ENGINEERING TRIPOS PART IIB

Thursday 25 April 2013 2 to 3.30

Module 4A11

TURBOMACHINERY II

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Aattachment: extract from Compressible Flow Tables, 1 page.

STATIONERY
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

version lx-05

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1 A low speed axial flow compressor stage is to be designed for a compressor test rig.

The compressor has a constant casing diameter of $0.8\,\text{m}$ and a constant hub-to-tip ratio $H/T = 0.6$. At midspan, the stage should have flow coefficient $\phi = V_x / U_m = 0.5$, and a loading coefficient $\psi = \Delta h_0 / U_m^2 = 0.4$. The blade speed is specified as 60 ms^{-1} at the rotor tip. The flow is uniform and axial at the stage inlet with standard sea level atmospheric condition. A free vortex distribution is to be used for the design. When leaving the stage, the flow has zero swirl. All symbols have their usual meanings.

(a) Estimate the minimum power required to drive the stage. [20%]

(b) Determine the velocity triangles at hub, mid-span and tip sections. [30%]

(c) At the design condition, a uniform 1° of negative incidence for both blades has been selected along the spans, and a deviation distribution linearly varying between 6° to 4° from hub to tip is also specified for both blades. Determine the blade turning angles and leading edge and trailing edge metal angles for three sections at hub, mid-span and tip. [15%]

(d) Calculate variations of the reaction and the loading coefficient along the span. [15%]

(e) Identify the section with the highest aerodynamic loading and comment on any potential difficulties in the blade design for this section. Suggest measures you might take to relieve these problems through modification of the design. [20%]

2 A two dimensional section of a transonic compressor rotor with straight blade section has the following design parameters:

Blade stagger angle $\beta = -62.76^\circ$; chord $C = 100$ mm ; thickness $\cong$ 0; solidity $\sigma = 1.4$; and blade Mach number $M_b = 1.2625$.

Absolute flow conditions at the inlet are: $p_{o,1}$ = 101330 Pa; $T_{o,1}$ = 288 K; Inlet Mach number M_∞ = 0.65, and the flow is axial. For air, $R = 287$ $\text{Jkg}^{-1}\text{K}^{-1}$ and $\gamma = 1.4$.

Assume a weak and negligible leading edge shock wave and that the compression is achieved through a strong, normal passage shock wave. The surface static pressure, *p*, distribution on the blade surfaces is as shown as in Fig. 2.

(a) By calculating appropriate angles, draw the velocity triangles up- and downstream of the blade section. [20%]

(b) Sketch the shock wave pattern in the blade section below. [15%]

(c) Assuming negligible viscous loss on the blade surface, calculate the static and stagnation pressure ratios and the isentropic efficiency of the blade section. [25%]

(d) Assuming a dissipation coefficient due to the surface boundary layer of 0.003 for both surfaces, estimate the additional loss of efficiency due to viscous effects. [15%]

(e) Describe, with the help of sketches, the change of the shock wave pattern with increasing back pressure; highlight the conditions for the near peak efficiency and near stall conditions. [25%]

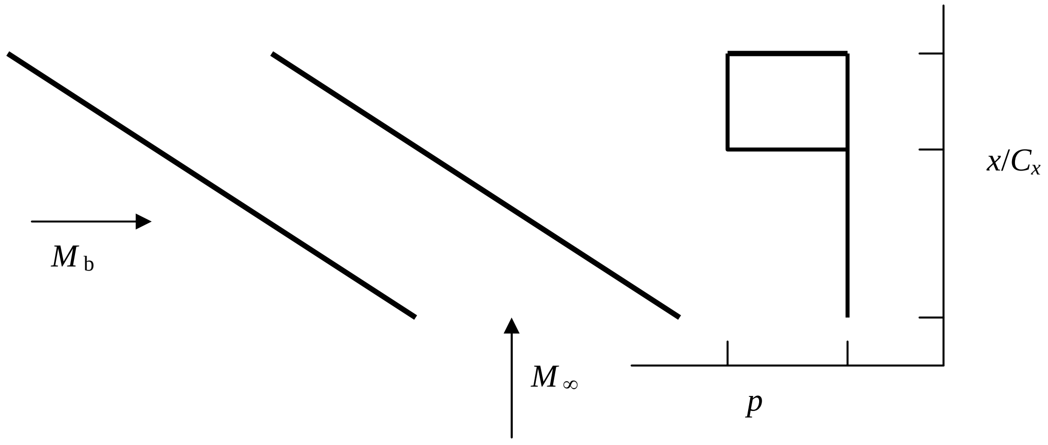

Fig. 2

3 (a) For the time accurate solution of the flow of a compressible inviscid fluid through a two-dimensional domain:

