ENGINEERING TRIPOS PART IIB

Monday 22 April 2013 9:30 to 11

Module 4F6

SIGNAL DETECTION AND ESTIMATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

wjf03

1 (a) Prove the *Neyman-Fisher Factorisation* theorem. [20%]

(b) The *scalar exponential family* of probability density functions for a random variable x may be written as

$$p(x|\theta) = \exp(A(\theta)B(x) + C(x) + D(\theta))$$

(i) If data $x(n)$ for $n = 0, 1, 2, \ldots, N-1$ are observed which are *iid* and whose probability density function belongs to this family, show that a *sufficient statistic*, $T(x)$, for the parameter θ is given by

$$T(x) = \sum_{n=0}^{N-1} B(x(n))$$

[40%]

(ii) Show that both the Gaussian and the exponential probability density functions belong to the *scalar exponential family* and that the sufficient statistics in both cases are given by

$$T(x) = \sum_{n=0}^{N-1} x(n)$$

[40%]

2 (a) Outline a proof of the *Cramer-Rao Lower Bound* inequality and show that

$$E\left(\left(\frac{\partial \ln p(x|\theta)}{\partial \theta}\right)^2\right) = -E\left(\frac{\partial^2 \ln p(x|\theta)}{\partial \theta^2}\right)$$

using the standard notation. [35%]

(b) Calculate the Cramer-Rao lower bounds for the estimation of the slope and intercept of a straight line model fitted to data, $d(n)$, given by

$$d(n) = A + Bn + w(n)$$

where $n = 0, 1, 2, \ldots, N-1$ and $w(n)$ is white Gaussian noise of variance σ^2. [30%]

(c) Show that for $N \geq 3$ it is easier to estimate B than it is to estimate A. [35%]

3 (a) Describe, in detail, the three main decision rules used in detection theory and discuss the advantages and disadvantages of each of these rules. [20%]

(b) Multiple observations are made, using a radar system, and when a target is present, the received signal samples are given by

$$y(n) = s(n) + e(n)$$

where $n = 1, 2, \ldots, N$, $s(n)$ is the known transmitted signal and $e(n)$ is zero mean white Gaussian noise with variance σ^2. With no target present, the received signal is just noise.

(i) Show that the likelihood ratio detector is given by

$$\mathbf{y}^\mathbf{T}\mathbf{s} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{1}{2}\mathbf{s}^\mathbf{T}\mathbf{s} + \sigma^2 \ln k$$

where k is the appropriate threshold depending on the decision rule chosen. [60%]

(ii) Give a new expression for the detector if the noise is coloured. [20%]

4 (a) Describe, in detail, the Neyman-Pearson decision rule applied to detection theory and discuss the advantages and disadvantages of this decision rule over the MAP and Bayes' criteria. [30%]

(b) Show that the value of the threshold for the Neyman-Pearson test for a single observation is given by the slope of the receiver operating characteristic (ROC) at the required false alarm probability. [40%]

(c) Based on n statistically independent Gaussian samples with variance σ^2, determine the likelihood ratio test to choose between the hypotheses that their mean is zero (H_0) or that their mean is one (H_1). [30%]

END OF PAPER