

Q1

- (a) Reflectors improve neutron economy by reflecting back into the core some of the neutrons that would otherwise be lost through leakage. This reduces the critical size or the enrichment required to achieve criticality, and also results in a flatter flux profile with lower form factors.

A reflector material needs a high scattering cross-section and a low capture cross-section – the same characteristics as a moderator. [15%]

(b)
$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{\partial^2 \phi}{\partial z^2} + B^2 \phi = 0$$

Assume that $\phi(r, z) = F(r)Z(z)$

$$\therefore \frac{Z}{r} \frac{\partial}{\partial r} \left(r \frac{\partial F}{\partial r} \right) + F \frac{\partial^2 Z}{\partial z^2} + B^2 FZ = 0$$

So, dividing through by FZ

$$\frac{1}{Fr} \frac{\partial}{\partial r} \left(r \frac{\partial F}{\partial r} \right) + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} + B^2 = 0$$

This implies that

$$\frac{1}{Fr} \frac{d}{dr} \left(r \frac{dF}{dr} \right) + \alpha^2 = 0 \quad (1.1)$$

and

$$\frac{1}{Z} \frac{d^2 Z}{dz^2} + \beta^2 = 0 \quad (1.2)$$

with

$$\alpha^2 + \beta^2 = B^2$$

Equation (1.2) is a Simple Harmonic Motion (SHM) equation, so its general solution is

$$Z(z) = A \sin(\beta z) + C \cos(\beta z)$$

From symmetry considerations, $A = 0$, thus

$$Z(z) = C \cos(\beta z)$$

The general solution of equation (1.1) is

$$F(r) = PJ_0(\alpha r) + QY_0(\alpha r)$$

where P and Q are arbitrary constants and J_0 and Y_0 are ordinary Bessel functions of zero order.

As $r \rightarrow 0$, $Y_0(\alpha r) \rightarrow -\infty$. This would give infinite flux at the centre of the reactor, which is physically impossible, so $Q = 0$. Thus

$$F(r) = PJ_0(\alpha r)$$

and combining these results

$$\phi(r, z) = F(r)Z(z) = \phi_0 J_0(\alpha r) \cos(\beta z)$$

where $\phi_0 = PC$

[45%]

(c) The general solution of the PDE follows the same approach as in (b), leading to

$$\frac{1}{Fr} \frac{\partial}{\partial r} \left(r \frac{\partial F}{\partial r} \right) + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} - \frac{1}{L^2} = 0$$

Hence
$$\frac{1}{Fr} \frac{d}{dr} \left(r \frac{dF}{dr} \right) + \alpha'^2 = 0$$

and
$$\frac{1}{Z} \frac{d^2 Z}{dz^2} + \beta'^2 = 0$$

with
$$\alpha'^2 + \beta'^2 = -\frac{1}{L^2}$$

If $\alpha' = \alpha$, β'^2 must be negative, i.e. $\beta'^2 = -\gamma^2$, and

$$\gamma^2 = \alpha^2 + \frac{1}{L^2}$$

The equation for z is then
$$\frac{1}{Z} \frac{d^2 Z}{dz^2} - \gamma^2 = 0$$

which has a general solution

$$Z(z) = A' \exp(\gamma z) + C' \exp(-\gamma z) \quad [15\%]$$

(d) In the core
$$\phi(r, z) = \phi_0 J_0(\alpha r) \cos(\beta z)$$

In the reflector
$$\phi(r, z) = \phi_1 J_0(\alpha r) \exp(-\gamma z)$$

So, in the core
$$\frac{\partial \phi}{\partial z} = -\beta \phi_0 J_0(\alpha r) \sin(\beta z)$$

and in the reflector
$$\frac{\partial \phi}{\partial z} = -\gamma \phi_1 J_0(\alpha r) \exp(-\gamma z)$$

Flux continuity gives at the core-reflector interface gives

$$\phi_0 \cos\left(\frac{\beta H}{2}\right) = \phi_1 \exp\left(-\frac{\gamma H}{2}\right) \quad (1.3)$$

Current continuity gives

$$\beta \phi_0 D_c \sin\left(\frac{\beta H}{2}\right) = \gamma \phi_1 D_r \exp\left(-\frac{\gamma H}{2}\right) \quad (1.4)$$

Dividing equation (1.4) by equation (1.3) gives

$$\beta D_c \tan\left(\frac{\beta H}{2}\right) = \gamma D_r$$

$$\therefore \gamma = \frac{D_c}{D_r} \beta \tan\left(\frac{\beta H}{2}\right) \quad [25\%]$$

Principal Assessor's Comments:

A very popular question attempted by 84% of candidates, and done very well by many of them. The only common error was that several candidates wasted time finding expressions for α and β in part

(b), even though this was not required.

