

ENGINEERING TRIPOS PART IIB

Monday 22 April 2013 09:30 to 11.00

Module 4C2

DESIGNING WITH COMPOSITES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:

4C2 datasheet (6 pages).

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering Data Book

CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) A unidirectional carbon fibre-reinforced composite consists of carbon fibres ($E_f = 340$ GPa) in an epoxy resin matrix ($E_m = 3.40$ GPa). Sketch a graph that shows how the axial and transverse stiffnesses of this composite are expected to vary as a function of the fibre volume fraction, f ($0 \leq f \leq 1$). Calculate the axial and transverse stiffnesses of a composite of this material with $f = 0.4$. [15%]

(b) Describe what is meant by a laminate and explain briefly why laminates are often preferred to simple unidirectionally reinforced composite materials. Define a balanced symmetric laminate. What is the main significance of whether or not a laminate is balanced symmetric in terms of its elastic response to various types of loading? [15%]

(c) A $[45 / -45]_s$ symmetric angle-ply laminate is made up from 0.2 mm thick unidirectional carbon/epoxy laminae with the following elastic constants: $E_1 = 138$ GPa, $E_2 = 9$ GPa, $G_{12} = 6.9$ GPa and $\nu_{12} = 0.3$.

(i) Determine the laminate extensional stiffness matrix $[A]$. Comment on its form. [45%]

(ii) Taking values for the laminate coupling stiffness matrix $[B]$ and the laminate bending stiffness matrix $[D]$ given below, calculate the tip deflection for a cantilever beam made from the laminate, of length 0.1 m and of width 0.01 m, under an end load of 1 N.

$$[B] = 0 \quad [D] = \begin{pmatrix} 1.93 & 1.34 & 1.04 \\ 1.34 & 1.93 & 1.04 \\ 1.04 & 1.04 & 1.52 \end{pmatrix} \text{GPa mm}^3 \quad [25\%]$$

2 (a) List the process steps involved in filament winding. Outline the main parameters controlling the process. [15%]

(b) What is the Tsai-Hill failure criterion for composite materials? Why is the Tsai-Hill failure criterion for composite materials usually considered more reliable than the maximum stress criterion? [15%]

(c) An angle-ply glass-polyester composite is used to produce a gas pipeline with a diameter of 300 mm and a wall thickness of 10 mm. The composite is filament-wound, at $\pm 45^\circ$ to the hoop direction. A gas supply accident results in the line pressure rising to 2 MPa. Using the Tsai-Hill failure criterion, predict whether this would result in damage to the pipeline. Note: To simplify the analysis, treat one of the two types of ply as if they were the only one present, and assume that the failure of the laminate is governed by this ply.

Composite failure stresses for loading parallel, transverse and in shear relative to the fibre axis are 400 MPa, 20 MPa and 25 MPa respectively. [60%]

(d) Comment on the simplification of failure analysis in (c). Without carrying out any further calculations, explain how would you tackle the analysis more rigorously. [10%]

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3 Give the physical basis for the following observations:

(i) The axial tensile strength of a CFRP lamina is fibre-governed whereas the axial compressive strength is matrix governed. [25%]

(ii) The pull-out toughness G_p of a CFRP lamina increases with diminishing shear strength of the lamina τ_f . [30%]

(iii) A thick curved beam of initial curvature κ is made from unidirectional CFRP. When the beam is subjected to a bending moment M , the beam splits at mid-depth. [30%]

(iv) When a unidirectional CFRP lamina is made into a panel containing a central hole, and is loaded in tension, there is only a small reduction in its tensile strength due to the presence of the hole. [15%]

4 The vertical mast of a sailing boat is to be designed at the conceptual stage from either CFRP or aluminium, with properties as listed in Tables 1 and 2 on page 6 of the datasheet. The mast is idealised as a thin-walled circular tube of length $L = 3$ m, diameter $D = 40$ mm, and wall thickness t to be specified. It is built-in at its lower end, and is subjected to a transverse load of $F = 1$ kN at the top end, associated with wind action on the sail; the corresponding transverse tip deflection of the mast must be less than 0.5 m. If CFRP is used for fabrication of the mast, the tube wall is to comprise a laminate of plies $0^\circ : 45^\circ : 90^\circ$ in proportion $80\% : 10\% : 10\%$.

- (i) Calculate the wall thickness t for the two choices of material, based upon the strain allowables of the datasheet for the CFRP and a strain allowable of 0.3% for the aluminium. [25%]
- (ii) What is the required wall thickness for each material, in order to prevent the tip deflection from violating the maximum allowable value? [25%]
- (iii) Which choice of material maximises profit, assuming a cost premium of £100/kg for reduction in weight of the mast. [30%]
- (iv) Outline how the mast might be manufactured from CFRP for a production run of 1 or 1000. [20%]

END OF PAPER