4D10 Structural Steelwork 2013.

Q1. a) $(\sigma_y - \sigma)(\sigma_E - \sigma) = \eta \sigma \sigma_E$

Insert $\sigma = \sigma_L = \chi \sigma_y$

$$(\sigma_y - \chi\sigma_y)(\sigma_E - \chi\sigma_y) = \eta \chi \sigma_y \sigma_E$$

Now $\lambda = \sqrt{\dfrac{\sigma_y}{\sigma_E}} \rightarrow \dfrac{\sigma_y}{\sigma_E} = \lambda^2$

$$\therefore \quad (1-\chi)(1-\chi\,\sigma_y/\sigma_E) = \eta\chi$$

$$(1-\chi)(1-\chi\lambda^2) = \eta\chi$$

$$1-\chi-\chi\lambda^2+\chi^2\lambda^2-\eta\chi = 0$$

$$1-\underbrace{(1+\eta+\lambda^2)}_{=2\Phi}\chi+\lambda^2\chi^2 = 0$$

$$1-2\Phi\chi+\lambda^2\chi^2 = 0.$$

Easiest if solve for $1/\chi$.

$$\frac{1}{\chi^2} - 2\Phi\left(\frac{1}{\chi}\right) + \lambda^2 = 0$$

$$\frac{1}{\chi} = \frac{2\Phi \pm \sqrt{4\Phi^2 - 4\lambda^2}}{2} = \Phi \pm \sqrt{\Phi^2-\lambda^2}$$

Lowest root (for χ) is largest for $1/\chi$, so take +ve sign

$$\therefore \quad \chi_{\substack{\text{lowest}\\\text{root}}} = \frac{1}{\Phi + \sqrt{\Phi^2-\lambda^2}}$$

QED

4D10. Q1 b).

The DS1 buckling curves are normalised versions of the elastic-plastic interaction diagrams, which for the buckling of columns, had a plastic plateau, and an elastic Euler curve falling away as $(1/\text{slenderness})^2$, where slenderness was say L/r. ie. it falls away as $1/L^2$, and

the x-axis is proportional to length (for axial buckling).

$$x \text{ axis} = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{(A\sigma_y)}{(\pi^2 EI/L^2)}} \propto \underline{\underline{L}}$$

Can use same curves for LTB, since the choice of normalised axes leads to a sort of self-fulfilling prophecy, that buckling ~~with~~ value y will fall away as $1/x^2$.

(Since y axis $= \dfrac{\text{Design}}{\text{Plastic}}$ and x axis $= \sqrt{\dfrac{\text{Plastic}}{\text{Elastic}}}$

then on the buckling part of the curve, Design = Elastic

so $y = \dfrac{\text{Elastic}}{\text{Plastic}} = \dfrac{1}{x^2}$ by definition!)

It means that the x-axis for LTB does not scale as length – it is a more general version of slenderness.

$$\Big(\cdot\ M_{LTB} = \frac{\text{Konst}}{L}\sqrt{\text{const}}\left(1 + \frac{\text{Const}}{L^2}\right)^{1/2}$$

NOT proportional to $1/\text{length}^2$).

4D10

Q1(c) $457 \times 152 \times 74$ UB

$$M_{plastic} = \sigma_y Z_{p,major}$$
$$= 275 \times 10^6 \text{ N/m}^2 \times 1627 \times 10^{-6} \text{m}^3$$
$$= \underline{\underline{447.4 \text{ kNm}}}$$

$$M_{LT} = \frac{\pi}{L}\sqrt{GJEI_y}\left[1 + \frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}\right]^{1/2}$$

$J = 65.9 \text{ cm}^4$

$I_y = 1047 \text{ cm}^4$

$G = 81$ GPa

$E = 210$ GPa

$$M_{basic} = \frac{\pi}{L}\sqrt{GJEI_y} = \frac{\pi}{6}\sqrt{\frac{81}{210} J . I_y} \times 210 \times 10^9$$
$$= \frac{\pi}{6}\sqrt{\frac{81}{210}(65.9 \times 1047)} \times 210 \times 10^9 \times 10^{-8}$$
$$= \frac{\pi}{6}(163.1) \times 2100 = \underline{\underline{179.4 \text{ kNm}}}$$