(i) Describe qualitatively which equations must be solved. (There is no need to state the equations explicitly.) [10%]

(ii) State how many types of small amplitude characteristic waves exist within the two-dimensional flowfield. For each wave give a physical interpretation, its propagation speed, and its direction. [15%]

(iii) Explain how the information in (a)(ii) can be used to determine the number of inflow and outflow boundary conditions that are required. State clearly the commonly adopted boundary conditions for subsonic flow. [15%]

(iv) State the CFL condition, give a simple physical interpretation of it and explain why it is important for explicit computational fluid dynamic codes. [10%]

(b) A turbomachinery design engineer is trying to calculate the trajectory of the tip leakage flow in an unshrouded compressor rotor blade using a three-dimensional Euler solver. The calculations are not resolving the tip vortex.

(i) Describe, with sketches, the important features that would be expected to be associated with the tip leakage flow. [20%]

(ii) Comment on the likely effect on the run time and the quality of the solution that would result from doubling the mesh density in all three directions. [15%]

(iii) Comment on the computational benefits and costs that would be associated with using a Navier-Stokes solver for this problem. [15%]

END OF PAPER

Attachment: **Compressible Flow Tables (page 1 of 1)**

GAS FLOW TABLES (γ=1.400): SUBSONIC FLOW

M	$\frac{T}{T_0}$	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{V}{\sqrt{c_pT_0}}$	$\frac{\dot{m}\sqrt{c_pT_0}}{Ap_0}$	$\frac{\dot{m}\sqrt{c_pT_0}}{Ap}$	$\frac{F}{\dot{m}\sqrt{c_pT_0}}$	$\frac{4c_fL_{max}}{D}$	$\frac{\frac{1}{2}\rho V^2}{p_0}$
0.610	0.9307	0.7778	0.8357	0.3722	1.0887	1.3996	1.0867	0.4527	0.2026
0.620	0.9286	0.7716	0.8310	0.3779	1.0990	1.4242	1.0800	0.4172	0.2076
0.630	0.9265	0.7654	0.8262	0.3835	1.1090	1.4489	1.0737	0.3841	0.2127
0.640	0.9243	0.7591	0.8213	0.3891	1.1186	1.4736	1.0678	0.3533	0.2177
0.650	0.9221	0.7528	0.8164	0.3948	1.1280	1.4984	1.0621	0.3246	0.2226
0.660	0.9199	0.7465	0.8115	0.4003	1.1371	1.5233	1.0568	0.2979	0.2276
0.670	0.9176	0.7401	0.8066	0.4059	1.1459	1.5483	1.0518	0.2730	0.2326
0.680	0.9153	0.7338	0.8016	0.4115	1.1544	1.5733	1.0471	0.2498	0.2375
0.690	0.9131	0.7274	0.7966	0.4170	1.1626	1.5984	1.0426	0.2282	0.2424
0.700	0.9107	0.7209	0.7916	0.4225	1.1705	1.6237	1.0384	0.2081	0.2473

