ENGINEERING TRIPOS PART IIB

Thursday 25 April 2013 2 to 3.30

Module 4F2

ROBUST AND NONLINEAR CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed
Supplementary pages: One extra copy of Fig. 2 (Question 3).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

jmm05

1 (a) (i) How is the $\mathscr{L}_2$-norm, $\|u\|_2$, of a time domain signal $u(t)$ defined? [10%]

(ii) Let $G(s)$ be the transfer function of a stable linear time-invariant system. Show that

$$\sup_{u \neq 0} \frac{\|y\|_2}{\|u\|_2} = \|G(s)\|_\infty$$

where $y(t)$ denotes the output of the system for an input $u(t)$, and $\|\cdot\|_\infty$ denotes the H_∞ norm. [40%]

(b) Consider the lightly damped mechanical system with transfer function

$$G(s) = \frac{\omega_n^2}{s^2 + 2c\omega_n s + \omega_n^2} \quad \text{where} \quad \omega_n > 0 \quad \text{and} \quad c > 0.$$

(i) Show that $|G(j\omega)|$ has a non-zero maximising frequency at $\omega = \omega_n\sqrt{1-2c^2}$ when $c^2 < 1/2$. Hence deduce that

$$\|G(s)\|_\infty = \begin{cases} 1 & \text{if} \quad c^2 \geq \frac{1}{2} \\ \frac{1}{2c\sqrt{1-c^2}} & \text{if} \quad c^2 < \frac{1}{2} \end{cases}$$

[35%]

(ii) Briefly suggest a feedback control scheme which could reduce the H_∞ norm of the transfer function from input disturbance to output when c is small. [15%]

2 Let $G(s)$ be a $p \times m$ rational transfer function matrix.

(a) (i) Explain what is meant for $G = \tilde{M}^{-1}\tilde{N}$ to be a left coprime factorisation over H_∞, and state what is meant for the factorisation to be normalised. [20%]

(ii) Let $G = \tilde{M}^{-1}\tilde{N}$ be a normalised left coprime factorisation over H_∞. Suppose $G(s)$ is stabilised by a controller $K(s)$ in the standard positive feedback configuration and let $G(s)$ be perturbed to

$$G_\Delta = \left(\tilde{M} + \Delta_M\right)^{-1}\left(\tilde{N} + \Delta_N\right)$$

where Δ_M, Δ_N belong to H_∞ with $\|[\Delta_M, \Delta_N]\|_\infty < \varepsilon$. With the aid of an appropriate block diagram and making use of the small gain theorem show that the closed loop system remains stable for all such perturbations if and only if

$$b(G,K) = \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} \tilde{M}^{-1} \right\|_\infty^{-1} \geq \varepsilon.$$

[25%]

(b) Let

$$G(s) = \frac{3}{s-4}.$$

(i) Find a normalised left coprime factorisation of $G(s)$. [25%]

(ii) For the constant controller $K(s) = -2$ calculate $b(G,K)$. [30%]

3 (a) Explain what is meant by a *sector nonlinearity* in the context of nonlinear systems. [10%]

(b) State the *Circle criterion* for the nonlinear feedback system shown in Fig.1, in which $\psi(\cdot)$ is the gain of a sector nonlinearity satisfying $\alpha \leq \psi(\cdot) \leq \beta$. [15%]

(c) It is known that if the transfer function $G(s)$ which appears in Fig.1 is strictly positive-real, and $\psi(\cdot) \geq 0$, then the closed loop is globally asymptotically stable. By considering the transformation

$$\psi = \frac{\tilde{\psi} - \alpha}{\beta - \tilde{\psi}}$$

or otherwise, derive the Circle criterion. [40%]

(d) A linear system with transfer function

$$G(s) = \frac{24}{(s+1)(s+2)(s+3)}$$

has the Nyquist diagram shown in Fig.2 (shown for both negative and positive frequencies). This system is connected in feedback with a sector nonlinearity $\psi(\cdot)$, as shown in Fig.1. This nonlinearity lies in the sector $-k < \psi(\cdot) < k$. Find the largest value of k for which the global asymptotic stability of the closed-loop system can be assured. [35%]

A second copy of Fig.2 is provided, which can be handed in with your solution.