$\Gamma \simeq \frac{ID^2}{4}$ $D =$ dist between flange centroids
$= 462.0 - 17 = \underline{\underline{445 \text{ mm}}}$

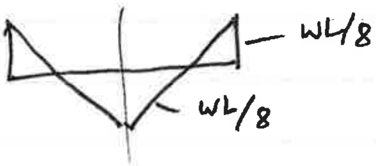
$$\frac{\Gamma}{J} = \frac{1047 \text{ cm}^4}{65.9 \text{ cm}^4}\frac{(0.445)^2 \text{m}^2}{4} = 0.7865 \text{ m}^2$$

$$\frac{\pi^2}{L^2}\frac{E\Gamma}{GJ} = \frac{(0.7865)\text{m}^2}{6^2 \text{ m}^2}\left(\frac{210}{81}\right)\pi^2 = 0.5591$$

$$\sqrt{1 + (\ \ )} = \sqrt{1 + 0.5591} = \underline{\underline{1.249}}$$

$$\therefore M_{LT} = 179.4 \text{ kNm} \times 1.249 = \underline{\underline{224 \text{ kNm}}}$$

Now, applied loads.



$\therefore \psi = -1$

$\therefore C_{unequal} = 0.4$

$$\therefore M_{cr} = \frac{M_{LT}}{0.4} = \underline{\underline{560 \text{ kNm}}}$$

Q1 (c) cont'd.

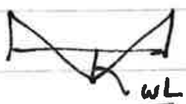
$$\lambda = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{447.4}{560}} = \underline{\underline{0.894}}$$

Which curve? $\underline{\text{Rolled}}$. $\frac{h}{b} = \frac{462}{154.4} = 2.99 > 2$

$\therefore$ $\underline{\text{curve b)}}$.

$\therefore$ From DS1, curve b), $\lambda = 0.894 \rightarrow \chi = 0.66$

$$\therefore \quad M_{Design} = (0.66)\, M_{plastic}$$
$$= (0.66)(447.4)$$
$$= \underline{\underline{295.3 \text{ kNm}}}$$

$M_{applied}$ =  $\frac{wL}{8}$ $= \frac{(100)(12)}{8} = \underline{\underline{150 \text{ kNm}}}$

$\therefore$ Beam is adequate, (+ comfortably so).

4D10 STRUCTURAL STEELWORK.

Q2. 305 x 305 x 158 UC. L_{eff} = 2.8m (major)
σ_y = 235 MPa. = 3.5m (minor) (+LTB)

$D = 327.1$ mm
$B = 311.2$ mm.
$t_{web} = 15.8$ mm
$t_{flange} = 25.0$ mm.

$I_{maj} = 38750 \text{ cm}^4$
$I_{min} = 12570 \text{ cm}^4$
$Z_{p\,major} = 2680 \text{ cm}^3$
$J = 378 \text{ cm}^4$
$A = 201 \text{ cm}^2$.

Axial effects.

$$P_{plastic} = \sigma_y A$$
$$= 235 \times 10^6 \text{ N/m}^2 \times 201 \times 10^{-4} \text{ m}^2$$
$$= \underline{\underline{4723 \text{ kN}}}$$

$$P_{E,\,major} = \frac{\pi^2 EI}{L_{eff}^2} = \frac{\pi^2 (210 \times 10^9)(38750 \times 10^{-8})}{(2.8)^2}$$
$$= \underline{\underline{102{,}441 \text{ kN}}}$$

$$P_{E,\,minor} = \frac{\pi^2 EI}{L_{eff}^2} = \frac{\pi^2 (210 \times 10^9)(12570 \times 10^{-8})}{(3.5)^2}$$
$$= \underline{\underline{21{,}267 \text{ kN}}}$$

$$\lambda_{major} = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{4723}{102 \times 10^3}} = 0.2147$$

$$\lambda_{minor} = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{4723}{21267}} = 0.4713$$

Curves? $\frac{h}{b} = \frac{327.1}{311.2} = 1.05 \leq \underline{1.2}$. and $t_f < 100$ mm (=25)

$\therefore$ Buckling curve **b)** for major, c) for minor.

