ENGINEERING TRIPOS PART IIB

Tuesday 7 May 2013 2 to 3.30

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4C15 datasheet (3 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain briefly what is meant by the term Hertzian when applied to a nominal point contact. What physical effects can lead to deviations from Hertzian behaviour when dealing with contacts at the microscale? [20%]

(b) Figure 1 shows a spherical indenter of radius R in contact with a plane surface. The contact modulus is $E*$. Burnham, Colton and Pollock (BCP) have suggested that the relation between the normal contact force P and the radius of the resulting circular contact patch a can be expressed as

$$P = \frac{4E*}{3R}a^3 - \sqrt{2\pi wE*}\, a^{3/2} - \pi wR$$

where w is the work of adhesion of the two materials concerned.

(i) Comment briefly on the likely physical origin of each of the terms on the right hand side by comparing with the corresponding relationship arising from the Johnson-Kendall-Roberts (JKR) analysis. [20%]

(ii) When the contact force P falls to zero, JKR suggests that the contact spot will be of radius $\left(9\pi wR^2/2E*\right)^{1/3}$. Will BCP predict a larger or smaller contact radius under these conditions? [20%]

(iii) For both JKR and BCP what is the relevance of the value of P when $\frac{dP}{da} = 0$? [20%]

(iv) For given values of radius R and work of adhesion w which analysis predicts the larger pull-off force and by what factor do they differ? [20%]

(cont.

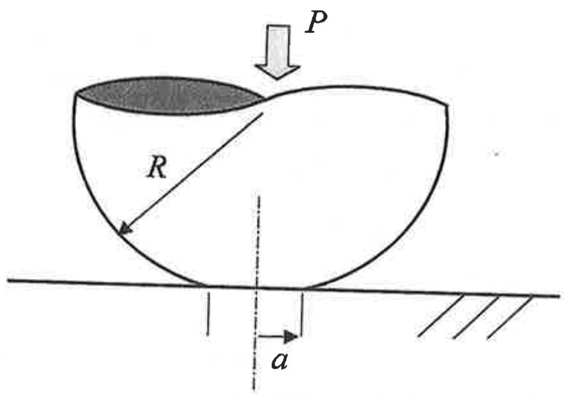


Fig. 1

(TURN OVER

2 A comb drive actuator is attached to a suspended proof mass as shown in schematic in Fig. 2. The thickness of the structural layer is t , the gap between adjacent electrodes is g and the number of actuator gaps is N . The comb drive actuator is used to generate linear motion in a specified drive direction for a proof mass compliant along two axes. The spring constant in the drive direction is k_x and the spring constant in the orthogonal direction is k_y as shown. A voltage V is applied between the comb drive and the proof mass. The nominal overlap of the electrodes is a when V=0.

(a) Derive the expression for the force generated by a comb drive actuator along the drive direction from first principles. [20%]

(b) Derive an expression for the force generated by the comb drive for motion in the direction orthogonal to the drive direction. [20%]

(c) The static displacement of the proof mass is associated with a sideways instability in the direction orthogonal to the drive direction. Derive an expression for the critical voltage at which sideways instability is first observed. [40%]

(d) Write down an expression for the displacement of the proof mass in the drive direction at the onset of sideways instability. What device parameters must be optimised to allow for large displacements? [20%]

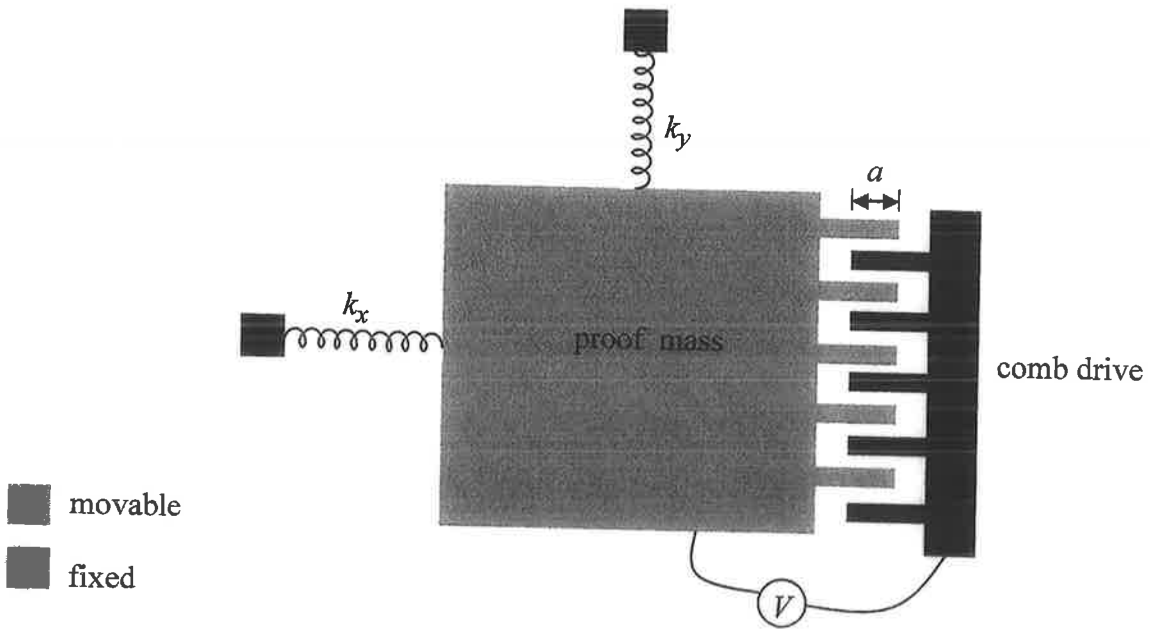


Fig. 2

3 A single-axis vibratory MEMS gyroscope is designed in a surface micromachining process. The natural frequencies of the gyroscope in the drive and sense directions are ω_x and ω_y respectively and the Quality factors in the drive and sense directions are Q_x and Q_y respectively. The effective mass of the device for motion along both directions is m. The Coriolis force couples the motion of the mass between the drive and sense directions for an input rotation about an axis orthogonal to both the drive and sense directions. All other coupling forces are assumed to be negligible for this analysis.

(a) Write down the equation of motion for the mass along the sense direction by assuming that an external force acts along the drive direction to produce constant amplitude harmonic motion along that direction, and a constant input rotation with angular rate Ω is applied about a third axis orthogonal to both drive and sense axes. [10%]

(b) Derive an expression for the mechanical sensitivity of the device. This expression will relate the sense mode displacement y to the constant input rotation rate Ω. The displacement along the drive direction x is assumed to be harmonic and coincident with the resonant frequency in the drive direction such that $x = A\cos(\omega_x t)$ where A is a constant. [30%]

(c) Assume now that the input rotation is not constant such that $\Omega = \Omega_o \cos(\lambda t)$ where λ is a constant. Obtain a modified expression for mechanical sensitivity and use this result to discuss the impact of the bandwidth-sensitivity trade-off for vibratory gyroscopes. [30%]

(d) Discuss the origin of thermo-mechanical noise. Derive an expression for the thermo-mechanical noise floor of the device in terms of the given device parameters, the ambient Temperature T and the Boltzmann constant k_B. Comment on the device parameters to be optimised to minimise the impact of thermo-mechanical noise on device accuracy. [30%]

4 A laterally driven microresonator is designed with an effective mass m, resonant frequency ω_r and Quality factor Q_r . A parallel plate electrode of overlap area A is employed for actuation and a comb drive configuration is employed for sensing the capacitive current resulting from the motion of the resonator. The structural thickness of the device is t and the nominal gap between electrodes is g. A dc polarization voltage V_p is applied to the microresonator body while the sense electrode is grounded through a load resistor R_L. Additionally, a small signal ac voltage v_{ac} is applied to the resonator drive electrode to actuate the device dynamically such that $|v_{ac}| << V_p$.

(a) Derive an expression for the transfer function relating resonator current to the input drive voltage.

[20%]

(b) Construct an equivalent electrical circuit model for the microresonator and find expressions for the resonator motional parameters.

[30%]

(c) Discuss the scaling of motional resistance with resonant frequency and discuss the design and process parameters that are critical in minimising motional resistance.

[20%]

(d) Discuss the limits to motional amplitude and the physical origins of non-linear effects.

[10%]

(e) The resonator describes an electro-static spring softening effect. Derive expressions for the variation of resonant frequency and Quality factor as a function of the applied resonator polarisation voltage.

[20%]

END OF PAPER

ENGINEERING TRIPOS Part IIB

Module 4C15 Data Sheet

Elastic Hertzian point contact under load P

Reduced radius R given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ (Suffixes 1, 2 refer to the two bodies in contact)

Contact modulus E^* given by $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

Radius of contact circle $a = \left\{\frac{3PR}{4E^*}\right\}^{1/3}$

Maximum contact pressure $p_0 = \frac{3P}{2\pi a^2} = \left\{\frac{6PE^{*2}}{\pi^3 R^2}\right\}^{1/3}$

Mean contact pressure $\bar{p} = \frac{2}{3} p_0$

Approach of distant points $\delta = \frac{a^2}{R} = \left\{\frac{9P^2}{16RE^{*2}}\right\}^{1/3}$

Maximum shear stress is of magnitude $0.31 p_0$ and at depth $0.48a$.

