ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Monday 22 April 2013 2 to 3.30

Module 4M16

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4M16 data sheet (8 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

gtp03

1 (a) What are the advantages of using reflectors in thermal power reactors? What characteristics should the material used for a reflector have? [15%]

(b) The equation for the neutron flux ϕ in one-group steady-state diffusion theory for a source-free homogeneous *multiplying* medium in cylindrical symmetry can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{\partial^2\phi}{\partial z^2}+B^2\phi=0.$$

Show that the general solution to this equation for a right circular cylindrical core is

$$\phi(r,z)=\phi_0 J_0(\alpha r)\cos(\beta z)$$

where J_0 is an ordinary Bessel function of zero order and ϕ_0, α and β are constants determined by the boundary conditions. Assume that the origin of the coordinate system is at the centre of the core. What is the relationship between α, β and B? [45%]

(c) The equivalent equation for a source-free homogeneous *non-multiplying* medium can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{\partial^2\phi}{\partial z^2}-\frac{1}{L^2}\phi=0$$

where L is the diffusion length.

This equation is used to model the flux distribution in *axial reflectors*, placed at both ends of the cylindrical core considered in (b). Under certain conditions the resulting flux distribution within the reflector above the core ($z>0$) is given by

$$\phi(r,z)=\phi_1 J_0(\alpha r)\exp(-\gamma z)$$

where α is the same as in (b) and ϕ_1 and γ are constants determined by the boundary conditions. Find the relationship between α, γ and L. [15%]

(d) The core-reflector boundary conditions require that the flux ϕ and the neutron current $D\frac{\partial\phi}{\partial z}$ are continuous across the core-reflector interfaces at $z=\pm\frac{H}{2}$. Show that these conditions imply that for the case under consideration

$$\gamma=\frac{D_c}{D_r}\beta\tan\left(\frac{\beta H}{2}\right)$$

where D_c and D_r are the diffusion coefficients in the core and reflector respectively. [25%]

2 (a) For axial coolant flow in a reactor with a 'chopped' cosine power distribution, Ginn's equation for non-dimensional temperature is

$$\theta = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L' is the flux half-length of the power distribution.

Show that the maximum non-dimensional temperature $\theta_{\max}$ occurs at a distance along the channel

$$x = \frac{2L'}{\pi}\tan^{-1}\left(\frac{1}{Q}\right)$$

and is given by

$$\theta_{\max}{}^2 = 1 + Q^2$$ [35%]

(b) The central channel of a Pressurised Water Reactor is rated at 32.0 MW(th). The fuel assembly there contains 264 fuel pins, each extending the length of the channel. The temperature of the coolant rises from 285 °C at inlet to 315 °C at outlet. Each fuel pin consists of a UO_2 solid pellet of 9.3 mm diameter and 0.6 mm thick zircaloy cladding. The axial power distribution is cosinusoidal over the 4 m active fuel length. The flux half-length of the power distribution $L' = 2.5$ m.

(i) Sketch the form of the variation along the channel of the coolant temperature and the fuel pin temperature. [10%]

(ii) Show that the quantity $\dot{m}c_p$ is just over $4\ \mathrm{kWK^{-1}}$ for the central channel, where $\dot{m}$ is coolant mass flow rate per pin and c_p is the specific heat capacity of the coolant. [10%]

(iii) Find the maximum temperature in the fuel pins and its location axially along the channel. Assume there is no scale on the cladding. [45%]

Data:	Heat transfer coefficient to coolant	$h = 35\ \mathrm{kWm^{-2}K^{-1}}$
	Thermal conductivity of cladding	$\lambda_c = 12\ \mathrm{Wm^{-1}K^{-1}}$
	Bond heat transfer coefficient, fuel to cladding	$h_b = 25\ \mathrm{kWm^{-2}K^{-1}}$
	Thermal conductivity of fuel	$\lambda_f = 3\ \mathrm{Wm^{-1}K^{-1}}$

3 (a) In a 'lumped' model of the kinetic behaviour of a reactor operating at low power, the equations for the neutron population $n(t)$ and the precursor population $c(t)$ can be written as

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where s is an independent source rate. Identify β, λ and Λ. What major simplifying assumptions underlie this model? [20%]

(b) Estimate the ratio of precursors to neutrons in steady-state operation in a typical Pressurised Water Reactor (PWR) for which $\beta = 0.007$, $\lambda = 0.1\ \text{s}^{-1}$ and $\Lambda = 10^{-4}$ s. [10%]

(c) A critical, source-free PWR has been operating in steady state and is subject to a step increase in reactivity at time $t = 0$. Show that the subsequent variation of the neutron population predicted by the *prompt jump approximation* is

$$n(t) = \frac{\beta}{\beta - \rho} n_0 \exp\left(\frac{\rho\lambda}{\beta - \rho} t\right)$$

where n_0 is the steady-state neutron population prior to the increase in reactivity and ρ is the reactivity level after the increase. [40%]

