

**Engineering Tripos Part IIA: Module 3C2**  
**Manufacturing Engineering Tripos Part I: Paper P4B**  
**Materials process modelling and failure analysis**  
**CRIB - 2004/5**

1 (a)(i) There are several effects operating here. The thermal cycling introduces differential stresses; the wet conditions can corrode the glass fibres; water which penetrates the composite can introduce stresses when it freezes; sunshine (UV) may degrade the matrix; the surface of the matrix may suffer wear (abrasion). Some polymers are plasticised by water; however, neither PP nor PVC is significantly affected.

The large difference in thermal expansion coefficient between the glass and the matrix introduces stresses on thermal cycling. PP is below its  $T_G$  at  $-30^\circ\text{C}$  but above at  $40^\circ\text{C}$ ; it may crack at the lower temperature and creep at the higher. PVC is below  $T_G$  at both temperatures. At the higher temperature, the matrix will be put into compression by the fibres expanding less. PP may creep to relieve the stress, which may cause some distortion. At the lower temperature, the matrix will be put into tension by the fibres; cracking of the matrix is likely to occur in both matrix materials. How much cracking can be expected at this lower temperature depends on the elastic modulus of the material and the strength and toughness. We cannot properly compare the materials at  $-30^\circ\text{C}$  because the databook properties are for room temperature, so will not be correct for PP which is below its  $T_G$  at  $-30^\circ\text{C}$ .

Cracking of the matrix may lead to delamination failure and cracking of the fibre-matrix interface; both may allow wicking of water into the interior of the structure. The formation of ice would enhance this effect. A major difference between the two materials is in their resistance to UV light. PP is rated D (poor); PVC is A (very good).

Applied stress (people using the seat) will introduce additional stresses and may encourage crack propagation. However, most street furniture of the type is robustly designed, so it is unlikely that these stresses will be significant. Wear is not likely to be a problem (the composite is more resistant than the matrix alone, on account of the hard fibres).

[40%]

(ii) The tests omit UV effects, which will degrade the matrix (particularly PP). They also omit applied stress. Because the thermal cycling was dry, it will not pick up wicking, nor will it pick up effects from freezing water expanding. This expansion will tend to force open delamination cracks and cracks at the fibre-matrix interface, so accelerating damage.

There should be additional UV tests. The thermal cycling should be done under wet conditions. Fatigue loading might be tested, particularly at stress concentrations or joints.

[15%]

(iii) PVC likely to be the better matrix material, because resistance to UV light is important for outdoor applications. The fact that the material does not cycle through its glass transition temperature also adds some dimensional stability.

[10%]

(b) Temper embrittlement occurs in such steels under these conditions: segregation of P, Sn, Sb, As to grain boundaries leads to fracture along grain boundaries. The effect requires the elements to reach a high enough concentrations. Reducing the grain size increases the amount of grain boundary area, so may reduce the concentrations below the critical value.

[20%]

(c) Stress corrosion cracking in the presence of chloride ions; the pre-requisite tensile strain is present. Precipitate free zones around the grain boundaries have a lower local yield stress than the rest of the material (though this doesn't much affect macroscopic tensile properties), so the material forms slip steps there which fracture the protective oxide layer which is normally present on aluminium alloys. In the presence of chloride ions the oxide cannot re-form, so the material cracks and appears to show brittle failure (although there is in fact likely to be microductility).

[15%]

[The main ideas were well addressed, but high marks required more details than were generally given, for example inclusion of wicking in (a), particular relevant where ice might form and examples of segregation in (c). The importance of thermal cycling under wet conditions was overlooked by many.]

2 (a) (i) Differentiating thermal cycle equation to find time to peak:

$$\frac{\partial T}{\partial t} = 0 = \frac{(q/v)}{2\pi\lambda} \left( \frac{-1}{t^2} \exp\left(-\frac{r^2}{4at}\right) + \frac{1}{t^3} \left(\frac{r^2}{4a}\right) \exp\left(-\frac{r^2}{4at}\right) \right)$$

$$0 = -1 + \left(\frac{r^2}{4at}\right) \Rightarrow t_p = \frac{r^2}{4a}$$

Hence the exponential term may be written:  $\exp(-t_p/t)$ , which tends to unity as  $(t/t_p)$  becomes large. So for  $t \gg t_p$ , the temperature solution simplifies to:

$$T - T_o = \frac{(q/v)}{2\pi\lambda t} \quad \text{or} \quad t = \frac{(q/v)}{2\pi\lambda(T - T_o)}$$

Setting  $T = 800^\circ\text{C}$  and  $T = 500^\circ\text{C}$  in turn gives the times at which these temperatures are passed during cooling (provided  $T_p$  was much greater than  $800^\circ\text{C}$ , i.e.  $t \gg t_p$ ). Subtracting these times gives  $\Delta t_{8-5}$ :

$$\Delta t_{8-5} = \frac{(q/v)}{2\pi\lambda} \left( \frac{1}{(500 - T_o)} - \frac{1}{(800 - T_o)} \right)$$

[40%]

(a) (ii) In microalloyed steels the key microstructural evolution is as follows:

- dissolution of precipitates such as NbC and VC in the austenite;
- grain growth in the austenite, which can occur once the pinning effect of the carbides is lost, and is progressively more severe as the fusion boundary is approached (higher temperature and longer diffusion times for precipitate dissolution and grain growth);
- phase transformations in the austenite in the cooling part of the thermal cycle – of most importance is the formation of brittle martensite, which is influenced by the austenite grain growth increasing the hardenability of the steel.

[20%]

(b) (i) Stress concentrations around the toes of the weld bead will be severe because of the change in section thickness. Cracks are expected in the pressure vessel, spreading from the weld toe; these will grow under the fatigue loading. The effects can be reduced by reducing the contact angle, by grinding out the weld bead; grinding marks can act as surface cracks so direction is important (though stresses are biaxial in the vessel, so difficult to avoid this effect). Hammer peening the bead will put it into compression, so reducing crack nucleation in the bead. However, this type of repair weld is to be avoided if possible, since some cracking is almost inevitable.

[20%]

(ii) Corrosion is likely to occur. Water will seep through the crack in the pressure vessel and crevice corrosion will be found. The galvanised steel plate will be protected by its zinc coating, but will induce an anodic region adjacent to itself, and so accelerate the corrosion on the pressure vessel underneath the plate. Hydrogen embrittlement won't be a problem because the mild steel won't have the required high yield strength. The temperature of  $80^\circ\text{C}$  is only will speed up

corrosion., but is not high enough for LME effects which might have been found if the vessel had been heated to 500°C.

[20%]

[The numerical part was well answered, but again marks were dropped by considering only a few of the relevant factors in the descriptive parts. A common mistake in (a)(ii) was to neglect to mention how the cooling affects martensite formation.]

3 (a) Equivalence of cooling rate is important in heat treatment of steels because the same cooling conditions lead to the same microstructure and properties – notably the proportion of martensite formed, prior to tempering. Standard geometries can then be used to capture the response of the steel to a range of cooling rates (e.g. Jominy end-quench, or long cylinders of various diameters). Locations in real components with a given cooling rate will then have the corresponding microstructure and properties from the standard tests.

[15%]

(b) (i) For the slab, with  $x = 0$ : 
$$T(x, t) = T_0 + (T_1 - T_0) \frac{4}{\pi} \exp\left(-\frac{\pi^2 a t}{4l^2}\right)$$

Differentiating: 
$$\frac{\partial T}{\partial t} = (T_1 - T_0) \frac{4}{\pi} \exp\left(-\frac{\pi^2 a t}{4l^2}\right) \left(-\frac{\pi^2 a}{4l^2}\right)$$

Substituting  $(T - T_0)$  from original solution: 
$$\frac{\partial T}{\partial t} = -(T - T_0) \left(\frac{\pi^2 a}{4l^2}\right)$$
 [30%]

(b) (ii) For the Jominy: 
$$\frac{\partial T}{\partial t} \approx -\frac{(T - T_0)}{\sqrt{\pi}} \left(\frac{C^2 4 a}{z^2}\right) \exp(-C^2) \quad \text{where } C = \frac{T - T_0}{T_1 - T_0}$$

Hence if the two cooling rates are the same: 
$$\left(\frac{\pi^2}{4l^2}\right) = \frac{1}{\sqrt{\pi}} \left(\frac{C^2 4}{z^2}\right) \exp(-C^2)$$

For a given temperature at which the cooling rate is defined (i.e.  $C = \text{constant}$ ),  $z/l = \text{constant}$ , i.e.  $z$  against  $l$  will be linear. The gradient is given by:

$$\left(\frac{z}{2l}\right) = \left(\frac{4C^2}{\pi^2 \sqrt{\pi}} \exp(-C^2)\right)^{1/2}$$

For the temperatures given: 
$$C = \frac{T - T_0}{T_1 - T_0} = \frac{500 - 20}{840 - 20} = 0.585$$

Hence the gradient is:

$$\left(\frac{z}{2l}\right) = \left(\frac{4 \times 0.59^2}{\pi^2 \sqrt{\pi}} \exp(-0.59^2)\right)^{1/2} = 0.236$$

For  $z = 25\text{mm}$ ,  $2l = 25/0.24 = 106\text{mm}$ .