Major: (DS1) $\chi = 0.99$ Minor, $\chi = 0.86$

$P_{Design\,(maj)} = \underline{\underline{4676 \text{ kN}}}$ $P_{Design\,(minor)} = 0.86(4723) = \underline{\underline{4062 \text{ kN}}}$

Q2. cont'd.

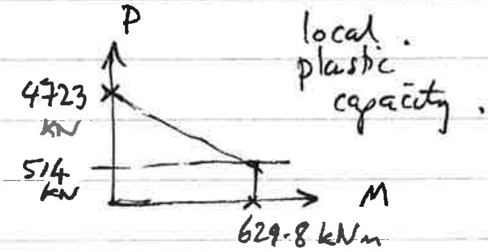
Moment effects. (Major axis applied only).

$$M_{plastic} = \sigma_y Z_p$$
$$= 235 \times 10^6 \ N/m^2 \ . \ 2680 \times 10^{-6} \ m^3$$
$$= \underline{\underline{629.8 \ kNm}}$$

Can now do plastic capacity envelope.

Half web factor $a/2$ $\qquad a =$ 

$$a = \frac{(15.8)mm \times (327.1 - 2(25.0))mm}{201 \times 10^2 \ mm^2}$$
$$= 0.2178$$
$$\therefore \underline{\underline{a/2 = 0.1089}}$$



$$\therefore \ a/2 \times P_{plastic} = 0.1089 \times (4723)$$
$$= \underline{\underline{514.4 \ kN}}$$

Now need LTB.

$$M_{basic} = \frac{\pi}{L}\sqrt{GJEI_{min}} = \frac{\pi}{3.5}(210 \times 10^9)\sqrt{\left(\frac{81}{210}\right) 378 \times 12570 \times 10^{-8}}$$
$$= 1885\sqrt{\quad} = 1885(1354) =$$
$$= \underline{\underline{2552 \ kNm}}$$

Warping term: $\Gamma \simeq \dfrac{ID^2}{4}$, $D =$ dist betw. flange centroids
$$= 327.1 - 25 = 302.1 \ mm$$

$$\frac{\Gamma}{J} = \frac{12\,570 \ cm^4}{378 \ cm^4}\ \frac{(0.3021)^2 \ m^2}{4} = 0.7587 \ m^2$$

4D10

Q2 cont'd.

$$\frac{\pi^2 E\Gamma}{L^2 GJ} = \frac{\pi^2}{(3.5)^2 m^2}\left(\frac{210}{81}\right)(0.7587)m^2$$
$$= 1.585.$$

$$\sqrt{1+(\;)} = \sqrt{1+1.585} = \underline{1.608}$$

$$\therefore\; M_{LT} = \underbrace{2552\,kNm}_{M_{basic}} \times 1.608 = \underline{\underline{4103}}\;kNm.$$

Equal + opposite end moments $\Rightarrow$ $\psi = 1$ $\Rightarrow$ $C_{unequal} = 1$.

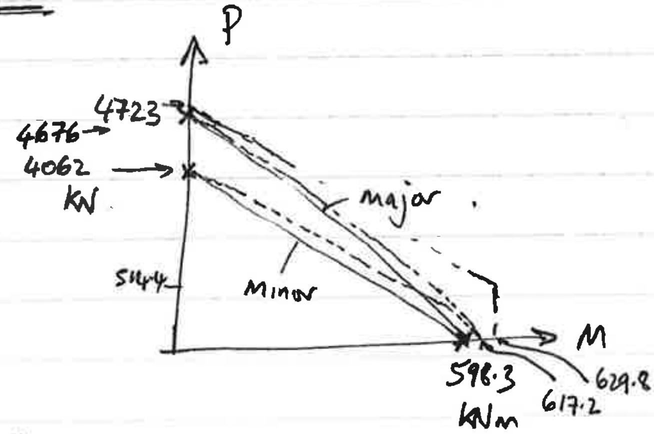
$$\lambda = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{629.8}{4103}} = 0.3918.$$

Which curve? Rolled. $\frac{h}{b} = \frac{327.1}{311.2} < 2$ $\therefore$ Curve a).