(d) Find the dominant time constant for the excursion predicted by this model of PWR kinetics for the case where $\rho = 0.003$. If the same reactivity change was made in an Advanced Gas-Cooled Reactor and in a Fast Breeder Reactor, explain qualitatively how the dominant time constant of the excursions would differ from that calculated for the PWR. [30%]

4 (a) Describe four ways of treating gaseous, liquid and solid radioactive wastes giving the advantages and disadvantages of each process in terms of applicability, cost and operator dose uptake. [60%]

(b) An aqueous waste stream arising at a rate of $0.1\ m^3hr^{-1}$ contains $20\ Bq\ g^{-1}$ of Ag-110m. Ag-110m decays to Cd-110 (stable) with a half-life of 252 days. The effluent is collected in a tank with a working capacity of $48\ m^3$ after which it is stored for 25 days before being passed through an ion exchanger with a decontamination factor of 10.

Stating any assumptions made, calculate the final Ag-110m activity of the effluent and comment on the result. [40%]

END OF PAPER

MODULE 4M16

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	$299.792458 \times 10^{6}\ \mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.62606957 \times 10^{-34}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892 \times 10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}\ \mathrm{kg}$ $(931.5016\ \mathrm{MeV})$
Electron volt	eV	$1.6021892 \times 10^{-19}\ \mathrm{J}$
Curie	Ci	$3.7 \times 10^{10}\ \mathrm{Bq}$
Barn	barn	$10^{-28}\ \mathrm{m^2}$

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

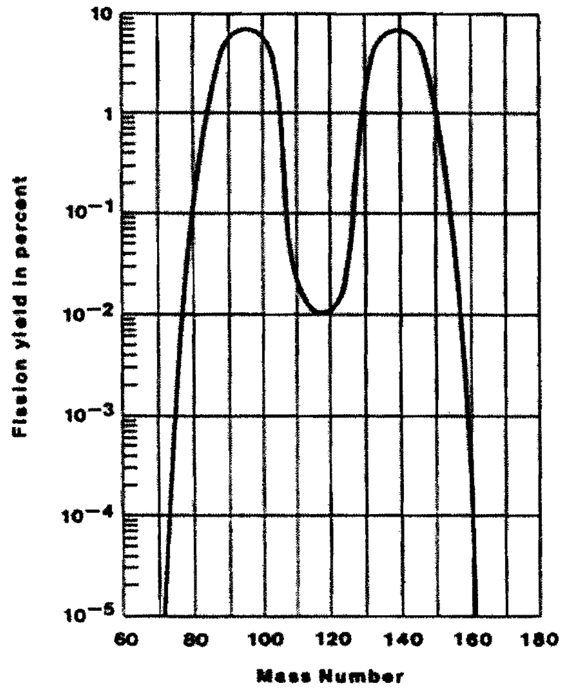
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90 $\xrightarrow{33\text{ s}}$ Rb-90 $\xrightarrow{2.7\text{ min}}$ Sr-90 $\xrightarrow{28\text{ yr}}$ Y-90 $\xrightarrow{64\text{ hr}}$ Zr-90

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131 $\xrightarrow{23\text{ min}}$ Te-131 $\xrightarrow{24\text{ min}}$ I-131 $\xrightarrow{8\text{ days}}$ Xe-131

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135 $\xrightarrow{6.7\text{ hr}}$ Xe-135 $\xrightarrow{9.2\text{ hr}}$ Cs-135 (nearly stable) Fission yield $\gamma = 0.064$

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn.

Pm-149 $\xrightarrow{54\text{ hr}}$ Sm-149 Fission yield $\gamma = 0.014$

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn.

Se-87 $\xrightarrow{17\text{ s}}$ Br-87 $\xrightarrow{55\text{ s}}$ either {Kr-86 + n} or {Kr-87 $\xrightarrow{78\text{ min}}$ Rb-87}

This chain leads to a "delayed neutron".

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	Total
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where n = neutron concentration

c = precursor concentration

β = delayed neutron precursor fraction $= \sum \beta_i$

λ = average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\overline{\mu})}$$

with $\overline{\mu}$ = the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $R(r)=A_1J_0(r)+A_2Y_0(r)$

$J_0(0)=1$; $Y_0(0)=-\infty$;

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $g\,cm^{-3}$	Σ_a cm^{-1}	D cm	$L^2=D/\Sigma_a$ cm^2
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$

Enrichment of Isotopes

Value function: $v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)$ for small x

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $R_n\equiv\frac{L_n''}{P}=\frac{x_p-x_{n+1}'}{x_{n+1}'-x_n''}$

Stripping section reflux ratio: $R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x_{n+1}'-x_w}{x_{n+1}'-x_n''}\right]$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i

λ_j = decay constant of nuclide j

P = parent nuclide production rate

T = filling time

τ = decay hold-up time after filling

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity