Version GRH/3

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 1 May 2019   9.30 to 11.10

Module 4A10

FLOW INSTABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachment: 4A10 Flow Instability data sheet (2 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Two inviscid liquids in a potentially unstable configuration are contained within an infinitely long vertical circular tube of radius $r = a$. The lower liquid extends from $z = 0$ to $z = -\infty$ and has density ρ_l. The upper liquid extends from $z = 0$ to $z = \infty$ and has density $\rho_u > \rho_l$. Initially the liquids are at rest, separated by a horizontal interface at $z = 0$ with interfacial tension γ. The tube is subjected to a small perturbation and any motion in the liquids, initiated from rest, is then irrotational. Adopting the cylindrical polar coordinate system (r, θ, z) with origin at the centre of the undisturbed interface, velocity potentials are sought of the form

$$\phi_u = Ce^{st} e^{-kz} \cos(n\theta)\, J_n(kr)$$
$$\phi_l = Ce^{st} e^{kz} \cos(n\theta)\, J_n(kr)$$

where C is a constant, s is the growth rate, t is time, k is the radial wavenumber, n is the circumferential wavenumber and $J_n(kr)$ is the Bessel function of the n^{th} order. The wall boundary condition requires that the radial wavenumber k satisfies

$$\frac{d}{dr} J_n(kr) = 0 \quad \text{at} \quad r = a \tag{1}$$

A linear stability analysis yields the result

$$s^2 = \frac{|\,k\,|\,\gamma}{\rho_l + \rho_u} \left[\frac{g(\rho_u - \rho_l)}{\gamma} - k^2 \right] \tag{2}$$

where g is the acceleration due to gravity.

(a) Explain the principles and approach behind a linear stability analysis such as that required to derive Equation (2), stating the relevant governing equations and boundary conditions. Include a description of why a normal mode analysis may be regarded as enabling the influence of all possible small amplitude disturbances to be assessed. [15%]

(b) Show that the configuration is unstable if

$$(ka)^2 < \frac{g(\rho_u - \rho_l)a^2}{\gamma}$$

[5%]

(c) Explain the physical significance of the quantity $g(\rho_u - \rho_l)a^2/\gamma$ and, with reference to the result given in (b), describe in words the conditions for the configuration to be unstable. [30%]

(d) In experiments conducted over a wide range of liquid densities and tube diameters, both axisymmetric and non-axisymmetric modes of flow have been observed to develop in the tube when perturbed. Clearly explain why the predictions of the linear stability analysis are consistent with these observations. Hint: you may assume that Equation (1) has the following roots:

$n = 0$, $ka = 3.83, 7.02, 10.17...$;

$n = 1$, $ka = 1.84, 5.33, 8.53...$;

$n = 2$, $ka = 3.05, 6.70, 9.97...$. [50%]

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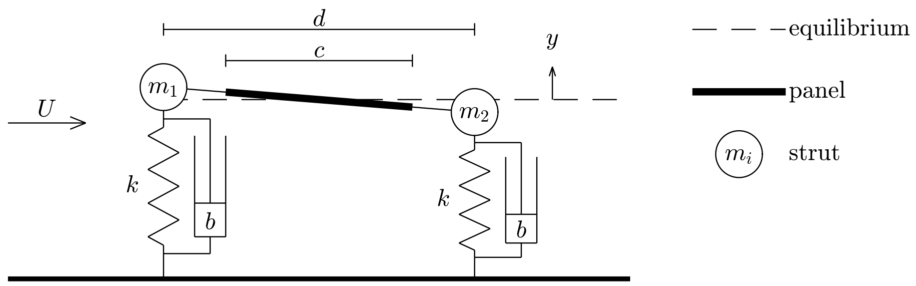

Fig. 1

2 As shown in Fig. 1, a canopy is to be modelled as a single panel of width c, attached to two tall heavy struts, distance d apart, with mass m_i, $i = 1,2$. The panel has negligible mass and its motion is constrained so as to be purely vertical. The vertical displacement of each strut from its equilibrium position is y_i, where $|y_i| \ll c$. The stiffness of the structure is modelled by assuming that each strut is held in position by a vertical spring of stiffness k and a damper with constant b such that the damping force is $-b\dot{y}_i$. Air moves parallel to the equilibrium position of the panel at speed U and can flow freely through the spaces between the struts and the panel.