Q2

$$(a) \quad \theta = \sin\left(\frac{\pi x}{2L'}\right) + Q \cos\left(\frac{\pi x}{2L'}\right)$$

θ is a maximum when $\frac{d\theta}{dx} = 0$

$$\therefore \frac{\pi}{2L'} \cos\left(\frac{\pi x}{2L'}\right) - \frac{\pi Q}{2L'} \sin\left(\frac{\pi x}{2L'}\right) = 0$$

$$\therefore Q \sin\left(\frac{\pi x}{2L'}\right) = \cos\left(\frac{\pi x}{2L'}\right)$$

$$\therefore \tan\left(\frac{\pi x}{2L'}\right) = \frac{1}{Q} \Rightarrow x = \frac{2L'}{\pi} \tan^{-1}\left(\frac{1}{Q}\right)$$

$$\text{Let } \tan^{-1}\left(\frac{1}{Q}\right) = \frac{\pi x}{2L'} = \phi$$

$$\therefore \theta_{\max} = \sin(\phi) + Q \cos(\phi)$$

$$\therefore \frac{\theta_{\max}}{\cos(\phi)} = \tan(\phi) + Q = \frac{1}{Q} + Q = \frac{1+Q^2}{Q}$$

$$\therefore \frac{\theta_{\max}^2}{\cos^2(\phi)} = \frac{(1+Q^2)^2}{Q^2}$$

Now $\sin^2(\phi) + \cos^2(\phi) = 1$

$$\therefore \frac{1}{\cos^2(\phi)} = 1 + \tan^2(\phi) = 1 + \frac{1}{Q^2} = \frac{1+Q^2}{Q^2}$$

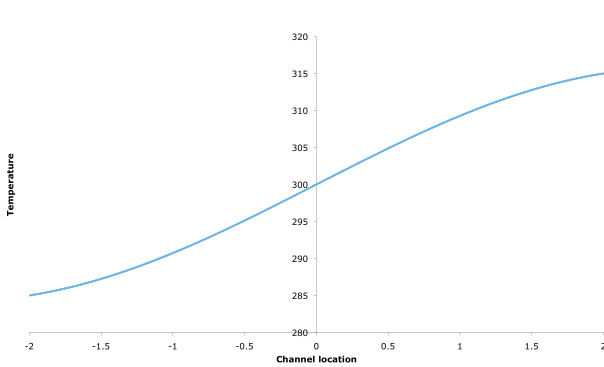
$$\therefore \theta_{\max}^2 \left(\frac{1+Q^2}{Q^2}\right) = \frac{(1+Q^2)^2}{Q^2}$$

$$\therefore \theta_{\max}^2 = 1+Q^2$$

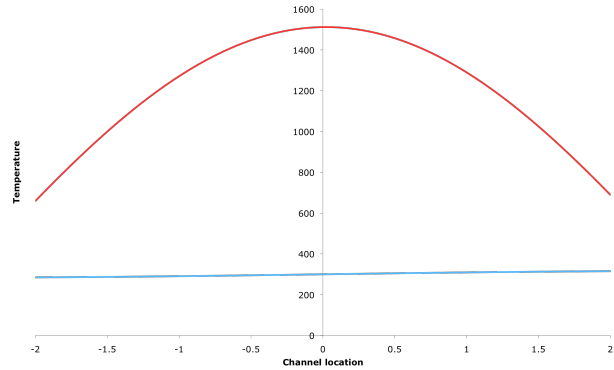
[35%]

(b)(i)

[10%]