Curve a), $\lambda = 0.3918 \rightarrow \chi = 0.95$

$$\therefore\; M_{Design} = 0.95 \times 629.8$$
$$= \underline{\underline{598.3\;kNm}}$$

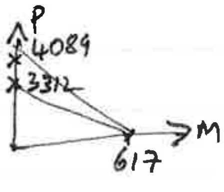
$\therefore$ Add lines.



b). Same direction end moments $\rightarrow$ $\psi = -1$ $\rightarrow$ $C_{unequal} = 0.4$

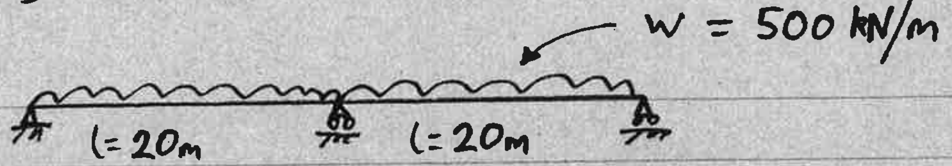
$\therefore\; M_{LTB} = \frac{4103}{0.4} = 10257.\,kNm.$ $\Rightarrow$ $\lambda = \sqrt{\frac{629.8}{10257}} = 0.25.$

$\rightarrow \chi = 0.98$ $\Rightarrow$ $M_{Design} = 0.98 \times 629.8 = 617.2\,kNm$

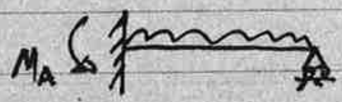


4D10, 2013

3(a)



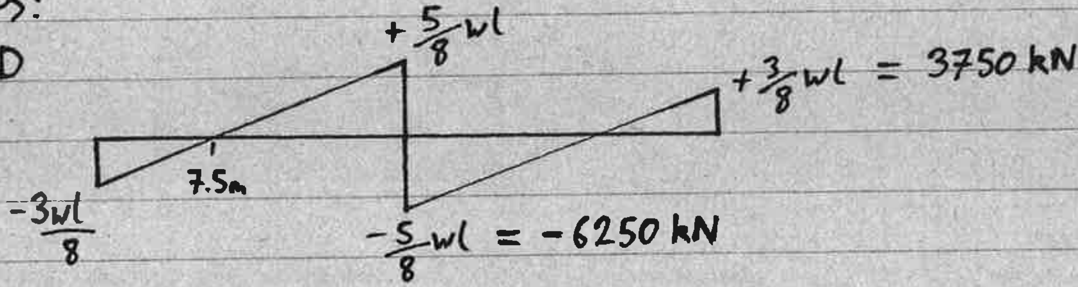
Data book p.7 4.5.3



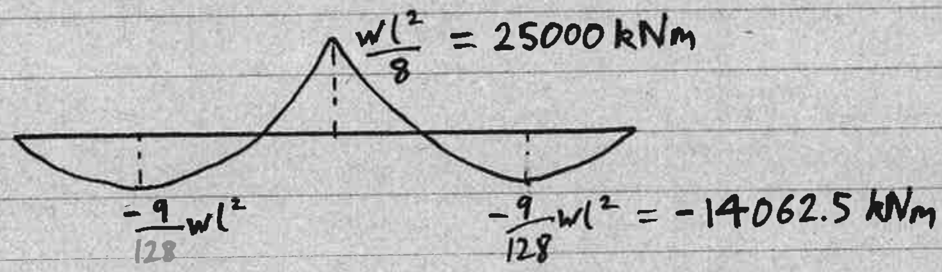
$M_A = \frac{wl^2}{8}$

Thus:

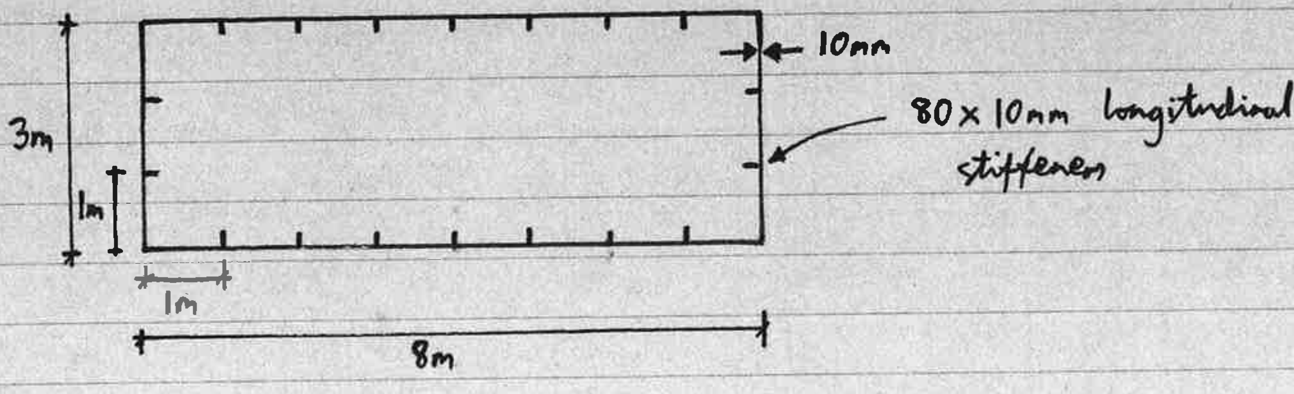
SFD



BMD



(b) Box girder cross-section. Cross-frames @ 2m centres



Check slenderness: Stiffeners $\lambda = \frac{80}{10}\sqrt{\frac{355}{355}} = 8 \leq 8$ ∴ compact

Flange & web plates $\lambda = \frac{1000}{10}\sqrt{\frac{355}{355}} = 100 > 24$
> 56

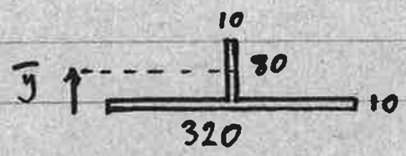
∴ non-compact in compression & bending

3) (b) (i) Most highly loaded longitudinal stiffeners are in the bottom flange of the cross-section on either side of the central support.

Effective width of flange: $b_e = K_c b$ (DS4) $\lambda = 100$

$= 0.32 \times 1000 = 320 \, mm$

Hence T-section:



$$320 \times 10 \times 5 + 10 \times 80 \times 50 = (320 \times 10 + 10 \times 80)\bar{y}$$

$$16000 + 4000 = \underbrace{4000}_{A_T} \bar{y} \qquad \therefore \bar{y} = 14 \, mm$$

$$I_T = \frac{10 \times 80^3}{12} + \frac{320 \times 10^3}{12} + 10 \times 80 \times 36^2 + 320 \times 10 \times 9^2$$

$$= (426.7 + 26.7 + 1036.8 + 259.2) \times 10^3$$

$$= 1.749 \times 10^6 \, mm^4$$

Radius of gyration $r = \sqrt{\frac{I_T}{A_T}} = \sqrt{\frac{1.749 \times 10^6}{4000}} = 20.91 \, mm$

Slenderness of stiffener as pin-ended column between cross-frames:

$$\bar{\lambda} = \sqrt{\frac{N_{pl}}{N_{el}}} = \sqrt{\frac{\sigma_y A_T}{\pi^2 E I_T / L_E^2}} = \frac{L_E}{r} \frac{1}{\pi} \sqrt{\frac{\sigma_y}{E}} = \frac{2000}{20.91} \frac{1}{\pi} \sqrt{\frac{355 \times 10^6}{210 \times 10^9}}$$

$$= 1.25$$

(DS2) Use curve 'c' of DS1

(DS1) $\bar{\lambda} = 1.25 \Rightarrow \chi = 0.42$

$\therefore$ Each stiffener can carry a max. compressive force of

$$0.42 \times A_T \sigma_y = 0.42 \times 4000 \times 355 = 596.4 \, kN$$

3) (b) (i) $^{(cont.)}$ 'Smear' stiffeners in flange and web to find equivalent thicknesses:

$$8000 \times 10 + 7 \times 10 \times 80 = 8000\, t_f \quad \therefore \quad t_f = 10.7\,\text{mm}$$

$$3000 \times 10 + 2 \times 10 \times 80 = 3000\, t_w \quad \therefore \quad t_w = 10.53\,\text{mm}$$

$$I \simeq 2\left[\frac{10.53 \times 3000^3}{12} + 8000 \times 10.7 \times 1500^2\right]$$

$$= (47.385 + 385.2) \times 10^9 = 432.6 \times 10^9\,\text{mm}^4$$

$$= 0.4326\,\text{m}^4$$

Compressive stress in bottom flange @ central support:

$$\sigma_{max} = \frac{M}{I} y_{max} = \frac{25000 \times 10^3}{0.4326} \times 1.5 = 86.7\ \text{MPa}$$

Thus total force in compression flange

$$\simeq 86.7 \times 8000 \times 10.53 = 7304\ \text{kN}$$

i.e. 913 kN per stiffener (1m spacing × 8)

$\therefore$ Stiffeners are very overloaded $913 > 596.4$ kN

<u>NOT OK</u>

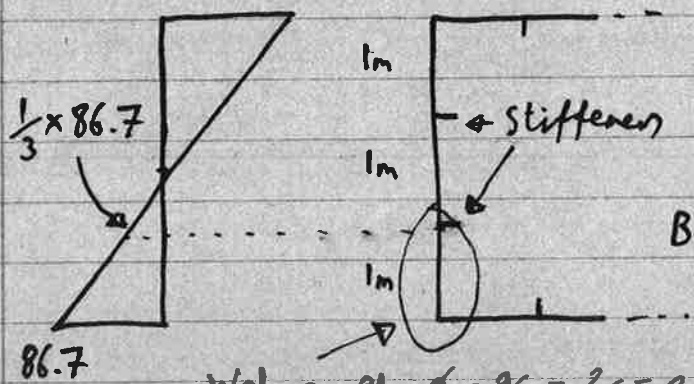
(b) (ii) Most highly loaded panels are in bottom flange and the lowest web panels either side of the central support.

Flange panel: Strength: $\dfrac{\sigma_c}{\sigma_y} = \dfrac{86.7}{355} < 1 \;\therefore$ OK

Stability: $\dfrac{\sigma_c}{\sigma_{cc}} = \dfrac{\sigma_c}{K_c \sigma_y} = \dfrac{86.7}{0.32 \times 355} = \dfrac{86.7}{113.6} = 0.76 < 1$

$\therefore$ OK

3) (b) (iii) Web panel



Web panel $\phi = a/b = 2/1 = 2$

Mean stress : $\sigma_c = \frac{2}{3} \times 86.7 = 57.8$ MPa

Bending stress : $\sigma_b = \frac{1}{3} \times 86.7 = 28.9$ MPa

Shear stress in web $\quad \tau = \dfrac{6250 \times 10^3}{2 \times 3000 \times 10} = 104$ MPa

Strength : $\left(\dfrac{\sigma}{\sigma_y}\right)^2 + \left(\dfrac{\tau}{\tau_y}\right)^2 = \left(\dfrac{86.7}{355}\right)^2 + \left(\dfrac{104}{205}\right)^2 = 0.32 < 1 \; \therefore$ OK

$\dfrac{\sigma_y}{\sqrt{3}} = 355/\sqrt{3} = 205$ MPa

Stability : $\dfrac{\sigma_c}{\sigma_{cc}} + \left(\dfrac{\sigma_b}{\sigma_{bc}}\right)^2 + \left(\dfrac{\tau}{\tau_c}\right)^2 = \dfrac{57.8}{113.6} + \left(\dfrac{28.9}{365}\right)^2 + \left(\dfrac{104}{127}\right)^2$