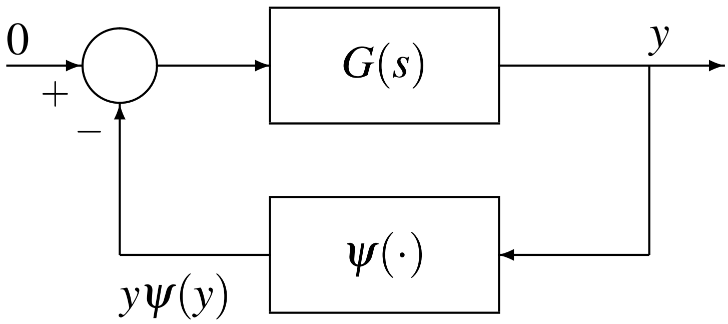
0
+
−
$G(s)$
y
$\psi(\cdot)$
$y\psi(y)$

Fig. 1

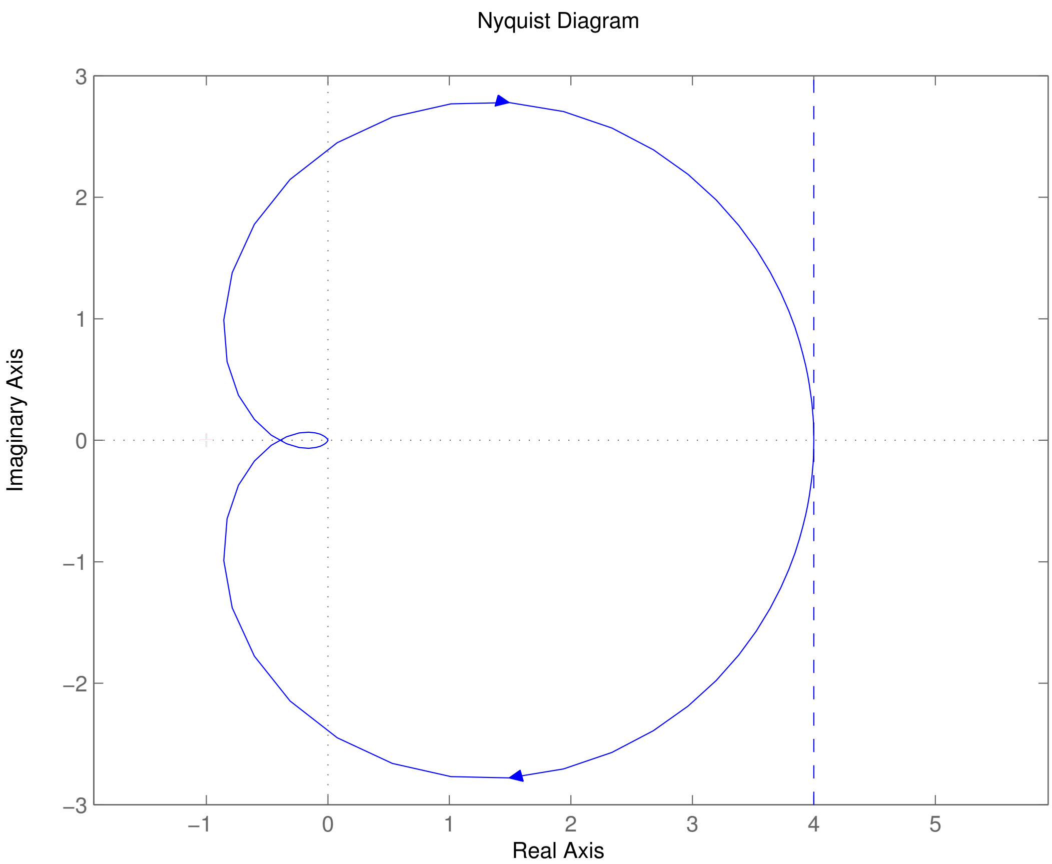
Nyquist Diagram
Imaginary Axis
Real Axis
3
2
1
0
−1
−2
−3
−1
0
1
2
3
4
5

