ENGINEERING TRIPOS PART IIB

Tuesday 7 May 2013 2 to 3.30

Module 4A10

FLOW INSTABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4A10 data sheet (two pages);
Copy of figure 4.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

mpj05

1 Consider a two-dimensional inviscid mixing layer with a flow profile as shown in figure 1. The flow velocity in the x direction is U_1 for $z > 0$ and U_2 for $z < 0$. Investigate the temporal stability of this flow by considering small disturbances to the interface of the vortex sheet of the form $z = \eta(x,t) = \eta_0 \exp(st) \exp(\mathrm{i}kx)$.

(a) Show that the dispersion relationship is given by

$$s = -\frac{1}{2}\mathrm{i}k(U_1 + U_2) \pm \frac{1}{2}k(U_1 - U_2)$$

[80%]

(b) Determine the phase speed of the disturbance. [10%]

(c) Comment on the stability of the system. [10%]

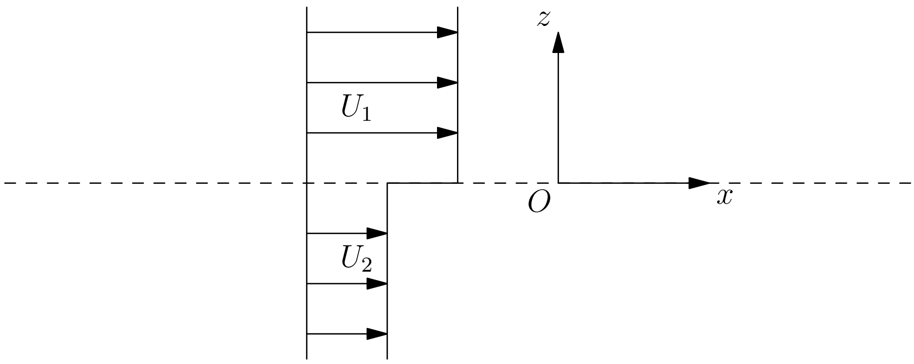

Fig. 1

2 For each of the flows (i)–(iv) listed at the end of this question,

(a) explain the physical mechanism that can lead to instability,

(b) state the stabilising influences,

(c) derive an appropriate nondimensional number to describe the susceptibility to instability,

(d) and describe the flow pattern just after the onset of instability.

The flows are:

(i) a boundary layer flow over a concave wall [25%]

(ii) a shear flow with velocity profile $U(z) = U_0 \tanh(z/\delta)$ [25%]

(iii) a thin layer of liquid between two horizontal plates heated from below [25%]

(iv) a thin layer of liquid with a free upper surface and heated from below. [25%]

3 Sometimes heat transfer from a structure can cause flow instability. Figure 2(a) shows a tube containing a hot grid. Air flows through the tube at average speed $\overline{U}$. The hot grid is placed a distance x_f from the upstream end, transferring heat to the flow at an average rate $\overline{Q}$. Acoustic velocity and pressure fluctuations, $u(x,t)$ and $p(x,t)$, cause fluctuations in the heat release rate at the grid, $q(t)$. The acoustic waves also cause isentropic compression and expansion of the air around the grid, as sketched in figure 2(b). If $q(t)$ is positive during moments of higher pressure and negative during moments of lower pressure then work is done on the flow over a cycle and the acoustic amplitude increases.