(a) With the aid of a diagram, show that the mass-spring-damper system in Fig. 1 can be represented by a translation spring with stiffness $k_y = 2k$ and a torsional spring with stiffness $k_\theta = kd^2/2$, both acting through the same point, called the elastic axis. Derive expressions for the corresponding damping coefficients b_y and b_θ and the moment of inertia around the elastic axis, I_θ. [20%]

(b) The force on each panel due to the wind can be assumed to act through the point $1/4$ of the panel length from the leading edge of the panel and to be proportional to the effective angle of attack of the leading edge of the panel. Stating your assumptions, write down expressions for the vertical force on the panel, F_y, and the moment around the elastic axis, F_θ, due to the wind. [20%]

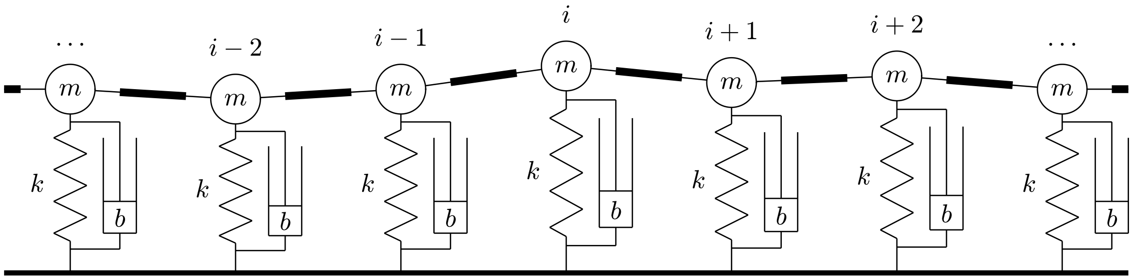

Fig. 2

(c) If S_x is defined such that the centre of mass is a distance S_x/m behind the elastic axis, where $m = m_1 + m_2$, then the translational and torsional equations of motion are:

$$m\ddot{y} + S_x\ddot{\theta} + b_y\dot{y} + k_y y = F_y \tag{3}$$

$$I_\theta\ddot{\theta} + S_x\ddot{y} + b_\theta\dot{\theta} + k_\theta\theta = F_\theta \tag{4}$$

Without further calculations, describe how to derive an algebraic expression for the parameter values at which the system is unstable. [20%]

(d) Without further calculations, describe how one could obtain the phase between the torsional and translational motions and how one could calculate the work done by the wind over a cycle. [20%]

(e) Without further calculation, describe how the analysis would differ for a series of N connected identical canopies, as shown in Fig. 2. How would the solution for N identical canopies differ qualitatively from that for an infinite number of identical canopies? [20%]

It may be assumed, without proof, that at least one solution of the quartic

$$C_0S^4 + C_1S^3 + C_2S^2 + C_3S + C_4 = 0$$

has a positive real part when

$$C_1C_2C_3 < C_0C_3^2 + C_4C_1^2$$

for constants C_0, C_1, C_2, C_3 and C_4.

3 (a) A long slender horizontal cylindrical liquid thread is moving at a steady velocity in still air. The liquid has density ρ and the surface tension is γ. Initially the thread has a radius $r = a$. When perturbed from this initial state, wave-like disturbances form, so that the radius of the thread varies as $r = \alpha + \beta \cos(kx)$, for some constant α. The coordinate x is measured along the axis of the thread, and β and k (≥ 0) denote the amplitude and wavenumber of the disturbance, respectively, where $\lambda = 2\pi k$ is the wavelength. For small amplitude disturbances, show that the ratio of the surface potential energy in the perturbed state to that in the initial state may be written as

$$\frac{PE_{perturbed}}{PE_{initial}} = 1 + \frac{\beta^2}{4a^2}(k^2a^2 - 1)$$

Hence, or otherwise, determine the wavelengths of disturbances that grow with time. [65%]

(b) Consider the Taylor-Couette problem, namely, of an inviscid liquid in the narrow annular gap formed between two long concentric co-rotating circular cylinders. Provide a physical argument to establish that the flow is stable provided that throughout the liquid

$$\frac{d}{dr}(\Omega r^2)^2 \geq 0$$

where Ω denotes the angular velocity of the fluid. [35%]

4 (a) A long cylindrical rigid rod is suspended horizontally from two springs in a wind tunnel. The air moves horizontally, perpendicular to the axis of the rod. The rod is constrained so as to move solely in the vertical direction, remaining horizontal. Describe the motion of the rod as the speed of the air increases from zero, noting critical values of the behaviour in terms of the wind speed U, the rod diameter d, the spring stiffnesses k, and the mass of the rod m. Repeat this description as the speed of the air decreases from a large value. [40%]

(b) Explain the above behaviour in terms of the absolute and convective instability of the velocity profile behind the rod. Illustrate your answer by sketching contours of complex angular frequency in the complex wavenumber plane for an absolutely unstable region and a convectively unstable region. [40%]

(c) With reference to your answers to (a) and (b), explain how and why these oscillations can be reduced by changing the shape of the rod. [20%]

END OF PAPER

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