ENGINEERING TRIPOS PART IIB

Tuesday 23 April 2013 9.30 to 11.00

Module 4C8

APPLICATIONS OF DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C8 datasheet, 2013 (5 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

dc02

1 (a) A 'bicycle' model of a car, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 1. The car moves at steady forward speed u. It has mass m, yaw moment of inertia about its centre of gravity I, and lateral creep coefficients C_f and C_r at the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure. The car is subject to a steady side wind of magnitude Y which acts a distance x forward of the centre of gravity. Show that the equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v} + u\Omega) + (C_f + C_r)\frac{v}{u} + (aC_f - bC_r)\frac{\Omega}{u} = Y + C_f\delta$$

$$I\dot{\Omega} + (aC_f - bC_r)\frac{v}{u} + (a^2C_f + b^2C_r)\frac{\Omega}{u} = xY + aC_f\delta$$

State your assumptions. [30%]

(b) Determine an expression for the steer angle δ_{ss} needed to set the steady state yaw rate Ω_{ss} to zero. For low speeds (less than the 'critical speed'), explain how this steady-state steer angle varies with x and the vehicle parameters. [40%]

(c) Determine an expression for the steady state sideslip angle β_{ss} at the centre of gravity of the vehicle when $\Omega_{ss} = 0$ Explain how the motion of the vehicle varies with x and the vehicle parameters. [30%]

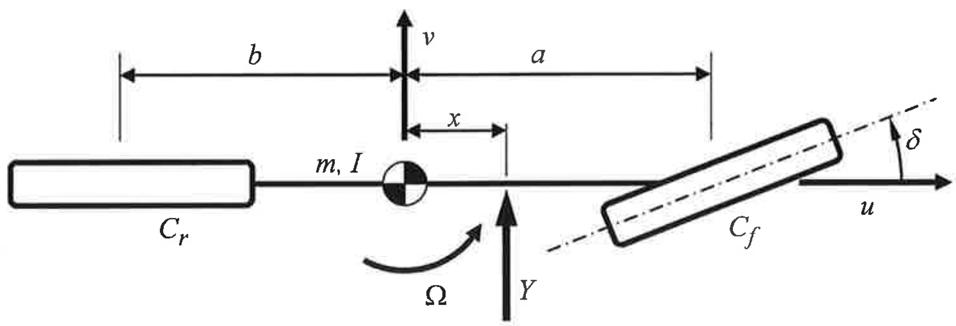

Fig. 1

2 (a) Explain the 'brush model' of rolling contact and describe qualitatively how it can be used to calculate the forces and moments generated by pneumatic tyres rolling with small lateral or longitudinal creep. [30%]

(b) The castered wheel of a baggage trolley is idealised as shown in plan view in Fig. 2. A wheel of mass m and diametral moment of inertia I, is free to rotate about a horizontal axis (with stiff bearings) through its mass centre D. The lateral creep coefficient of the wheel is C. The axle is attached to a light, rigid yoke BD, of length a which is free to rotate about a vertical axis at B, with yaw angle θ. The leg of the trolley is flexible and allows lateral movement x of point B. This is modelled as a linear spring of stiffness k joining B to the body of the trolley at A as shown. The body of the trolley moves in a straight line at constant speed u, and the floor is smooth and level.

(i) Derive equations for small lateral and yaw oscillations of the system in the horizontal plane (neglecting gyroscopic effects). [35%]

(ii) Use the Routh-Hurwitz criterion to determine the conditions for which the motion of the system would be stable. [35%]

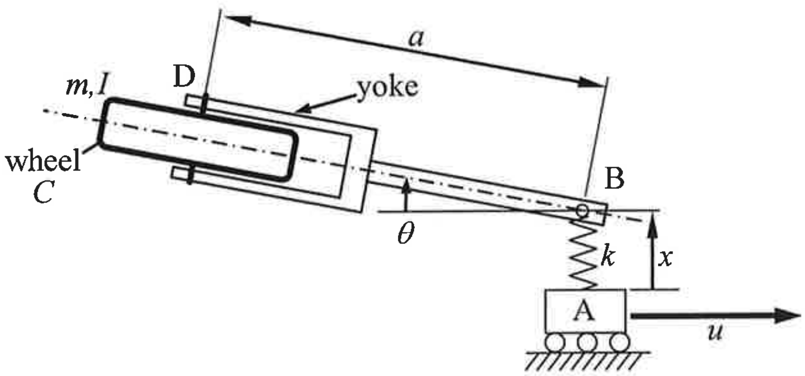

Fig. 2

3 (a) Figure 3(a) shows the *single-sided* mean square spectral density (as a function of wavenumber) of the vertical displacement profile of a road surface. Explain why this measured profile might be adequately represented mathematically by a white noise vertical velocity profile with magnitude S_0. [10%]