$= 0.51 + 0.006 + 0.67 = 1.19 > 1$

$\therefore$ **NOT OK**

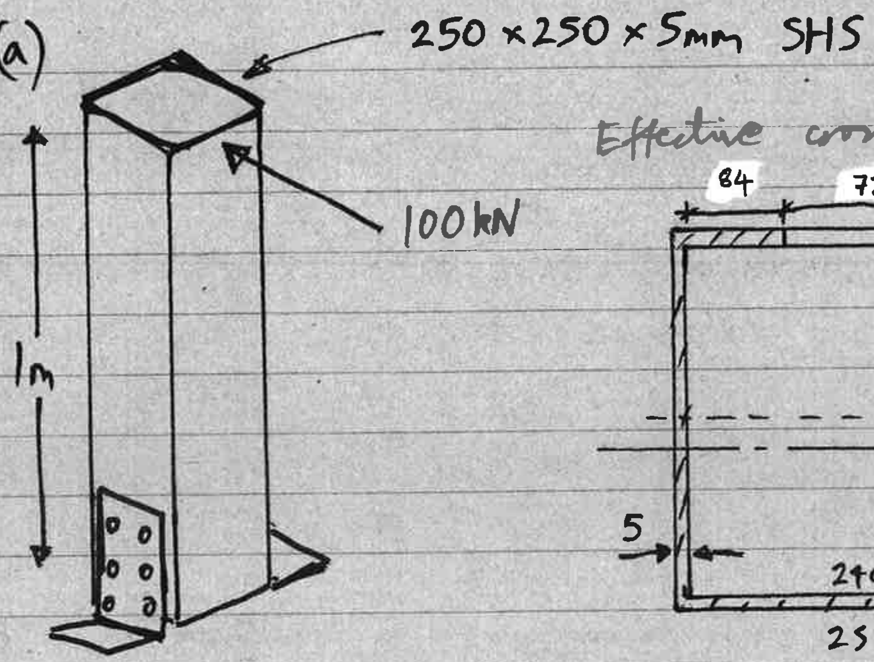
where $\sigma_{cc} = K_c \sigma_y = 0.32 \times 355 = 113.6$ MPa

$\sigma_{bc} = K_{bc} \sigma_y = 1.03 \times 355 = 365$ MPa

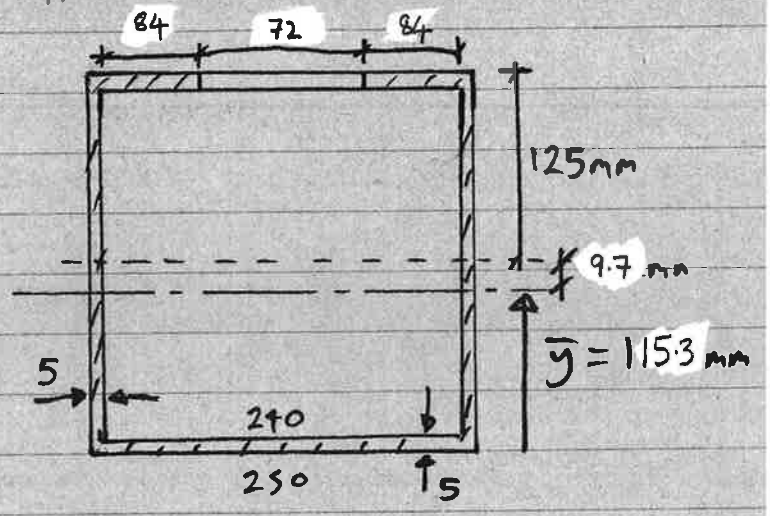
$\tau_c = K_q \tau_y = 0.62 \times 205 = 127$ MPa $\quad$ (DS4 p.6 $\phi = 2$)

(c) Reducing the cross frame spacing from 2m to, say, 1m in a region close to the central support will reduce the length L_E and prevent overloading of the longitudinal stiffeners. This will also improve the aspect ratio of the web panel from $\phi = 2$ to $\phi = 1$ with a corresponding increase in K_q. However τ is still too high so could also increase the web thickness to, say, 12mm.

