ENGINEERING TRIPOS PART IIB

Tuesday 30 April 2013 14.00 to 15.30

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

Candidates may bring their notebooks to the examination.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 The equation of motion that governs the flapping response of a helicopter rotor blade to wind turbulence of velocity $V(t)$ has the form

$$\ddot{\beta} + \alpha\Omega\dot{\beta} + \Omega^2\beta = \gamma\Omega V(t)$$

where β is the blade flap angle, Ω is the rotor rotation rate, and α and γ are constants. The wind velocity is a broad-banded random process whose double sided spectrum $S_{VV}(\omega)$ is reasonably flat over the frequency range $0 \le |\omega| \le 3\Omega$.

(a) Derive an expression for the probability that the flap angle will exceed a level b within a time T. Find the value of Ω for which this probability is a maximum for the case $\alpha = 0.1$, $\gamma = 0.1$, $b = 0.8$ and $S_{VV}(\Omega) = 5.0$ (SI units throughout). For this value of Ω, calculate the probability that the flap angle will exceed b during 50 rotations of the rotor. [70%]

(b) The stress in the rotor blade is given by $s(t) = a\beta$, where a is a constant. The S-N law for fatigue failure of the blade has the form

$$N = cs^{-r}$$

where N is the number of cycles to failure at stress level s. Show that the fatigue damage accumulated in a time T under random wind loading is proportional to Ω^n, and express the value of the exponent n in terms of r. [30%]

2 A random process $x(t)$ is simulated over a time interval T by using the equation

$$x(t) = \sum_{n=1}^{N} \cos(\omega_n t + \varepsilon_n)$$

where $\omega_n = n\pi / T$ and the phase angles ε_n are statistically independent and chosen randomly from a uniform distribution between 0 and 2π .

(a) Explain by using a sketch what is meant by the ensemble of the random process. [15%]

(b) Calculate the ensemble average of $x(t)$, and comment on whether the process is ergodic for: (i) large N, and (ii) N=1. [30%]

(c) Show that the autocorrelation function of $x(t)$ is given by

$$R_{xx}(\tau) = \frac{1}{2} \sum_{n=1}^{N} \cos(\omega_n \tau)$$ [30%]

(d) Show that if N is large then the probability density function of $x(t)$ at any time t is given by

$$p(x) = \sqrt{\frac{1}{\pi N}} \exp\left(-\frac{x^2}{N} \right)$$

Explain why this result is valid only for large N. [25%]

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3 A nonlinear vibratory system is described by the equation:

$$\ddot{x} + p^2 x + \varepsilon \alpha_1 x^2 + \varepsilon^2 \left(\mu \dot{x} + \beta_1 x^3 + \beta_2 \dot{x}^3 \right) = 0$$

where ε represents a small parameter and μ, p, α_1, β_1 and β_2 are constants.

(a) By using the method of perturbation, write down a series of equations which govern the response of the system up to second order accuracy in ε. [20%]

(b) Solve for the response of the system up to first order accuracy in ε, assuming that the amplitude of the solution is bounded. [40%]

(c) By considering bounded amplitude behaviour for the second order correction to the system response, show that limit cycle oscillations will exist if β and μ are of opposite sign. In this case obtain analytical expressions for the amplitude and frequency of the oscillations. [40%]

4 The nonlinear vibrations of a system are described by the equation

$$\ddot{x} + \alpha \dot{x} + \dot{x}^3 + x - x^3 = 0$$

where α is a real valued constant.

(a) Identify the singular points of the system. [20%]

(b) Determine the type and stability of each singular point. Comment on the nature of the singular points as α is varied. [30%]

(c) Sketch the behaviour of the system in the phase plane for the case where $-2 < \alpha < 2$. [40%]

(d) The system describes a bifurcation as α passes through zero. Discuss the nature of this bifurcation and comment on the physical basis for this behaviour. [10%]

END OF PAPER