Fig. 2

4 (a) State *LaSalle's Theorem*. [20%]

(b) Explain why LaSalle's Theorem is a useful extension to Lyapunov's theorems for proving the stability of some equilibrium points. [20%]

(c) A body of mass m moves along a straight-line track under the influence of a nonlinear friction force $b(\dot{x})$ and a nonlinear 'spring' force $c(x)$, where x is the position of the mass along the track. Its equation of motion is

$$m\ddot{x} + b(\dot{x}) + c(x) = 0 \tag{1}$$

The nonlinear functions satisfy the conditions: $b(0) = 0$, $c(0) = 0$, $vb(v) > 0$, $vc(v) > 0$ if $v \neq 0$.

Find any equilibrium points of this system. [20%]

(d) By considering the total energy

$$V(x, \dot{x}) = \frac{m\dot{x}^2}{2} + \int_0^x c(v)dv$$

show that the system described by equation (1) is globally asymptotically stable. [40%]

END OF PAPER

Candidate number

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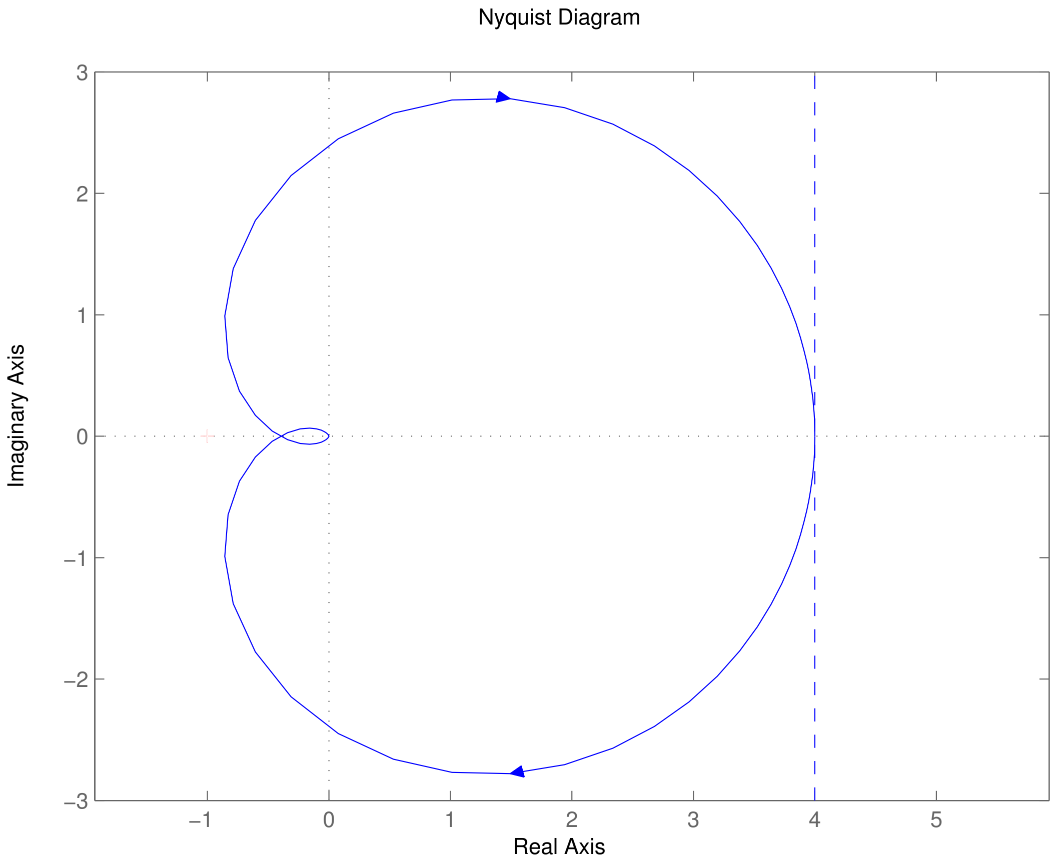
Thursday 25 April 2013, Module 4F2, Question 3.



This copy of Fig.2 may be handed in with your solution to Question 3.