ENGINEERING TRIPOS PART IIB

Thursday 2 May 2013 9.30 to 11

Module 4C6

ADVANCED LINEAR VIBRATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C6 Advanced Linear Vibration data sheet (10 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

jw03

1 (a) Sketch a typical testing arrangement for experimental modal analysis. Your sketch should include items such as the impulse hammer, accelerometer, any amplifiers and filters and the data acquisition system. [20%]

(b) Explain the importance of the following parameters:

(i) the impulse hammer mass and tip stiffness;

(ii) the charge amplifier gain and high-pass filter.

Where appropriate, use formulae, sketches or calculations to illustrate your answers. [30%]

(c) An impulse is applied at one point on a structure and an accelerometer is fixed to another point. The sampling rate of the data acquisition system is 5000 Hz and there are 8192 data points per channel. A mode is identified at a frequency of 400 Hz with a Q-factor of 50. The peak of the magnitude of the frequency-response function at this mode is 0.002 m/N. For the contribution of this mode alone, give careful sketches of:

(i) the magnitude of the frequency-response function; [20%]

(ii) the modal circle. [30%]

2 (a) Explain the distinction between *material damping* and *boundary damping*. Describe the circumstances under which one or the other would be expected to dominate. Give *two* examples of systems that you would expect to be dominated by material damping, and *two* likely to be dominated by boundary damping. [30%]

(b) A viscously damped system with N degrees of freedom has mass matrix M, stiffness matrix K and damping matrix C. In the absence of damping, the nth mode vector is $\mathbf{u}_n$ with natural frequency ω_n. If the damping matrix takes the particular form

$$C = \alpha M + \beta K$$

where α and β are constants, prove that the undamped eigenvector $\mathbf{u}_n$ still satisfies the damped equations of motion and find an expression for the corresponding damped natural frequency. [20%]

(c) If damping is light so that α and β are both small, find an approximate expression for the damped natural frequency. If it is expressed in the form $\omega_d(1+i\eta_n)$, find η_n and sketch a graph of its variation with frequency. [20%]

(d) A two degree of freedom system is shown in Fig. 1: it consists of two masses m connected by springs of stiffness k and s and dampers with damping constants c_1, c_2 and c_3. Write down the mass, stiffness and damping matrices. For what values of the damping constants does this system satisfy the condition of part (b)? For all cases that satisfy this condition, give expressions for the mode vectors and complex natural frequencies. [30%]

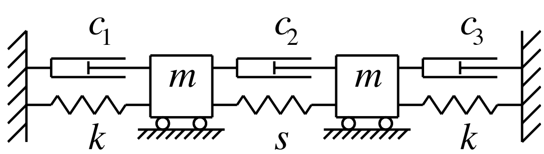

Fig. 1

3 (a) A circular membrane with radius a, tension per unit length T and mass per unit area m is fixed around its perimeter. You may assume that the vibration modes expressed in polar coordinates r, θ take the form

$$\left.\begin{matrix}\sin \\ \cos\end{matrix}\right\} n\theta \, J_n(kr), \qquad n = 0, 1, 2, 3 \cdots$$

where J_n is the Bessel function of order n, and that the corresponding natural frequency ω satisfies $k^2 = \dfrac{\omega^2 m}{T}$. Explain how the edge boundary condition allows the natural frequencies and mode shapes to be found. Sketch a few mode shapes. (Plots of the Bessel functions for $n = 0$–3 are given in Fig. 2.) [25%]

(b) The membrane from part (a) is now fixed around its edge to a rigid circular ring of mass M that is free to move. For the case of axisymmetric modes with $n = 0$, find the new boundary condition at the membrane edge. Hence obtain an equation that determines the new natural frequencies. What does the interlacing theorem lead you to expect about these frequencies? Test this expectation, using graphical means. [50%]

(c) Now consider the other natural frequencies of the membrane from (a), with $n > 0$. Explain which will change under the boundary conditions of (b), and which will not. For the ones that do change, what does the interlacing theorem tell you about them? [25%]

