ENGINEERING TRIPOS PART IIB

Tuesday 7 May 2013 9.30 to 11.00

Module 4A15

AEROACOUSTICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4A15 data sheet (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

aa03

1 A simple method for modifying the linear acoustic equations to simulate sound absorption is to add a force per unit volume of $-\rho_0 \alpha \mathbf{v}'$ to the momentum equation for linear acoustics, where α is a positive constant that leads to a drag force, ρ_0 is the ambient density and $\mathbf{v}'$ represents the acoustic velocity.

(a) What is the resulting form of the wave equation if such a sound absorption term is taken into account? [30%]

(b) If plane waves of the form $p' = Ae^{\mathrm{i}\omega t}e^{-\mathrm{i}kx}$ are to satisfy the wave equation derived in part (a), what should the complex wave number k be? Find the real and imaginary parts of k if $\alpha \ll 1$. Here t denotes time and x the distance from the origin. [30%]

(c) Show that the energy conservation equation for acoustic motion is given by

$$\frac{\partial w}{\partial t} + \nabla \cdot \mathbf{I} = -\mathscr{D}$$

where $\mathscr{D}$ is always non-negative. Determine the expressions for w, $\mathbf{I}$, and $\mathscr{D}$. [40%]

2 For an unsteady force per unit volume distribution $F_i(\mathbf{x},t)$, the wave equation is given by

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2\nabla^2\right)\rho'(\mathbf{x},t) = -\frac{\partial F_i}{\partial x_i}$$

(a) By assuming that the source distribution is spatially compact, show that the sound radiated in the acoustic far-field scales as

$$\rho' \sim \rho_0\beta_i\left(\frac{l}{x}\right)m^3$$

where ρ_0 is the ambient density, x is the distance from the source to the observer, $\beta = x_i/x$, x_i represents the component of $\mathbf{x}$ in the i-direction, l is the length scale of the source, $m = u'/c_0$, u' is the velocity scale of the source and c_0 represents the speed of sound.

Hint: $$\frac{\partial}{\partial x_i}\int F_i\left(\mathbf{y},t-\frac{x}{c_0}\right)\mathrm{d}\mathbf{y} = -\frac{1}{c_0}\left(\frac{x_i}{x}\right)\frac{\partial}{\partial t}\int F_i\left(\mathbf{y},t-\frac{x}{c_0}\right)\mathrm{d}\mathbf{y}$$

[60%]

(b) Use this result to find the scaling of the far-field acoustic power radiated by the source. [40%]

3 Consider the linear sound speed profile

$$c_0(x) = \alpha x + \beta$$

where α and β are constants and β is positive.

(a) Determine the path of the ray which passes through the origin at angle θ_0 to the x-axis, being careful to distinguish between the three cases $\alpha > 0$, $\alpha = 0$ and $\alpha < 0$. [80%]

(b) Explain briefly how the approximations of ray theory can be used to determine the variation of the amplitude of the acoustic pressure along the ray. [20%]

4 (a) Explain the meaning of the term "cut-off" in connection with acoustic modes in a duct. [20%]

(b) A 3-bladed fan of diameter 300 mm is to be operated in a cylindrical duct of circular cross-section of the same diameter. Table 1 shows the values of z_{mn}, the m^{th} zero of $dJ_n(z)/dz$, where J_n is the n^{th} order Bessel function of the first kind. For $|n| > 6$, use $z_{1n} \approx |n| + 0.80861|n|^{1/3}$. Use the data in Table 1 to determine R_{max}, the maximum number of revolutions per minute if all rotor alone modes are to be cut-off at atmospheric conditions. Formulae on the data sheet may be used without proof. [40%]

(c) The fan rotor in (b) is operated at $10,000$ rpm. Choose a suitable number of blades for a downstream stator row, explaining clearly the reasons for your choice. With your choice of stator blade number which, if any, of the rotor-stator interaction modes at the blade passing frequency (bpf) propagate? [40%]

	$n=0$	$n=\pm1$	$n=\pm2$	$n=\pm3$	$n=\pm4$	$n=\pm5$	$n=\pm6$
$m=1$	0.00000	1.84118	3.05424	4.20119	5.31755	6.41562	7.50127
$m=2$	3.83170	5.33144	6.70613	8.01524	9.28240	10.51986	11.73494
$m=3$	7.01558	8.53632	9.96947	11.34592	12.68191	13.98719	15.26818
$m=4$	10.17346	11.70600	13.17037	14.58585	15.96411	17.31284	18.63744
$m=5$	13.32369	14.86359	16.34752	17.78875	19.19603	20.57551	21.93172

Table 1

END OF PAPER