ENGINEERING TRIPOS PART IIB

Wednesday 1st May 2013 9:30-11:00

Module 4B5

NANOTECHNOLOGY

Answer not more than ***three*** questions.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1

(a) Discuss, with the aid of two examples how and why the properties of nanostructures differ from those of bulk materials. What is to be gained by attempting to control these properties? [30%]

(b) We describe quantum systems using *wave-functions*. What is their physical significance? In what ways do quantum systems differ from classical ones and, from a mathematical standpoint, what is the origin of these differences? [30%]

(c) Fig. 1 illustrates a time-sequence of the interaction of an electron wave-packet with a potential barrier structure. The electron has an energy, E, as indicated in Fig.1. The electron is incident from the left. Describe the evolution of the wave-function during the interaction. [40%]

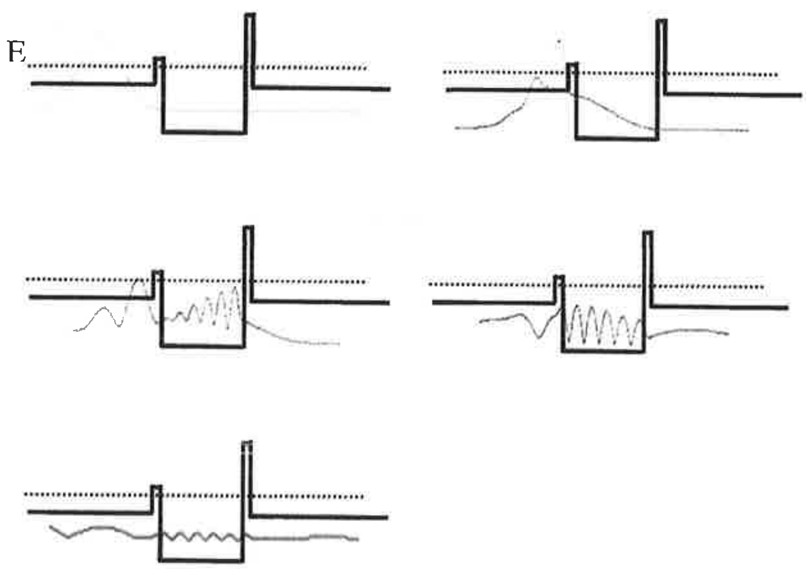

Fig. 1

2

(a) Derive the spectrum of energy eigenvalues for electrons in an infinitely-deep potential well that is 5 nm wide in GaAs, where the electron's effective mass is 0.067 times the free electron mass. Determine the normalisation constant for the wave-functions describing this system, and sketch the probability density for the first excited state, and find its energy. [50%]

(b) Now consider the case where the quantum well has a finite depth of 1 eV. On the basis of your answer to part (a) above, estimate the number of bound states you would expect the well to contain, and sketch their approximate wave-functions. When is it reasonable to approximate a finite quantum well as an infinite one? [40%]

(c) Show that if an electron is placed in any symmetric quantum well, then the corresponding wave-functions will be either symmetric or anti-symmetric. [10%]

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3

(a) Describe, with the aid of sketches, what happens when two quantum wells are brought into close enough proximity to each other that the wave-functions for the electrons in each well start to overlap. [25%]

(b) State the assumptions used to construct the nearly-free electron model, and how they differ from those of the free electron model. To first order, when we can approximate the potential as seen by electrons as $V(x) = V_0 + V_1 e^{i2\pi x/a} + V_1 e^{-i2\pi x/a}$, show that

$$[E - V_0 - (\hbar^2 k^2/2m)]\,[E - V_0 - \hbar^2 (k - 2\pi/a)^2/2m] = |V_1|^2$$ [50%]

(c) Sketch

(i) the dispersion relation for a nearly-free electron, using the reduced-zone scheme. [10%]

(ii) the potential as seen by the electrons, and the wave-functions at the conduction and valence band edges. [15%]

4

(a) Sketch the no-bias band diagrams for two different types of resonant tunnelling device. What are the key differences between these two devices, in terms of their operation and their fabrication? [35%]

(b) Sketch the typical current vs voltage characteristics for one of the above types of device, labelling the salient features. For each regime, draw the corresponding band diagram and briefly describe the conduction mechanism. [55%]

(c) Briefly explain why there is so much interest in trying to use molecular systems in electronic devices – what are the perceived benefits? [10%]

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5

(a) Deduce the Schrödinger equation for a quantum simple harmonic oscillator of stiffness k. Derive the spectrum of allowed energy levels, clearly showing the steps involved. [50%]

(b) Discuss the characteristics of the above solution, with particular emphasis on the differences between it and what one would expect using classical mechanics. As part of your answer, sketch the wave-functions for the first three states. [25%]

(c) In reality, what can the quantum simple harmonic oscillator be used to describe? What can it tell us about the behaviour of atoms in a material? [25%]

END OF PAPER

4B5 2012/13 short/numerical Answers

2. (a) $E_n = 0.223n^2$ eV, $E_1 = 0.223$ eV

(b) 2 bound states.