ENGINEERING TRIPOS PART IIB

Monday 6 May 2013 9.30 to 11.00

Module 4C9

CONTINUUM MECHANICS

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C9 datasheet (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

vsd02

1 The Kronecker delta is denoted by the symbol δ_{ij}.

(a) By direct expansion or otherwise evaluate/simplify the following

(i) $\delta_{ij}\delta_{ik}\delta_{jk}$; [10%]

(ii) $\delta_{ij}\delta_{ij}$; and [10%]

(iii) $\delta_{ij}\delta_{jk}$. [10%]

(b) (i) Show that the quadratic form $D_{ij}x_i x_j$ is unchanged if D_{ij} is replaced by its symmetric part. [25%]

(ii) Show that $D_{ij}x_i x_j = 0$ when D_{ij} is skew-symmetric, i.e. $D_{ij} = -D_{ji}$. [30%]

(c) A deformation field is specified by the displacements $u_1 = 4x_1 - x_2 + 3x_3$, $u_2 = x_1 + 7x_2$ and $u_3 = -3x_1 + 4x_2 + 4x_3$ in the x_1, x_2 and x_3 directions, respectively. Calculate the strain field ε_{ij} and also the principal strains. [15%]

2 (a) Consider a scalar function $\phi(r,\theta) = Ar^2\theta^m$ in polar co-ordinates (r,θ) as sketched in Fig. 1 (A and m are constants). Find all values of the constant m such that ϕ is an Airy stress function for a planar problem in elasticity. [20%]

(b) For a semi-infinite plate in the region $0 \le \theta \le \pi$ determine the tractions along the edges $\theta = 0$ and $\theta = \pi$ as given by the Airy stress function $\phi = Ar^2\theta$. Hence also find expressions for the stress field $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ in terms of the Cartersian co-ordinates (x, y) shown in Fig. 1. [40%]

(c) Using the above Airy stress function or otherwise solve for the shear stress σ_{xy} due to a normal uniform pressure p applied on the edge $y = 0$ in a band $-a \le x \le a$. Hence also determine the distribution of σ_{xy} on the plane $y = a$. [40%]

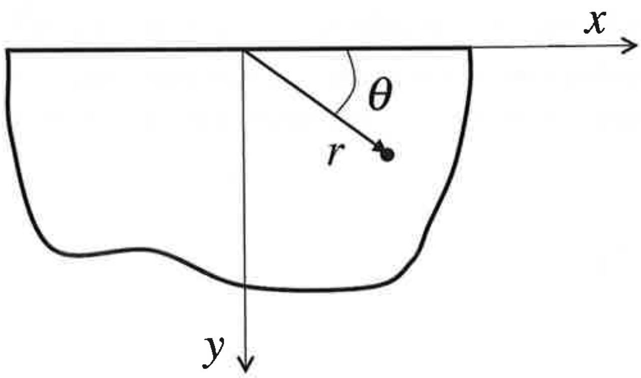

Fig. 1

3 (a) What assumptions are needed for Drucker's postulate to be valid? Why are the postulates important for the development of modern plasticity theory? [20%]

(b) For a metallic material satisfying the von Mises yield criterion, show that $\dot{J}_2 = s_{ij}\dot{\sigma}_{ij}$ where σ_{ij} is the stress tensor, $J_2 = 0.5 s_{ij} s_{ij}$ is the second invariant of the deviatoric stress s_{ij} and the overdot indicates the time derivative of the respective variable. [30%]

(c) A metallic solid has a Young's modulus E, an initial uniaxial yield strength σ_Y, and a post-yield tangent modulus E_t such that it obeys the following bilinear uniaxial stress σ versus strain ε relation,

$$\varepsilon = \begin{cases} \sigma / E & \text{for } 0 < \sigma < \sigma_Y \\ \sigma_Y / E + \left[(\sigma - \sigma_Y)/E_t\right] & \text{for } \sigma \geq \sigma_Y \end{cases}$$

This material is loaded such that σ_{12} is held fixed at a value $\sigma_Y / \sqrt{3}$ and σ_{11} increased gradually. Assuming that all other stress components equal zero derive an explicit formula for the strain component ε_{11} as a function of σ_{11} on the basis of J_2 flow theory. [50%]

Hint: $\int (1+x^2)^{-1}\, dx = \tan^{-1} x$

END OF PAPER