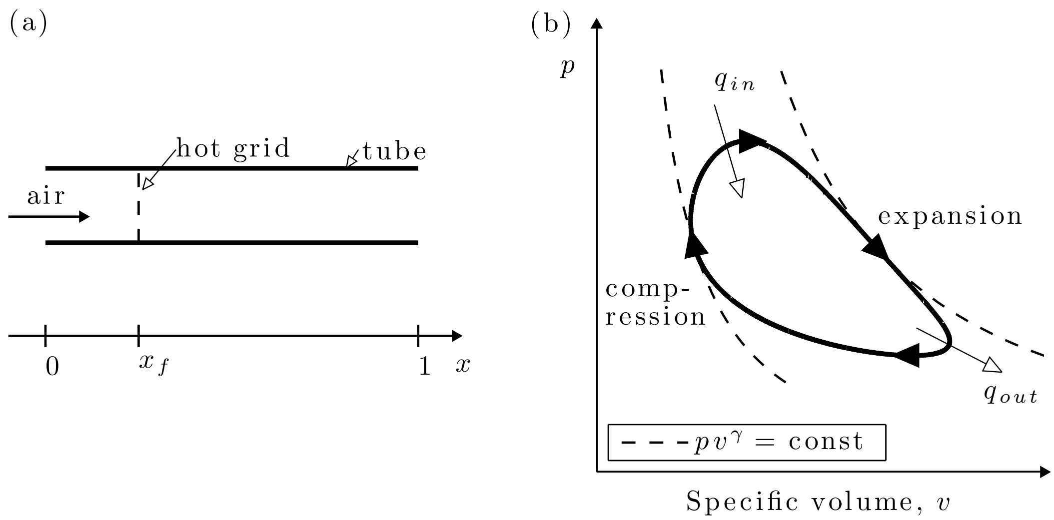

Fig. 2

(a) The boundary conditions are that p and $\partial u/\partial x$ are both zero at the upstream ($x = 0$) and downstream ($x = 1$) ends of the tube. Show that waves of the form $u(x,t) = U(t)\cos(\pi x)$ and $p(x,t) = P(t)\sin(\pi x)$ satisfy these boundary conditions. Sketch these waves. [10%]

(b) The heat release rate at the grid is a function of the velocity and pressure at the grid. This can be expressed as $q(t) = \alpha u(x_f,t) + \beta p(x_f,t)$, where α and β are unknown constants. The non-dimensional momentum and energy equations for the acoustic fluctuations are:

$$\dot{U} + \pi P = 0 \quad (1)$$

$$\dot{P} - \pi U = aU + bP \quad (2)$$

where $a = 2\alpha\cos(\pi x_f)\sin(\pi x_f)$ and $b = 2\beta\sin(\pi x_f)\sin(\pi x_f)$. Assuming that $\alpha \ll 1$ and $\beta \ll 1$, find expressions for the frequency and growth rate of oscillations in terms of a and b. [50%]

(c) With reference to figure 2(b), explain why the growth rate is affected by β but not by α. [20%]

(d) Explain physically why b is zero when the grid is placed at either end of the tube, and why a is also zero when the grid is placed in the centre. [20%]

4 Figure 4 shows the velocity field, $\mathbf{U}(x,z)$, around a cylinder at Re $= 50$. A local stability analysis is performed on the velocity profile at $x = 1.33$. In this analysis, permitted pairs of (k,ω) are calculated for perturbations of the form $\mathbf{u}(z)\exp\{\mathrm{i}(kx-\omega t)\}$. Figure 4 shows the contours of $\omega_i(k)$.

(a) (i) Using figure 4 as a guide, sketch the result of a temporal stability analysis applied to this flow. On the attached copy of figure 4, indicate the value of k at which the temporal growth rate is a maximum. [10%]

(ii) On the attached copy of figure 4, indicate the value of k at which the group velocity is zero. Is the flow stable, convectively unstable, or absolutely unstable? [10%]

(iii) Using figure 4 as a guide, sketch the result of a spatial stability analysis applied to this flow. Explain why a spatial stability analysis has no physical meaning for this flow. [20%]

(b) (i) Sketch the contours of $\omega_i(k)$ that you would expect when the local stability analysis is performed on the velocity profile at $x = 5$, where the flow is convectively unstable. Label the $\omega_i = 0$ contour. [10%]

(ii) Explain why a spatial stability analysis will work at $x = 5$ and sketch the result of a spatial stability analysis. [10%]

(c) In an experiment on the flow around this cylinder, the Reynolds number is increased gradually from 20, where the flow is globally stable, to 100, where the flow is globally unstable and oscillates at a Strouhal number 0.2. The Strouhal number is defined as fD/U, where f is the oscillation frequency in Hertz, D is the cylinder diameter and U is the flow speed. Sketch and describe the flow behind the cylinder at Re $= 100$. [10%]

(d) In a further set of experiments at Re $= 100$, the cylinder is vibrated in the z–direction at Strouhal numbers from 0.19 to 0.21 and at various amplitudes. With the aid of a diagram, describe the behaviour of the flow behind the cylinder during these experiments. [30%]