Coolant



Coolant & Fuel

(b)(ii)

$$\dot{m}c_p [T_{co} - T_{ci}] = q \text{ where } q \text{ is the pin power}$$

$$\text{From the data in the question, here } q = \frac{32 \times 10^6}{264} = 121.2 \text{ kW}$$

$$\text{and } [T_{co} - T_{ci}] = 315 - 285 = 30 \text{ }^\circ\text{C (or K)}$$

$$\therefore \dot{m}c_p = \frac{q}{[T_{co} - T_{ci}]} = \frac{121.2 \times 10^3}{30} = 4.04 \text{ kWK}^{-1}$$

[10%]

(b)(iii)

$$\text{From the 4M16 data sheet, for Ginn's equation } Q = \frac{\pi \dot{m}c_p L}{UA L'}$$

$$\text{Using } A = 4\pi r_o L \quad \therefore Q = \frac{\pi \dot{m}c_p L}{U4\pi r_o L L'} = \frac{\dot{m}c_p}{U4r_o L'}$$

Here $L' = 2.5 \text{ m}$ (given) and $L = 2.0 \text{ m}$ (half the active fuel length)

$$r_o = r_i + t_c = \frac{9.3}{2} + 0.6 = 5.25 \text{ mm}$$

$$\text{From the 4M16 data sheet: } \frac{1}{U} = \frac{1}{h} + \frac{t_c}{\lambda_c} + \frac{r_o}{h_b r_i} + \frac{r_o}{2\lambda_f} \left(1 - \frac{r^2}{r_i^2} \right)$$

The maximum fuel temperature will be on the centre-line of the fuel pin (at $r = 0$).

$$\therefore \frac{1}{U} = \frac{1}{h} + \frac{t_c}{\lambda_c} + \frac{r_o}{h_b r_i} + \frac{r_o}{2\lambda_f}$$

$$\therefore \frac{1}{U} = \frac{1}{35 \times 10^3} + \frac{6 \times 10^{-4}}{12} + \frac{5.25 \times 10^{-3}}{25 \times 10^3 \times 4.65 \times 10^{-3}} + \frac{5.25 \times 10^{-3}}{2 \times 3} = 9.987 \times 10^{-4} \text{ m}^2\text{KW}^{-1}$$

$$\therefore Q = \frac{\dot{m}c_p}{U4r_o L'} = 9.987 \times 10^{-4} \times \frac{4.04 \times 10^3}{4 \times 5.25 \times 10^{-3} \times 2.5} = 76.85$$

$$\therefore \theta_{\max}^2 = 1 + Q^2 = 1 + 76.85^2 \Rightarrow \theta_{\max} = 76.86$$

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) \Rightarrow T = T_{c1/2} + \frac{(T_{co} - T_{c1/2})}{\sin\left(\frac{\pi L}{2L'}\right)} \theta$$

$$\text{Here } T_{c1/2} = \frac{1}{2}(T_{co} + T_{ci}) = \frac{1}{2}(315 + 285) = 300 \text{ } ^\circ\text{C}$$

$$\therefore T_{\max} = T_{c1/2} + \frac{(T_{co} - T_{c1/2})}{\sin\left(\frac{\pi L}{2L'}\right)} \theta_{\max} = 300 + \frac{(315 - 300)}{\sin\left(\frac{\pi \times 2}{2 \times 2.5}\right)} \times 76.86 = 1512 \text{ } ^\circ\text{C}$$

$$\text{This occurs at } x = \frac{2L'}{\pi} \tan^{-1}\left(\frac{1}{Q}\right) = \frac{2 \times 2.5}{\pi} \tan^{-1}\left(\frac{1}{76.85}\right) = 0.0207 \text{ m, i.e. just past half way [45\%]}$$

Principal Assessor's Comments:

Another very popular question attempted by 85% of candidates, and done very well by many of them. Common mistakes were: physically unrealistic temperature variation sketches in part (b)(i); omission of the $\sin(\pi L/2L')$ term in the formula for the dimensionless temperature θ ; calculating location of the maximum temperature using an argument in degrees rather than radians; and belief that the correct value for the maximum temperature (1512 °C) must be wrong (even though the melting point of UO_2 is 2865 °C).