(b) Figure 3(b) shows a linear model with two degrees of freedom for predicting the vertical vibration response of a vehicle travelling along a road surface. It has sprung mass m_s, unsprung mass m_u, tyre stiffness k_t, suspension stiffness k and suspension damping c. The mean square dynamic tyre force in response to a white noise vertical velocity input at the tyre-road contact is given by

$$E\left[\left(k_t\left(z_r - z_u\right)\right)^2\right] = \frac{\pi S_0\left\{\left(m_u + m_s\right)^3 k^2 - 2\left(m_u + m_s\right) m_u m_s k_t k + \left(m_u + m_s\right)^2 k_t c^2 + m_u m_s^2 k_t^2\right\}}{m_s^2 c}.$$

(i) State why the mean square dynamic tyre force is an important criterion, and show that the value of suspension stiffness k that minimizes it is given by:

$$k = \frac{m_u m_s}{\left(m_u + m_s\right)^2} k_t .$$ [30%]

(ii) If k is as given in (b)(i) and $m_u << m_s$, determine an approximate value of suspension damping c that minimizes the mean square dynamic tyre force. [50%]

(iii) Comment on the absence of m_s in the result of (b)(ii). [10%]

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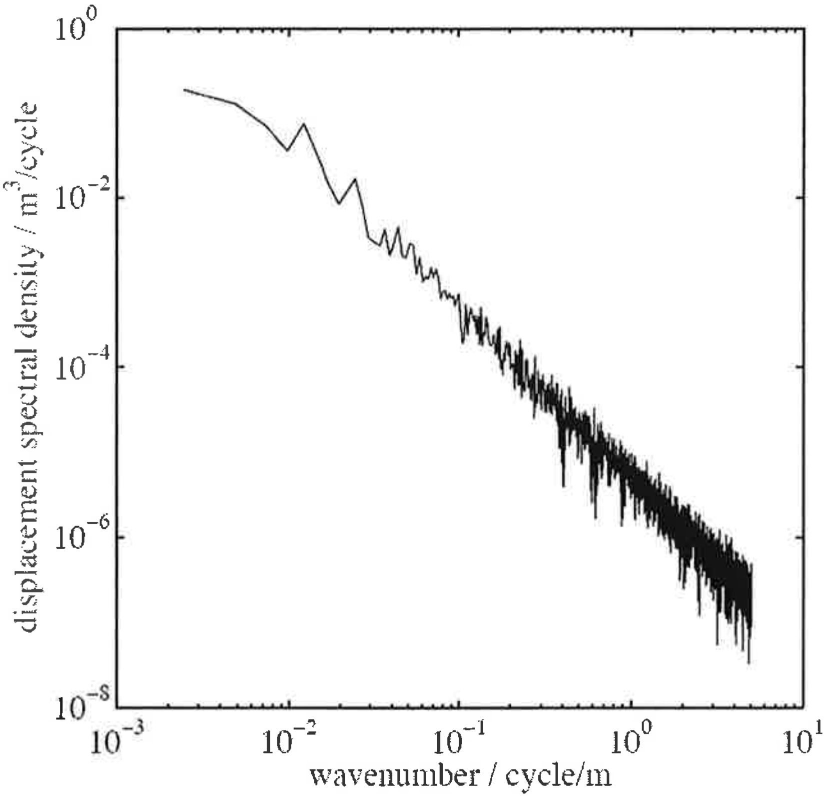
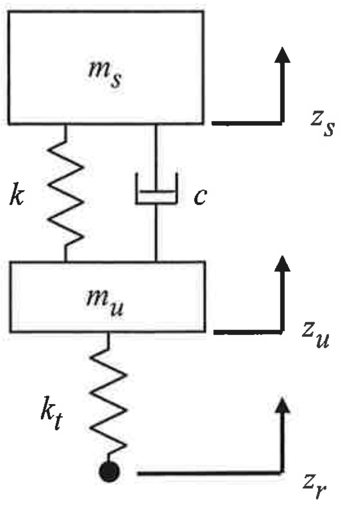


displacement spectral density / m³/cycle
10^0
10^-2
10^-4
10^-6
10^-8
10^-3
10^-2
10^-1
10^0
10^1
wavenumber / cycle/m

Fig. 3(a)



m_s
z_s
k
c
m_u
z_u
k_t
z_r

Fig. 3(b)

4 Figure 4 shows a pitch-plane model of a vehicle, with mass $m = 1000$ kg, moment of inertia $I = 1500$ kg m^2, distance $a = 1.2$ m, stiffness $k = 80$ kN m^{-1} and damping $c = 3$ kN s m^{-1} at each axle. The vehicle travels with constant speed $U = 9.6$ m s^{-1} over a road surface with random roughness.