4) (a)



Effective cross-section:



Slenderness of flange & web of cross-section:

$$\lambda = \frac{240}{5}\sqrt{\frac{235}{355}} = 48 \times 0.814 = 39.05 > 24 \text{ but } < 56$$

∴ NON-COMPACT in compression but OK in bending (webs)

DS4 page 5 $\quad \lambda = 39.05 \Rightarrow K_c = 0.7 \quad \therefore \frac{b_e}{2} = \frac{0.7 \times 240}{2} = 84$ mm

Neutral axis (elastic):

$$240 \times 5 \times 2.5 + 2 \times 5 \times 250 \times 125 + 2 \times 84 \times 5 \times 247.5 = \bar{y}(240 \times 5 + 2 \times 5 \times 250 + 2 \times 84 \times 5)$$

$$\therefore \bar{y} = \frac{523\,400}{4540} = 115.3 \text{ mm}$$

A_{eff}

$$I_{eff} \simeq 2 \times \left(\frac{5 \times 250^3}{12} + 5 \times 250 \times 9.7^2\right) + 240 \times 5 \times (115.3 - 2.5)^2 + 2 \times 66 \times 5 \times (125 + 9.7 - 2.5)^2$$

$$= (13.256 + 15.269 + 11.535) \times 10^6$$

$$= 40.06 \times 10^6 \text{ mm}^4$$

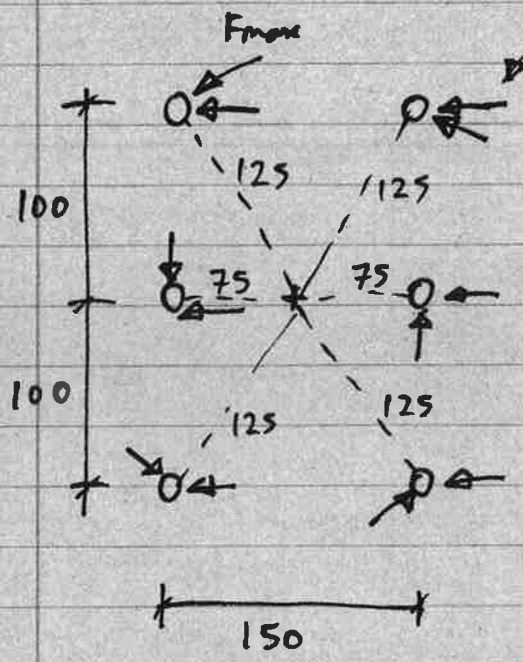
Max. allowed moment is 1st yield

$$M_y = \frac{\sigma_y I_{eff}}{y_{max}} = \frac{235 \times 40.06 \times 10^6}{125 + 9.7} = 69.9 \text{ kNm} < 100 \text{ kNm}$$

∴ NOT OK

4)(b) Bolt group

Shear divided by 2 × 6 bolts $= \frac{100}{12} = 8.33\text{ kN}$

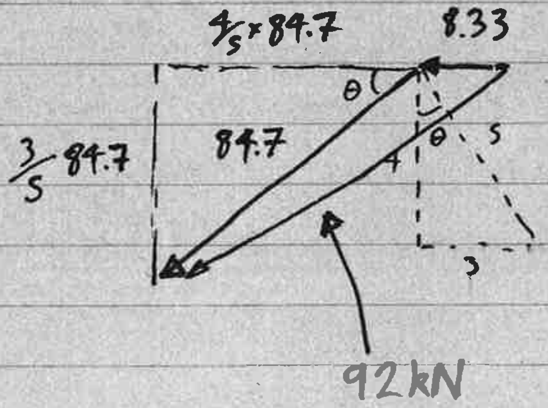


Moment divided between 2 bolt groups

$= \frac{100}{2} = 50\text{ kNm}$

$$F_{max} = \frac{50}{\sum r_i^2 / r_{max}} = \frac{50 \times 125}{4 \times 125^2 + 2 \times 75^2}$$

$$= 84.7\text{ kN}$$



Need an M36 grade 4.6 bolt (DS5)

(c) Check spacing between bolt axes (100mm):

$100 \leq 32 \times (5 + ?)$ and $100 \geq 2.5 \times 36 = 90$ ∴ OK

angle cleat thickness

End distance (below lowest bolts) must be > 90mm

Check bearing stress in SHS:

$$\frac{92 \times 10^3}{36 \times 5} = 511\text{ MPa} > 235\text{ MPa} \quad \therefore \text{ NOT OK}$$

∴ Increase wall thickness to, say, 7mm.