ENGINEERING TRIPOS PART IIB

Wednesday 24 April 2013 9.30 to 11

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) When an image $g(u_1,u_2)$ is sampled on a rectangular grid (spacings Δ_1 and Δ_2) the sampled image $g_s(u_1,u_2)$ may be written as

$$g_s(u_1,u_2) = s(u_1,u_2)g(u_1,u_2)$$

where $s(u_1,u_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \delta(u_1 - n_1\Delta_1, u_2 - n_2\Delta_2)$

(i) By writing s as a Fourier series, find $G_s(\omega_1,\omega_2)$, the Fourier transform of g_s, in terms of $G(\omega_1,\omega_2)$, the Fourier transform of g. Hence explain the phenomenon of *aliasing*. [30%]

(ii) Discuss the advantages and disadvantages of sampling on grids other than rectangular grids. [10%]

(iii) $g(u_1,u_2)$ is a 2-dimensional sinewave of the following form

$$g(u_1,u_2) = \sin(\Omega u_1)\ \sin(3\Omega u_2)$$

If the sample spacings, Δ_1 and Δ_2 in the u_1 and u_2 directions respectively, are $\Delta_1 = \Delta_2 = \frac{\pi}{\Omega}$, sketch the spectrum of the sampled signal. [20%]

(b) Consider the ideal bandpass filter, $H(\omega_1,\omega_2)$, shown in Fig. 1, with $H = 1$ in the shaded regions and 0 otherwise.

Sampling is done on a rectangular grid with spacings of Δ_1 and Δ_2 in the u_1 and u_2 directions. Show that the ideal impulse response, $h(n_1,n_2)$, of this filter is given by

$$h(n_1,n_2) = K_1 \mathrm{sinc}(\Omega_{U1}n_1\Delta_1)\,\mathrm{sinc}(\Omega_{L2}n_2\Delta_2) - K_2 \mathrm{sinc}(\Omega_{L1}n_1\Delta_1)\,\mathrm{sinc}(\Omega_{U2}n_2\Delta_2)$$
$$-K_3 \mathrm{sinc}(\Omega_{L1}n_1\Delta_1)\,\mathrm{sinc}(\Omega_{L2}n_2\Delta_2)$$

and give the values of the constants K_i, $i = 1,2,3$, in terms of the sampling intervals and frequency bounds. Comment on whether we are correct in assuming there is no aliasing in this scenario. [40%]

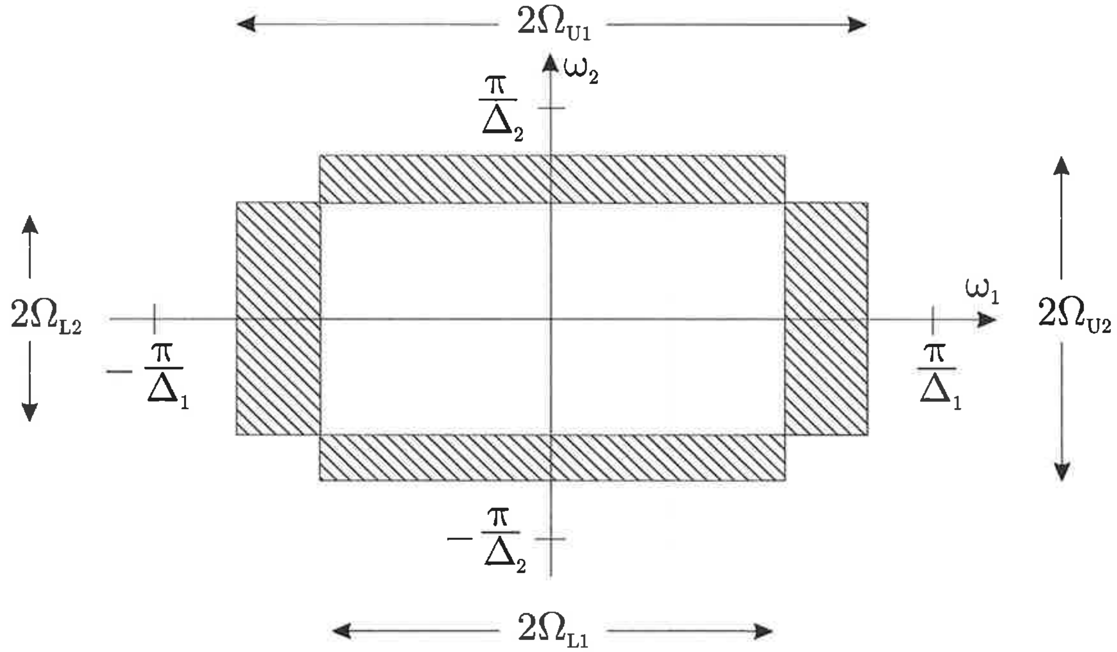
$2\Omega_{U1}$
ω_2
$\frac{\pi}{\Delta_2}$
$2\Omega_{L2}$
$-\frac{\pi}{\Delta_1}$
ω_1
$\frac{\pi}{\Delta_1}$
$2\Omega_{U2}$
$-\frac{\pi}{\Delta_2}$
$2\Omega_{L1}$