Q3

(a) β is the fraction of neutrons that are delayed

λ is the precursor decay constant

Λ is the prompt neutron reproduction time

The major simplifications of this model are that it assumes there is no spatial variation in behaviour, whereas in practice the reactor core is highly heterogeneous and the neutron population varies spatially, and it also assumes that there is only one type of precursor, whereas in reality there are a large number of them with widely varying production rates and half-lives. [20%]

(b) In steady-state operation $\frac{dc}{dt} = 0$

$$\therefore \frac{\beta}{\Lambda} n_0 = \lambda c_0 \Rightarrow \frac{c_0}{n_0} = \frac{\beta}{\lambda \Lambda} = \frac{0.007}{0.1 \times 10^{-4}} = 700 \quad [10\%]$$

(c) The prompt jump approximation assumes that the neutron population remains in equilibrium with the precursor population, even when the latter is varying with time. Thus, for a source-free system:

$$\frac{dn}{dt} \approx 0 = \frac{\rho - \beta}{\Lambda} n + \lambda c \Rightarrow n = \frac{\Lambda \lambda c}{\beta - \rho} \quad (3.1)$$

Using this expression to substitute for n in the precursor equation gives:

$$\frac{dc}{dt} = \frac{\beta \lambda c}{\beta - \rho} - \lambda c = \frac{\rho \lambda}{\beta - \rho} c$$

which, by inspection, has a solution:

$$c = c_0 \exp\left(\frac{\rho\lambda}{\beta - \rho} t\right) \quad (3.2)$$

if $c = c_0$ at $t = 0$.

But, if the system was in equilibrium with $n = n_0$ and $\rho = 0$ before the change in ρ , then from part (b):

$$\beta n_0 = \Lambda \lambda c_0 \quad (3.3)$$

So, combining equations (3.1), (3.2) and (3.3), the variation in the neutron population after the change in ρ is given by:

$$n = \frac{\Lambda \lambda c}{\beta - \rho} = \frac{\Lambda \lambda}{\beta - \rho} c_0 \exp\left(\frac{\rho\lambda}{\beta - \rho} t\right) = \frac{\beta}{\beta - \rho} n_0 \exp\left(\frac{\rho\lambda}{\beta - \rho} t\right) \quad [40\%]$$

- (d) The dominant time constant $T = \frac{\beta - \rho}{\rho\lambda}$

$$\therefore T = \frac{0.007 - 0.003}{0.003 \times 0.1} = 13.3 \text{ s}$$

Delayed neutrons play a vital role in determining the dominant time constant T and thus the controllability of a reactor. Delayed neutrons will be present for both the AGR and the FBR. If it is assumed that the values of λ and β are the same as for the PWR – this is a reasonable assumption for the AGR as it is a thermal reactor like the PWR, but less good for the FBR where fission conditions are significantly different due to the fast neutron spectrum – then the value of T will not vary greatly between reactors – indeed according to the prompt jump approximation model $\left(T = \frac{\beta - \rho}{\rho\lambda}\right)$ it will be identical.

In practice, T will vary slightly due to variations in Λ , the prompt neutron reproduction time. The value of Λ depends on the effectiveness of the moderator.

In the AGR moderation is less effective than in a PWR and therefore Λ is longer (by about an order of magnitude) and in consequence T will be slightly longer than in the PWR.

In the FBR there is no moderator (by design) and therefore Λ is much (three orders of magnitude) shorter than in a PWR. This will result in T being slightly shorter than in the PWR. [30%]

Principal Assessor's Comments:

A popular question attempted by 76% of candidates. The quantitative/analytical parts were done very well in general, but the physical explanations and reasoning were less good, particularly in part (d). A few candidates did not use the result in part (c) to do part (d) but instead attempted to find the dominant time constant predicted by the exact in-hour equation, with varying degrees of success.

Q4

- (a) Hold-up and decay is the simplest and cheapest method of liquid and gaseous and solid waste treatment for short half-life nuclides. It can also be used for some medium-term half-life nuclides if the quantities are low. It simply involves storing the waste for sufficient time for the activity to decay to a safe level before discharge to the environment. At least two tanks/containers are normally needed, one filling and one holding the material for the required period of time. In the case of gases charcoal absorber beds can be used as an alternative

method of hold-up. Care may be needed with some daughter products as they can be more radioactive than the parent. Tanks must be shielded but otherwise operator dose uptake is low and little maintenance is required.