Lennard-Jones potential between point atoms

$$U(r) = -\frac{C}{r^6} + \frac{D}{r^{12}} = -4U_0\left\{\left(\frac{1.12r}{r_0}\right)^{-6} - \left(\frac{1.12r}{r_0}\right)^{-12}\right\}$$

where U_0 is bond energy and r_0 is bond length, i.e. spacing at which $U(r)$ is minimum.

Smooth surface adhesion $p(h) = \frac{8w}{3h_0}\left\{\left(\frac{h}{h_0}\right)^{-3} - \left(\frac{h}{h_0}\right)^{-9}\right\}$

w is the work of adhesion, in principal $w = \gamma_1 + \gamma_2 - \gamma_{12}$

Elastic spherical contact with adhesion, JKR $\frac{4E^* a^3}{3R} = P + 2\sqrt{2\pi w E^* a^3}$

Pressure drop across meniscus $\Delta p = \frac{\gamma}{r}$ for each liquid/vapour interface

Yield stress in shear $k \approx H/6$

Archard wear dimensional wear rate $\propto \frac{\text{pressure} \times \text{sliding speed}}{\text{hardness } H}$

SURFACE ENERGIES AT ROOM TEMPERATURE*

High energy solids

Material	Surface energy mJ m^{-2}
NaCl	160
Al_2O_3	641
Si	1280
Al	1120
Ag	1440
Fe	2400
W	4490

Low energy solids

Material	Surface energy mJ m^{-2}
nylon	46.5
polyvinyl chloride	38.9
polystyrene	33.0
polyethylene	30.4
paraffin wax	25.0
PTFE	18.3
Diamond-Like-Carbon	25~40

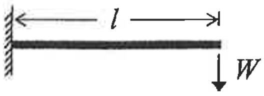
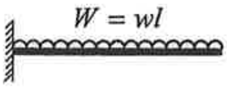
Liquids

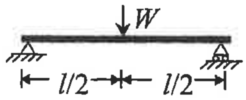
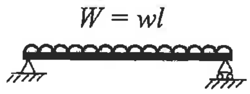
Material	Surface energy mJ m^{-2}
water	73.1
benzene	28.8
n-pentane	16.0
n-octane	21.6
n-dodecane ($C_{12}H_{26}$)	25.5
n-hexadecane ($C_{16}H_{34}$)	27.6
n-octadecane ($C_{18}H_{38}$)	28.0
Fomblin Zdol	20~25

* from: Adamson, A. W., *Physical Chemistry of Surfaces*, Wiley (1990)
and Israelachvili, J., *Intermolecular and Surface Forces*, Academic Press (1992)

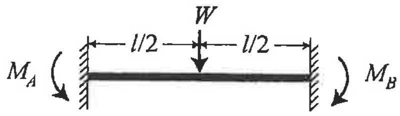
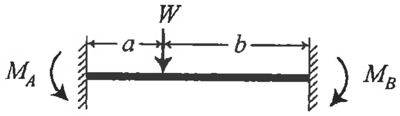
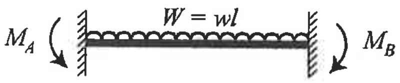
FORMULAE for ELASTIC ANALYSIS

Statically determinate structures

	end rotation	end deflection
	$\frac{Wl^2}{2EI}$	$\frac{Wl^3}{3EI}$
	$\frac{Wl^2}{6EI} = \frac{wl^3}{6EI}$	$\frac{Wl^3}{8EI} = \frac{wl^4}{8EI}$
	$\frac{Ml}{EI}$	$\frac{Ml^2}{2EI}$

	end rotation	central deflection
	$\frac{Wl^2}{16EI}$	$\frac{Wl^3}{48EI}$
	$\frac{Wl^2}{24EI} = \frac{wl^3}{24EI}$	$\frac{5Wl^3}{384EI} = \frac{5wl^4}{384EI}$
	$\theta = \frac{Ml}{3EI}$	

Clamping moments for statically indeterminate structures

	M_A	M_B
	$\frac{Wl}{8}$	$\frac{Wl}{8}$
	$\frac{Wb^2a}{l^2}$	$\frac{Wa^2b}{l^2}$
	$\frac{Wl}{12} = \frac{wl^2}{12}$	$\frac{Wl}{12} = \frac{wl^2}{12}$
	$\frac{6EI\delta}{l^2}$	$\frac{6EI\delta}{l^2}$
	$\frac{2EI\theta}{l}$	$\frac{4EI\theta}{l}$