GAS FLOW TABLES (γ = 1.400) : SUPERSONIC FLOW

M	$\frac{T}{T_0}$	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{V}{\sqrt{c_pT_0}}$	$\frac{\dot{m}\sqrt{c_pT_0}}{Ap_0}$	$\frac{\dot{m}\sqrt{c_pT_0}}{Ap}$	$\frac{F}{\dot{m}\sqrt{c_pT_0}}$	$\frac{4c_fL_{max}}{D}$	$\frac{\frac{1}{2}\rho V^2}{p_0}$	M_s	$\frac{P_{0s}}{P_0}$	$\frac{P_s}{P}$	$\frac{P_{0s}}{P}$	$\frac{T_s}{T}$	ν	M
1.010	0.8306	0.5221	0.6287	0.5821	1.2809	2.4532	0.9898	0.0001	0.3728	0.9901	1.0000	1.0235	1.9152	1.0066	0.04	1.010
1.020	0.8278	0.5160	0.6234	0.5869	1.2806	2.4817	0.9899	0.0005	0.3758	0.9805	1.0000	1.0471	1.9379	1.0132	0.13	1.020
1.030	0.8250	0.5099	0.6181	0.5917	1.2801	2.5103	0.9900	0.0010	0.3787	0.9712	1.0000	1.0711	1.9610	1.0198	0.23	1.030
1.040	0.8222	0.5039	0.6129	0.5964	1.2793	2.5390	0.9903	0.0018	0.3815	0.9620	0.9999	1.0952	1.9844	1.0263	0.35	1.040
1.050	0.8193	0.4979	0.6077	0.6011	1.2784	2.5678	0.9905	0.0027	0.3842	0.9531	0.9999	1.1196	2.0083	1.0328	0.49	1.050
1.060	0.8165	0.4919	0.6024	0.6058	1.2773	2.5967	0.9909	0.0038	0.3869	0.9444	0.9998	1.1442	2.0325	1.0393	0.64	1.060
1.070	0.8137	0.4860	0.5972	0.6104	1.2760	2.6258	0.9913	0.0051	0.3895	0.9360	0.9996	1.1691	2.0570	1.0458	0.80	1.070
1.080	0.8108	0.4800	0.5920	0.6151	1.2745	2.6549	0.9917	0.0066	0.3919	0.9277	0.9994	1.1941	2.0819	1.0522	0.97	1.080
1.090	0.8080	0.4742	0.5869	0.6197	1.2728	2.6842	0.9922	0.0082	0.3944	0.9196	0.9992	1.2195	2.1072	1.0586	1.15	1.090
1.100	0.8052	0.4684	0.5817	0.6243	1.2709	2.7136	0.9928	0.0099	0.3967	0.9118	0.9989	1.2450	2.1328	1.0649	1.34	1.100
1.110	0.8023	0.4626	0.5766	0.6288	1.2689	2.7432	0.9934	0.0118	0.3990	0.9041	0.9986	1.2708	2.1588	1.0713	1.53	1.110
1.120	0.7994	0.4568	0.5714	0.6333	1.2667	2.7728	0.9940	0.0138	0.4011	0.8966	0.9982	1.2968	2.1851	1.0776	1.74	1.120
1.130	0.7966	0.4511	0.5663	0.6379	1.2643	2.8026	0.9947	0.0159	0.4032	0.8892	0.9978	1.3231	2.2118	1.0840	1.94	1.130
1.140	0.7937	0.4455	0.5612	0.6423	1.2618	2.8325	0.9954	0.0182	0.4052	0.8820	0.9973	1.3495	2.2388	1.0903	2.16	1.140
1.150	0.7908	0.4398	0.5562	0.6468	1.2590	2.8626	0.9961	0.0205	0.4072	0.8750	0.9967	1.3763	2.2661	1.0966	2.38	1.150
1.160	0.7879	0.4343	0.5511	0.6512	1.2562	2.8927	0.9969	0.0230	0.4090	0.8682	0.9961	1.4032	2.2937	1.1029	2.61	1.160
1.170	0.7851	0.4287	0.5461	0.6556	1.2531	2.9230	0.9978	0.0255	0.4108	0.8615	0.9953	1.4304	2.3217	1.1092	2.84	1.170
1.180	0.7822	0.4232	0.5411	0.6600	1.2500	2.9534	0.9986	0.0281	0.4125	0.8549	0.9946	1.4578	2.3500	1.1154	3.07	1.180
1.190	0.7793	0.4178	0.5361	0.6644	1.2466	2.9840	0.9995	0.0309	0.4141	0.8485	0.9937	1.4855	2.3786	1.1217	3.31	1.190
1.200	0.7764	0.4124	0.5311	0.6687	1.2432	3.0147	1.0004	0.0336	0.4157	0.8422	0.9928	1.5133	2.4075	1.1280	3.56	1.200
1.210	0.7735	0.4070	0.5262	0.6730	1.2396	3.0455	1.0014	0.0365	0.4171	0.8360	0.9918	1.5415	2.4367	1.1343	3.81	1.210
1.220	0.7706	0.4017	0.5213	0.6773	1.2358	3.0764	1.0024	0.0394	0.4185	0.8300	0.9907	1.5698	2.4663	1.1405	4.06	1.220
1.230	0.7677	0.3964	0.5164	0.6816	1.2319	3.1075	1.0034	0.0424	0.4198	0.8241	0.9896	1.5984	2.4961	1.1468	4.31	1.230