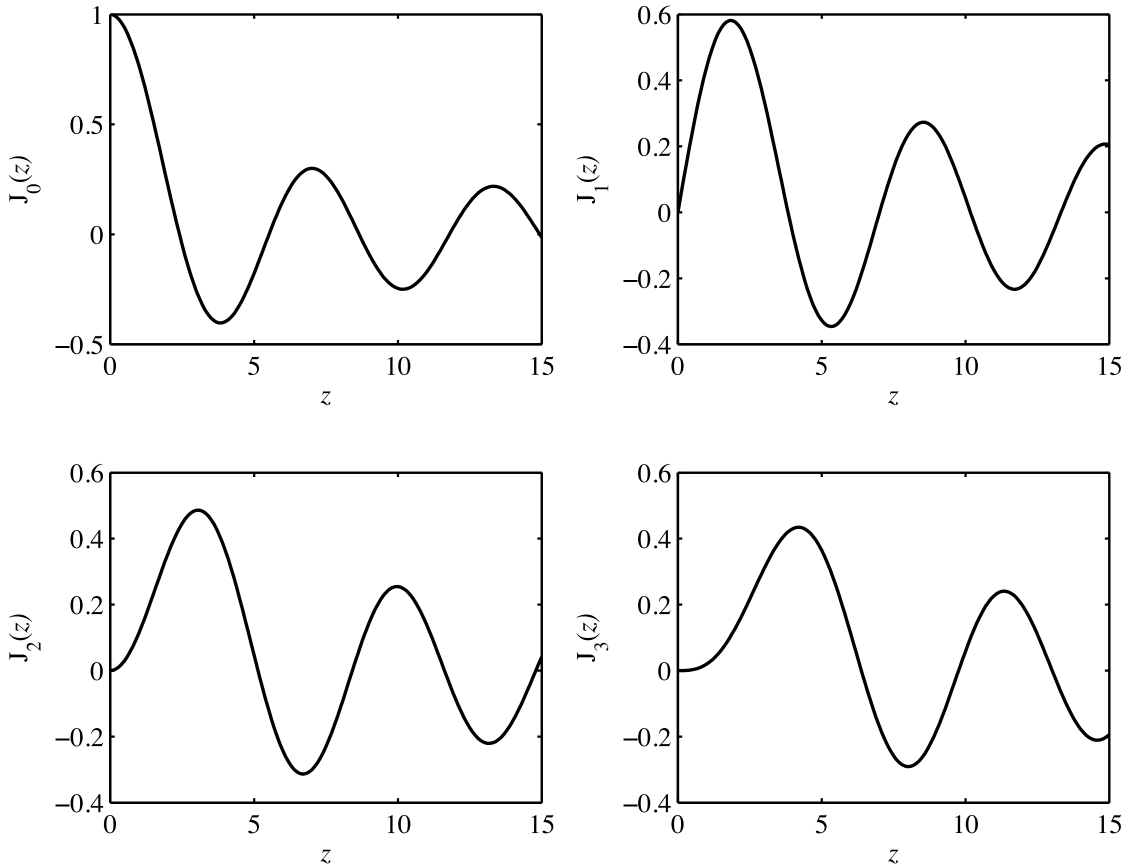

Fig. 2

4 A "bass reflex" loudspeaker can be idealised as shown in Fig. 3. A piston of mass M and area A (the loudspeaker cone) is supported by a spring of stiffness K, and is enclosed by a rigid box of volume V which has a single opening in the form of a neck of area S and effective length L (including any end corrections).

(a) Explain briefly how the air in the neck region can be treated approximately as a rigid mass, and write down the Helmholtz resonance frequency that would be found if the spring K were infinitely stiff. [15%]

(b) The piston is driven electrically with a harmonic force $Fe^{i\omega t}$. By considering the pressure change arising within the volume V, show that the equations of motion that govern small displacements u of the neck air-mass and x of the piston have the form

$$\begin{bmatrix} \rho SL & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{x} \end{bmatrix} + \frac{1}{V} \begin{bmatrix} \rho c^2 S^2 & \rho c^2 AS \\ \rho c^2 AS & KV + \rho c^2 A^2 \end{bmatrix} \begin{bmatrix} u \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ Fe^{i\omega t} \end{bmatrix}$$

where c is the speed of sound in air, and ρ is the air density. You may assume that pressure perturbations p' in the box are related to density perturbations ρ' by $p' = c^2 \rho'$. [35%]

(c) The output of the loudspeaker is the radiated sound pressure, which for low frequencies is determined by the change in net volume of the whole unit, including the neck air "piston". Show how the equations from part (b) can be used to obtain an expression for the frequency response function for this volume change in response to the given force amplitude F. What happens to this transfer function at very low frequency ω? Explain the physical basis of this behaviour. [50%]

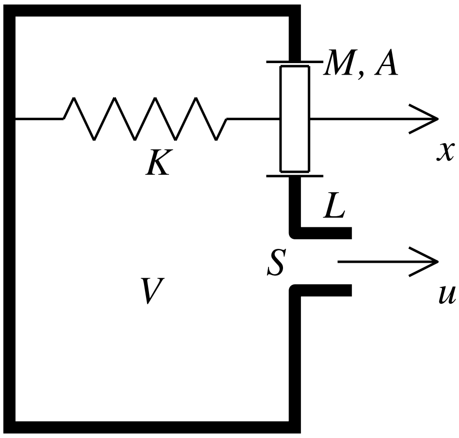

Fig. 3

END OF PAPER