Fig. 1

2 (a) Taking the inverse Fourier transform of an ideal zero-phase 2-D frequency response will normally create an impulse response with infinite support. *Windowing* can then be applied to produce a finite support filter.

(i) Outline the *product* and *rotation* methods for forming 2-D window functions from 1-D window functions and describe the effects of windowing on the ideal frequency response, noting the properties of a good windowing function. [25%]

(ii) Consider the 2-D window function $w(u_1,u_2)$ formed from the product of two 1-D window functions $w_i(u_i)$, for $i = 1,2$, where

$$w_i(u_i) = \begin{cases} t(u_i) + 0.5 & \text{if } |u_i| < U_i \\ 0 & \text{otherwise} \end{cases}$$

with $t(u_i)$ a triangular window centred on the origin of height 1 and width $2U_i$. Find the spectrum of the 2-D window function formed from $w_1(u_1)w_2(u_2)$. Sketch both the window and the spectrum for $w_1(u_1)$. [35%]

(iii) Comment on how the addition of the constant term to the triangular window effects the mainlobe and sidelobe behaviour. [10%]

(b) We have an observed image, $y(\mathbf{n})$, which can be modelled as a linear distortion of a true image, $x(\mathbf{n})$, plus additive noise, $d(\mathbf{n})$:

(i) If we can model the distortion via a point spread function $h(\mathbf{n})$, give an expression for y in terms of x, h and d. [10%]

(ii) We can obtain an estimate, $\hat{x}(\mathbf{n})$, of the true image via convolving the observed image with a linear filter $g(\mathbf{n})$, i.e. $\hat{x}(\mathbf{n}) = \sum_{\mathbf{q}\in\mathbf{Z}^2} g(\mathbf{q})y(\mathbf{n}-\mathbf{q})$. Describe the cost function we minimise to obtain the expression for $g(\mathbf{n})$ known as the *Wiener filter*. [10%]

(iii) Describe briefly why the Wiener filter often performs poorly and how we might improve deconvolution performance. [10%]

3 (a) The transform matrix T for a 4-point discrete cosine transform (DCT) can be represented as:

$$T = \begin{bmatrix} a & a & a & a \\ b & c & -c & -b \\ a & -a & -a & a \\ c & -b & b & -c \end{bmatrix}$$

Explain why it is desirable for the matrix to be orthonormal and calculate the values of the positive coefficients a, b and c. [25%]

(b) Show how T may be used to compute Y, the two-dimensional DCT of a 4×4 matrix of pixels X. Show also how X may be computed easily from a given Y. [15%]

(c) By considering matrices Y, comprising just one non-zero element of unit value (or otherwise), determine the first 6 of the 16 basis functions of the two-dimensional 4×4 DCT in terms of a, b and c. (The first 6 are those corresponding to coefficients in the upper left corner of Y. Use matrix symmetries where possible to simplify your answers.) [35%]