Ion exchange using either organic or inorganic media is very useful for some longer-lived nuclides in liquid waste. In this process the active ions such as cobalt-60 are exchanged for non-active ions such as sodium or g hydrogen in a process very similar to that used in water treatment. Unlike conventional ion exchange, the media is not regenerated when saturated but is usually encapsulated for long-term disposal. Decontamination factors of between 10 and 100 are possible with some nuclides but not all are suited to this form of treatment. It is relatively cheap and simple, the only problem being the long-term disposal of the spent media. Operator dose is again low as long as the vessels are shielded though the disposal of the spent media can give rise to higher doses. It is not effective on gases.

The most intractable, usually long half-life, liquid wastes may be treated by evaporation which produces a pure distillate that can be discharged to the environment and a highly active concentrate that can be immobilised by encapsulation or vitrification. It is very expensive in capital costs and also very energy-intensive and can give rise to high operator dose rates due to the need for maintenance. It is normally only used for very difficult materials, such as reprocessing wastes, or in locations where no radioactive discharge to the environment is permitted.

Solid wastes arise in a variety of forms and the need in all cases is to prevent the migration of the radioactive nuclides into the environment. For low-level waste all that is needed is to landfill them in a sealed pit isolated from the environment and monitor the run-off to ensure no contamination. This is not suitable for alpha or other long-lived nuclides. Medium active waste is usually encapsulated in cement and sealed in stainless steel drums for eventual storage in an underground repository. High-level wastes, such as fission products, are first evaporated to reduce volume then converted into glass for long-term storage. Cost and operator dose uptake will depend on the activity of the wastes and the complexity of the equipment some of which can require quite a bit of maintenance and hence operator dose. [60%]

(b) Flow rate $Q = 0.1 \text{ m}^3 \text{ hr}^{-1}$

Initial activity $A = 20 \text{ Bq g}^{-1}$

Tank capacity $V = 48 \text{ m}^3$

\therefore Collection time $T = 48 \div 0.1 = 480 \text{ hrs}$

Hold-up time $t = 25 \text{ days} = 25 \times 24 = 600 \text{ hrs}$

Assume effluent density $\rho = 1000 \text{ kg m}^{-3}$

Ag-110m half-life = 252 days = $252 \times 24 = 6048 \text{ hrs}$

$\therefore \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{6048} = 1.146 \times 10^{-4} \text{ hr}^{-1}$

Ag-110m rate of arising: $P = \frac{QA\rho \times 3600 \times 1000}{\lambda}$

$\therefore P = \frac{0.1 \times 20 \times 1000 \times 3600 \times 1000}{1.146 \times 10^{-4}} = 6.283 \times 10^{13} \text{ atoms hr}^{-1}$

Decay of Ag-110m:
$$N = \frac{P}{\lambda} (1 - \exp(-\lambda T)) \exp(-\lambda t)$$

$$\therefore N = \frac{6.283 \times 10^{13}}{1.146 \times 10^{-4}} (1 - \exp(-1.146 \times 10^{-4} \times 480)) \exp(-1.146 \times 10^{-4} \times 600)$$

$$\therefore N = 2.739 \times 10^{16} \text{ atoms}$$

The specific activity of the effluent is given by:

$$A_e = \frac{N\lambda}{\rho V \times 3600 \times 1000}$$

$$\therefore A_e = \frac{2.739 \times 10^{16} \times 1.146 \times 10^{-4}}{1000 \times 48 \times 3600 \times 1000} = 18.16 \text{ Bq g}^{-1}$$

After ion exchange with a decontamination factor of 10:

$$\therefore A_f = 1.816 \text{ Bq g}^{-1}$$

This shows that little is gained by hold-up and decay due to the relatively long half-life of Ag-110m but that ion exchange is effective. [40%]

Principal Assessor's Comments:

The least popular question, but nevertheless attempted by 57% of candidates. Most candidates gave reasonable answers to the descriptive part (a), although many answers did not properly address the issue of the applicability of the treatments to different waste forms (gas, liquid, solid) or wastes of different half-lives. Attempts at the quantitative part (b) were more variable in quality. Many candidates did not appreciate the fact that the long half-life of Ag-110m means that hold-up and decay would not be a very effective treatment, and therefore did not recognise that their answers which showed it was effective must be wrong.