1.240	0.7648	0.3912	0.5115	0.6858	1.2279	3.1387	1.0045	0.0455	0.4211	0.8183	0.9884	1.6272	2.5263	1.1531	4.57	1.240
1.250	0.7619	0.3861	0.5067	0.6901	1.2238	3.1700	1.0055	0.0486	0.4223	0.8126	0.9871	1.6563	2.5568	1.1594	4.83	1.250
1.260	0.7590	0.3809	0.5019	0.6943	1.2195	3.2015	1.0066	0.0517	0.4233	0.8071	0.9857	1.6855	2.5875	1.1657	5.09	1.260
1.270	0.7561	0.3759	0.4971	0.6984	1.2152	3.2331	1.0077	0.0549	0.4244	0.8016	0.9842	1.7151	2.6186	1.1720	5.36	1.270
1.280	0.7532	0.3708	0.4923	0.7026	1.2107	3.2648	1.0089	0.0582	0.4253	0.7963	0.9827	1.7448	2.6500	1.1783	5.63	1.280
1.290	0.7503	0.3658	0.4876	0.7067	1.2061	3.2967	1.0100	0.0615	0.4262	0.7911	0.9811	1.7748	2.6816	1.1846	5.90	1.290
1.300	0.7474	0.3609	0.4829	0.7108	1.2014	3.3287	1.0112	0.0648	0.4270	0.7860	0.9794	1.8050	2.7136	1.1909	6.17	1.300
1.310	0.7445	0.3560	0.4782	0.7149	1.1965	3.3608	1.0124	0.0682	0.4277	0.7809	0.9776	1.8355	2.7459	1.1972	6.44	1.310
1.320	0.7416	0.3512	0.4736	0.7189	1.1916	3.3931	1.0136	0.0716	0.4283	0.7760	0.9758	1.8661	2.7784	1.2035	6.72	1.320
1.330	0.7387	0.3464	0.4690	0.7229	1.1866	3.4255	1.0149	0.0750	0.4289	0.7712	0.9738	1.8971	2.8112	1.2099	7.00	1.330
1.340	0.7358	0.3417	0.4644	0.7270	1.1815	3.4581	1.0161	0.0785	0.4294	0.7664	0.9718	1.9282	2.8444	1.2162	7.28	1.340
1.350	0.7329	0.3370	0.4598	0.7309	1.1763	3.4907	1.0174	0.0820	0.4299	0.7618	0.9697	1.9596	2.8778	1.2226	7.56	1.350
1.360	0.7300	0.3323	0.4553	0.7349	1.1710	3.5236	1.0187	0.0855	0.4303	0.7572	0.9676	1.9912	2.9115	1.2290	7.84	1.360
1.370	0.7271	0.3277	0.4508	0.7388	1.1656	3.5566	1.0200	0.0890	0.4306	0.7527	0.9653	2.0231	2.9455	1.2354	8.13	1.370
1.380	0.7242	0.3232	0.4463	0.7427	1.1601	3.5897	1.0213	0.0926	0.4308	0.7483	0.9630	2.0551	2.9798	1.2418	8.41	1.380
1.390	0.7213	0.3187	0.4418	0.7466	1.1546	3.6229	1.0226	0.0962	0.4310	0.7440	0.9607	2.0875	3.0144	1.2482	8.70	1.390
1.400	0.7184	0.3142	0.4374	0.7505	1.1490	3.6563	1.0240	0.0997	0.4311	0.7397	0.9582	2.1200	3.0492	1.2547	8.99	1.400
1.410	0.7155	0.3098	0.4330	0.7543	1.1433	3.6899	1.0253	0.1033	0.4312	0.7355	0.9557	2.1528	3.0844	1.2612	9.28	1.410
1.420	0.7126	0.3055	0.4287	0.7581	1.1375	3.7236	1.0267	0.1069	0.4312	0.7314	0.9531	2.1858	3.1198	1.2676	9.57	1.420
1.430	0.7097	0.3012	0.4244	0.7619	1.1317	3.7574	1.0281	0.1106	0.4311	0.7274	0.9504	2.2191	3.1555	1.2741	9.86	1.430
1.440	0.7069	0.2969	0.4201	0.7657	1.1258	3.7914	1.0295	0.1142	0.4310	0.7235	0.9476	2.2525	3.1915	1.2807	10.15	1.440
1.450	0.7040	0.2927	0.4158	0.7694	1.1198	3.8255	1.0308	0.1178	0.4308	0.7196	0.9448	2.2863	3.2278	1.2872	10.44	1.450
1.460	0.7011	0.2886	0.4116	0.7732	1.1138	3.8598	1.0323	0.1215	0.4306	0.7157	0.9420	2.3202	3.2643	1.2938	10.73	1.460
1.470	0.6982	0.2845	0.4074	0.7769	1.1077	3.8942	1.0337	0.1251	0.4303	0.7120	0.9390	2.3544	3.3011	1.3003	11.02	1.470
1.480	0.6954	0.2804	0.4032	0.7805	1.1016	3.9287	1.0351	0.1288	0.4299	0.7083	0.9360	2.3888	3.3382	1.3069	11.32	1.480
1.490	0.6925	0.2764	0.3991	0.7842	1.0954	3.9634	1.0365	0.1324	0.4295	0.7047	0.9329	2.4235	3.3756	1.3136	11.61	1.490
1.500	0.6897	0.2724	0.3950	0.7878	1.0891	3.9983	1.0379	0.1361	0.4290	0.7011	0.9298	2.4583	3.4133	1.3202	11.91	1.500

End of the attachment