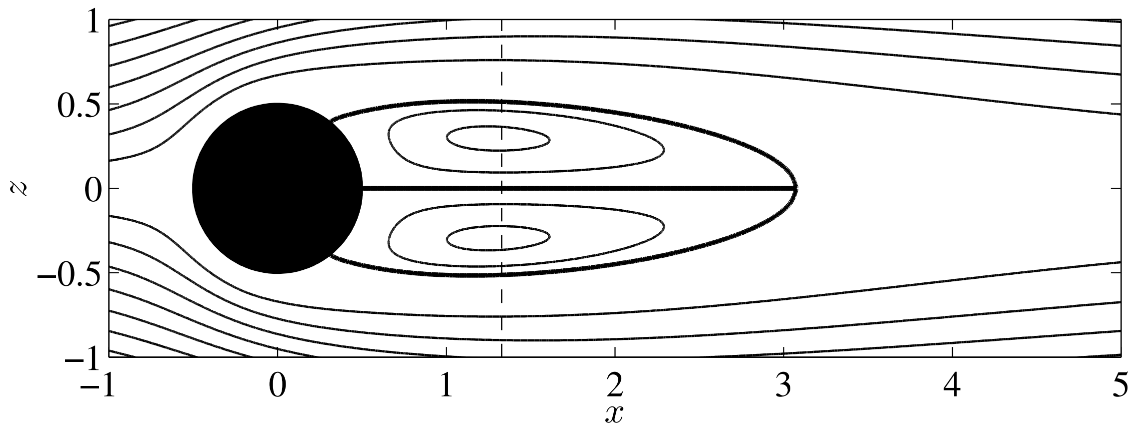
z
x

Fig. 3

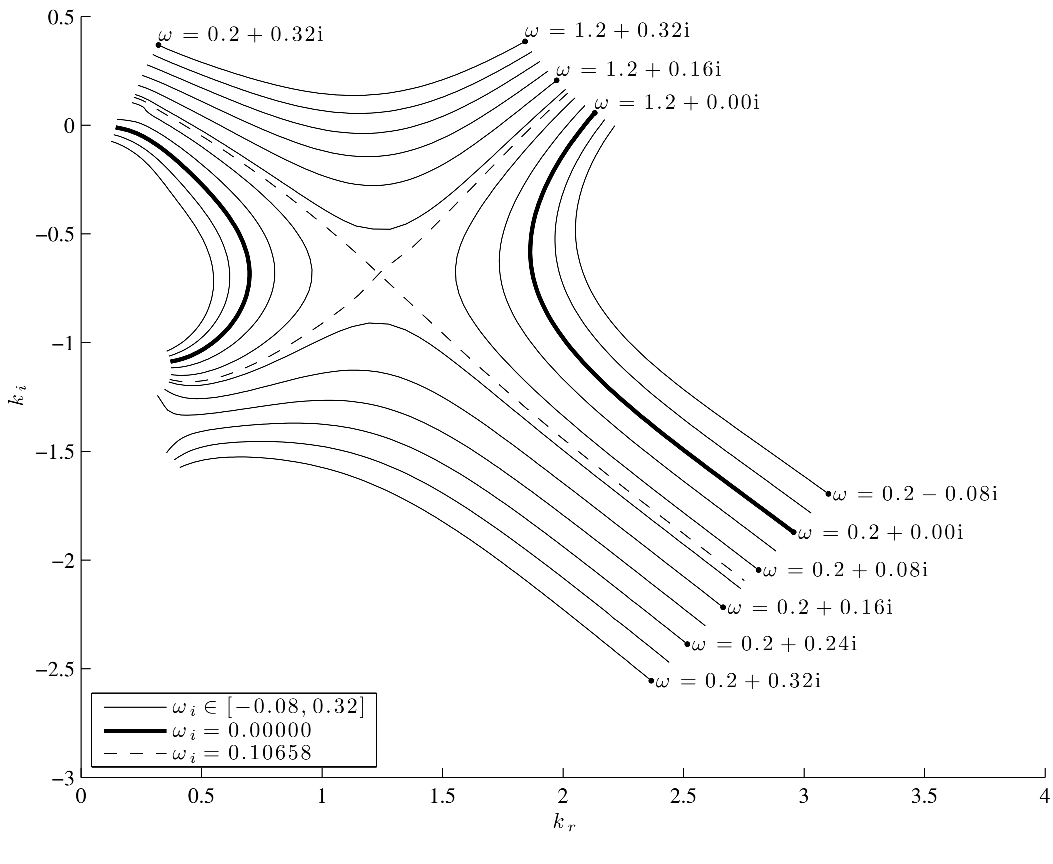
ω = 0.2 + 0.32i
ω = 1.2 + 0.32i
ω = 1.2 + 0.16i
ω = 1.2 + 0.00i
ω = 0.2 − 0.08i
ω = 0.2 + 0.00i
ω = 0.2 + 0.08i
ω = 0.2 + 0.16i
ω = 0.2 + 0.24i
ω = 0.2 + 0.32i
ω_i ∈ [−0.08, 0.32]
ω_i = 0.00000
ω_i = 0.10658
k_i
k_r

Fig. 4

END OF PAPER