frac{0.0035}{336} \times 150 = 0.00156$$ steel has <u>not</u> yielded

$$M = f_y \cdot A_{s1} \cdot 300 \times \underbrace{2}_{\text{tension + comp}} + \varepsilon_2 E_s A_{s2} \times 150 \times 2 + 0.6 \times 30 \times \underbrace{\frac{475 \times 475}{2}}_{A_c} \times \frac{1}{3} \times 336$$

$$= 400 \times 804.2 \times 300 \times 2 + 0.00156 \times 200000 \times 804.2 \times 2 \times 150 \times 2 + 0.6 \times 30 \times 475 \times \frac{336}{3} \times \frac{475}{2}$$

$$= 193.01 \times 10^6 + 150.55 \times 10^6 + 227.4 \times 10^6$$

$$= 570.96 \times 10^6\ N.mm$$

$$N = 0.6 \times 30 \times \frac{475^2}{2} = 2031\ kN$$

$$e = \frac{M}{N} = 281\ mm$$

4 b) ii)

$N = 2031\,kN$

$e_2 = \frac{l_0^2}{\pi^2} K_m$

$K_m = \frac{0.0035}{335.9} = 1.04 \times 10^{-5}/mm$

(estimate)

$e_{TOT} = 100mm + e_2 = 281\,mm$

$\therefore e_2 = 181\,mm$ for secondary effects

$l_0^2 = \frac{e_2 \pi^2}{K_m}$

$l_0 = \sqrt{\frac{e_2}{K_m} \pi^2} = \sqrt{\frac{181}{1.04 \times 10^{-5}} \pi^2}$

$= 13106\,mm$

$\approx 13\,m$

4D7 Examiner's comments to be attached to crib.

This paper falls into two halves and the candidates clearly favoured the more formulaic first half rather than the second half where they had to apply their knowledge. They were poor on the descriptive parts of the questions

Qu 1. The numerical part on characteristic and design loads was done well, with few errors, but the discussion part was less well done. There was little appreciation of the importance of quality control during manufacture (which the numerical part had shown them) and in the discussion relating to "are our structures too safe" only one of the candidates referred to the collapse in Dhaka, which had happened a couple of weeks before, which I had expected them to discuss at length.

Qu 2. Question relating to whole-life costing and to consider implications of factors such as whether or not to take account of traffic costs.

Qu 3. Yield line analysis of slabs. Done by only two candidates although probably the easier of the two concrete questions and fairly obviously done out of desperation.

Qu 4. The essay part was not done well - there seemed to be a tendency to think that any sketch of stress distribution would do whereas there actually was a point to asking how the stresses varied in the vicinity of a crack. There was some clear thinking in the numerical part although several candidates tried to work out response about two different axes, and very few checked the stresses in the different bars, some of which were not at yield. This was clearly the last question attempted by many candidates so the low mark reflects the shortage of time rather than lack of understanding. Two virtually perfect solutions from candidates who also did well in other questions so it could be done!

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May 2013