ENGINEERING TRIPOS PART IIB

Monday 6 May 2013 9.30 to 11

Module 4F1

CONTROL SYSTEM DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Formulae sheet (3 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed
Supplementary pages: Two extra copies of Fig. 1 (Question 3).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 An elementary model to study rider-bicycle stability gives the following transfer function from steering angle input to tilt angle:

$$\alpha V \frac{s+\beta V}{s^2-\gamma} \tag{1}$$

where α, β, γ are positive constants and V is the forward speed.

(a) For $V > 0$ sketch the root-locus diagram (for negative feedback with gain $k > 0$) in the two cases:

(i) $\beta V < \sqrt{\gamma}$, [10%]

(ii) $\beta V > \sqrt{\gamma}$. [10%]

Verify that the breakaway points are both in the left half plane in case (a)(ii). [5%]

(b) Assume $\beta V > \sqrt{\gamma}$. Sketch the general form of the Bode diagram of (1). [20%]

(c) Use your answer to part (b) to sketch the general form of the Nyquist diagram of (1). Use the Nyquist stability criterion to find the number of right half plane poles of the closed loop system when constant gain k is applied with negative feedback, as k varies. [20%]

(d) Comment briefly on the ease of control of this bicycle model. [10%]

(e) The transfer function (1) with $V < 0$ can be taken as a model of a rear-wheel steered bicycle. Explain with reference to a root-locus diagram why the model can never be stabilised with a controller $K(s)$ which has all its poles in the left half plane in the case that $\beta|V| < \sqrt{\gamma}$. What can be said about the ease of control of a rear-wheel steered bicycle? [25%]

2 (a) An uncertain system is modelled as

$$G(s) = (1 + \Delta(s))G_0(s) \tag{2}$$

where $G_0(s)$ is a known transfer function and $\Delta(s)$ is assumed only to be stable and to satisfy a bound $|\Delta(j\omega)| < h(\omega)$ for all ω. Let $K(s)$ stabilise $G_0(s)$ with unity gain negative feedback. Show that a necessary and sufficient condition for $K(s)$ to stabilise $G(s)$ is that

$$\left|\left(\frac{G_0(j\omega)K(j\omega)}{1 + G_0(j\omega)K(j\omega)}\right)\right| \leq h(\omega)^{-1} \tag{3}$$

for all ω. State clearly any results you use. [20%]

(b) An uncertain system has the transfer function

$$G(s) = \frac{as + b}{s^2 + 10s} \tag{4}$$

where the parameters a and b satisfy $-2 < a < 4$ and $1 < b < 3$. Define $G_0(s)$ by setting $a = 1$ and $b = 2$. Let $K(s)$ stabilise $G_0(s)$ and define

$$S(s) = \frac{1}{1 + G_0(s)K(s)} \quad \text{and} \quad T(s) = \frac{G_0(s)K(s)}{1 + G_0(s)K(s)}.$$

(i) Show that (4) can be contained within the class defined by (2) for [20%]

$$h(\omega) = \sqrt{\frac{9\omega^2 + 1}{\omega^2 + 4}}.$$

(ii) Using the fundamental relationship between $S(s)$ and $T(s)$, and assuming (3) holds, determine the frequencies at which it is impossible to achieve $|S(j\omega)| < 1/2$. [20%]

(iii) Let $K(s) = k$ be a positive constant. Show that (3) holds if and only if

$$0 \leq \omega^4 + \omega^2(-8k^2 + 16k + 100) + 3k^2 \tag{5}$$

for all ω. [20%]

(iv) For $K(s) = k$ it can be shown that $G_0(s)$ is stabilised and (5) holds if and only if $0 < k < 4.955$. Find directly the full range of k for which $K(s) = k$ will stabilise $G(s)$ and comment on the fact that this is a larger range. [20%]

3 Fig. 1 is the Bode diagram of a system $G(s)$ which is the transfer function relating forward speed to actuator input for a mobile aerial vehicle. A feedback compensator $K(s)$ in the standard negative feedback configuration is to be designed for the system. It is known that $G(s)$ has two poles satisfying $\mathrm{Re}(s) > 0$.