(d) In the JPEG XR image compression standard, the 4-point DCT is applied to 4×4 blocks of pixels in the input image, and then it is applied at a second level to blocks of certain coefficients from the first level. Describe which coefficients are transformed at the second level and discuss reasons for this choice. Additionally, suggest reasons why this 2-level approach, based on 4×4 DCTs, is preferred in JPEG XR over the single-level 8×8 DCT approach of basic JPEG. [25%]

4 (a) Figure 2(a) shows a digital filter, with transfer function $H(z)$, preceded by a 2:1 downsampler and followed by a 2:1 upsampler. Write down an expression relating the output samples, $\hat{y}(n)$, to the input samples, $x(n)$, and the filter impulse response coefficients, $h(k)$, $k = 1 \ldots K$. Hence show that the systems in Fig. 2(b) and Fig. 2(c) are both equivalent to the system in Fig. 2(a).

(You may assume that if $\hat{y}(n) = y(n)$ for n even, and $\hat{y}(n) = 0$ for n odd, then, in the z-domain, $\hat{Y}(z) = \frac{1}{2}[Y(z) + Y(-z)]$.) [30%]

(b) The following filters are used to form a perfect-reconstruction 2-band filter bank:

Analysis filter:

$$H_0(z) = \frac{1}{4}(-z^2 + 2z + 6 + 2z^{-1} - z^{-2}), \quad H_1(z) = \frac{1}{4}z^{-1}(-z + 2 - z^{-1})$$

Reconstruction filter:

$$G_0 = \frac{1}{4}(z + 2 + z^{-1}), \quad G_1(z) = \frac{1}{4}z(-z^2 - 2z + 6 - 2z^{-1} - z^{-2})$$

These analysis filters are used in a one-dimensional, two-level wavelet transform system. Derive expressions for the z-transfer functions from the input of the transform to the two level-2 outputs, $H_{00}(z)$ and $H_{01}(z)$. In addition, derive the equivalent transfer functions, $G_{00}(z)$ and $G_{01}(z)$, if the filter pairs were swapped between analysis and reconstruction. [30%]

(c) Explain how the z-transfer functions from part (b) can be combined to give impulse responses of 2-D wavelet transforms. [20%]

(d) By considering H_0 and H_1, or G_0 and G_1, given in part (b), determine whether the filters should be used as specified for analysis and reconstruction or swapped, in order to obtain optimum performance in an image compression system. [20%]

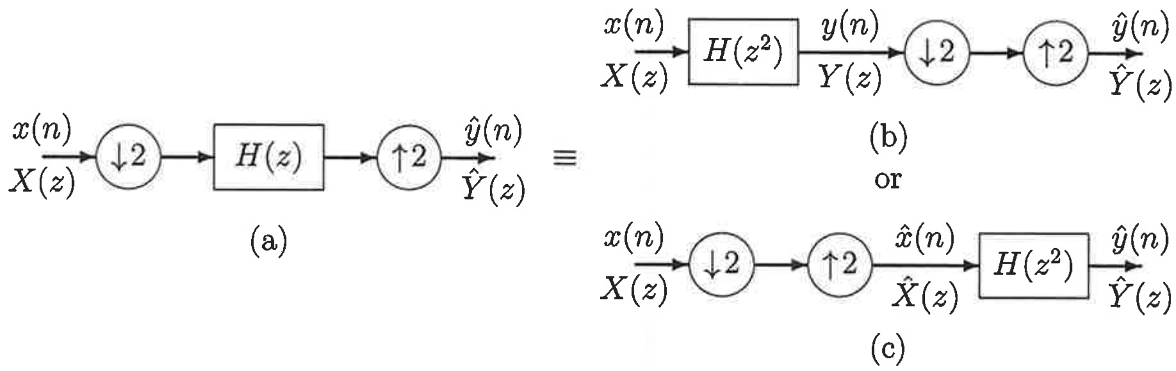

Fig. 2

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