(a) The model is used to predict the spectral density of vertical acceleration $\ddot{z}$ measured at the centre of mass of a real vehicle. Assuming that the parameter values of the model and the properties of the road surface have been identified correctly, state reasons why the prediction of the model might not agree with the measurement. The frequency range of interest is 0 Hz to 30 Hz. [20%]

(b) Estimate the natural frequencies and corresponding mode shapes of vibration of the vehicle model in Fig. 4. [20%]

(c) The transfer functions of the vehicle relating vertical road displacement inputs to vehicle displacement outputs are defined by

$$\begin{Bmatrix} z(j\omega) \\ \theta(j\omega) \end{Bmatrix} = \begin{bmatrix} H_z(j\omega) & H_z(j\omega) \\ -H_\theta(j\omega) & H_\theta(j\omega) \end{bmatrix} \begin{Bmatrix} z_{r1}(j\omega) \\ z_{r2}(j\omega) \end{Bmatrix}.$$

Taking account of the relationship between z_{r1} and z_{r2}, derive expressions for the transfer functions from z_{r1} to z and from z_{r1} to θ, in terms ω, a, U, $H_z(j\omega)$ and $H_\theta(j\omega)$. [20%]

(d) Sketch the magnitude of the transfer functions from $\dot{z}_{r1}$ to $\ddot{z}$ and from $\dot{z}_{r1}$ to $\ddot{\theta}$ over the frequency range 0 Hz to 20 Hz, indicating clearly the frequencies at which the magnitude is zero. [40%]

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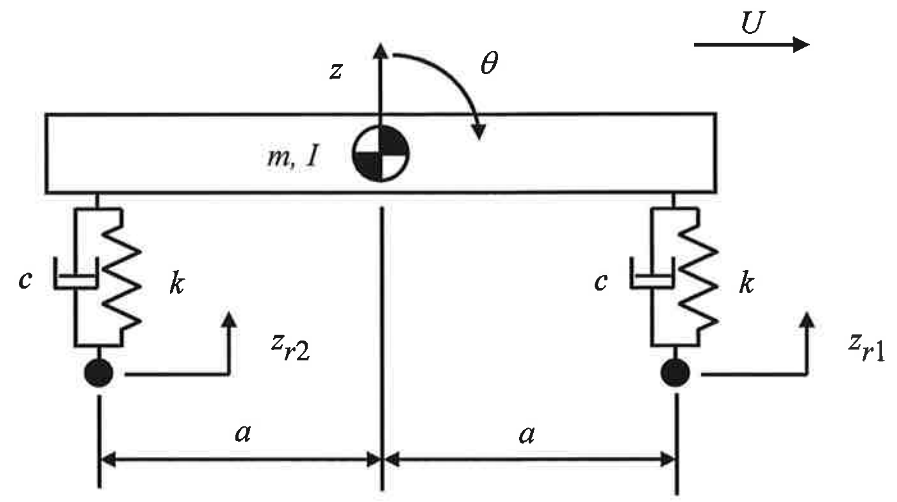

Fig. 4

END OF PAPER

ENGINEERING TRIPOS PART IIB
Module 4C8 Examination, 2013
Answers

1. (b) $\delta_{ss} = \dfrac{(s-x)Y}{(a-s)C_f}$ (c) $\beta_{ss} = \dfrac{(a-x)Y}{(a-s)C}$

2. (b)(i) $\begin{bmatrix} m & ma \\ ma & I_G + ma^2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c/u & ca/u \\ ca/u & ca^2/u \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k & c \\ 0 & ca \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix};$
   (b)(ii) Stable for $k > c/a$

3. (b)(ii) $c \approx \sqrt{m_u k_t}$

4. (b) Bounce $\omega_1 = \sqrt{\dfrac{2k}{m}} = 2.01$ Hz Pitch $\omega_2 = \sqrt{\dfrac{2ka^2}{I}} = 1.97$ Hz
   (c) $\dfrac{z}{z_{r1}} = H_z\left(1 + e^{-j\omega\frac{2a}{u}}\right)$ $\dfrac{\theta}{z_{r1}} = H_\theta\left(-1 + e^{-j\omega\frac{2a}{u}}\right)$

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