(a) Let $G(s) = G_m(s)B(s)$ where $G_m(s)$ is stable and minimum phase and $|B(j\omega)| = 1$ for all ω. Sketch on a copy of Fig. 1 the phase of $G_m(j\omega)$ and $B(j\omega)$. [15%]

(b) Use your sketch to find an estimate of the transfer function $B(s)$. [20%]

(c) Comment on any limitations that might be imposed on the achievable crossover frequency of the control system. [10%]

(d) Use a sketch of the Nyquist diagram of $G(s)$ to determine the number of right half plane poles of the closed-loop system for each k, for $K(s) = k$ (constant), as k varies over positive and negative values. [15%]

(e) Design a compensator $K(s)$ to stabilise the system and achieve a phase margin of at least 30°. Show the Bode diagram of your compensator and the resulting return ratio on a copy of Fig. 1. Justify that the system achieves closed-loop stability using a sketch of the modified Nyquist diagram. [40%]

Two copies of Fig. 1 are provided on separate sheets. These should be handed in with your answers.

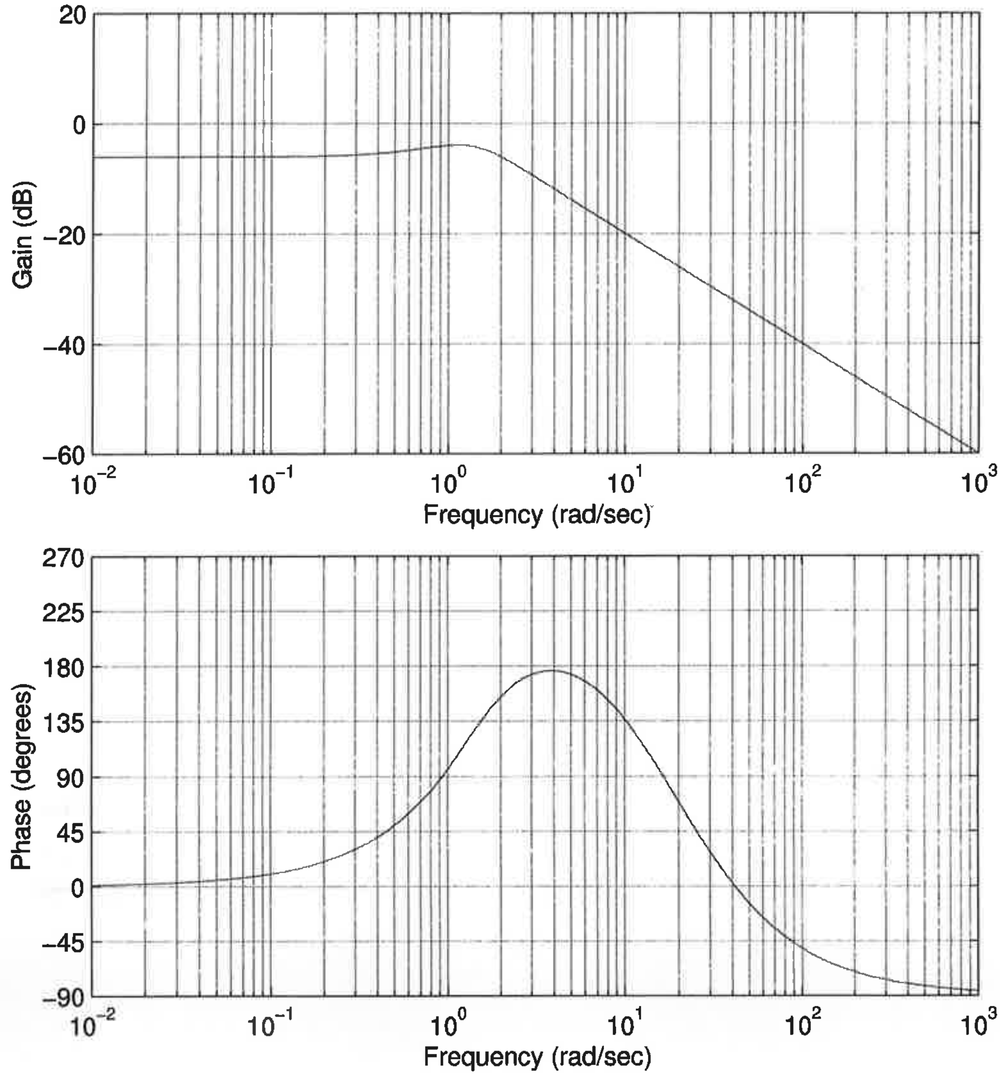

Fig. 1

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