[35%]

(b)(iii) The correlation is not unique because it depends on the temperature at which the cooling rate is defined (i.e. the value of  $C$ ), e.g. if  $C = 0.5$ , the gradient changes to 0.42.

Another choice of “the same cooling rate” is to fix the time taken to reach a particular fraction of the cooling interval – e.g. equivalent diameters are defined by making all cooling curves take the same time to reach the value  $C = 0.35$ . [20%]

[A slightly lower average on this question than expected, as failure to put the answer to (a)(i) in terms of  $T_1$ ,  $T$  and  $T_0$  as requested (i.e. eliminating  $t$ ) made it difficult to do part (b)(ii). Few people gave a good explanation of why the correlation is not unique ((b)(iii).]

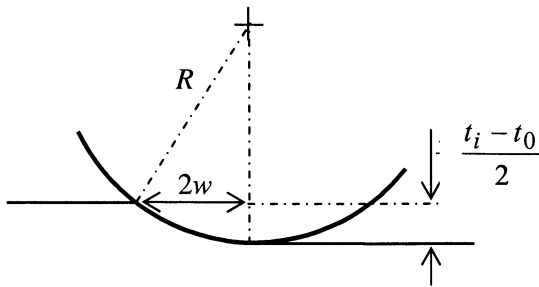
4. (a) The steps are as follows:

1. Identify a suitable element for equilibrium. Assume that the stress state is uniform in the height direction (so the element can have height  $2h$ ) but can vary in the width direction. Also assume plane strain with the principal directions perpendicular to the workpiece surfaces.
2. Write down the equilibrium equation for the element. This gives a governing differential equation. Assume either sticking friction or Coulomb friction, or a friction factor at the tooling contact surfaces (here Coulomb friction was assumed).
3. Relate the principal stresses using a yield criterion (here Tresca was used but von Mises could equally be used) and thus eliminate all stresses except the surface normal pressure  $p$  from the governing equation. It is assumed that the material is yielding throughout.
4. Solve the differential equation for the pressure distribution  $p$  and use boundary conditions at the free surfaces. With Coulomb friction, this gives an exponential pressure distribution.
5. Integrate the pressure distribution over the tooling surface to get the forging force. To get the given expression, it has also been assumed that  $w \ll h$  so that only terms up to second order in the power series expansion for the exponential function have been kept. [25%]

(b) In the analogy let  $2h = (t_i + t_o)/2$  and  $2w$  be the width of the roll bite. Write this in terms of

the roll radius  $R$  by noting the roll geometry:  $(2w)^2 + \left(R - \frac{t_i - t_o}{2}\right)^2 = R^2$ ,

but  $t_i - t_o \ll R$  giving  $2w \approx \sqrt{R(t_i - t_o)}$



Substitute for  $2h$  and  $2w$  in the given expression for  $F/D$  to get:

$$\frac{F}{D} \approx Y \left( \sqrt{R(t_i - t_o)} + \frac{\mu R(t_i - t_o)}{(t_i + t_o)} \right)$$

The roll torque is estimated by  $T = F.w$  (assuming the forces acts in the middle of the bite) giving

$$T \approx \frac{DY}{2} \left( R(t_i - t_o) + \frac{\mu(R(t_i - t_o))^3}{(t_i + t_o)} \right).$$

[20%]

(c) Five ways to reduce the rolling torque are listed in the table below. Any 3 will do.

Method	Disadvantage	Way to overcome disadvantage
Reduce roll radius $R$	Roll bending	Backup rolls (4 high mill etc.) Roll bending actuators.
Reduce friction coefficient $\mu$ (lubricate)	Rolls slip if $\mu$ is too low	Front tension and continuous process control.
Reduce yield stress $Y$ (anneal or hot work)	Annealing adds an extra step, whilst hot rolling gives relatively poor dimensional accuracy.	Use hot rolling for reduction of ingot to sheet but cold rolling for finishing.
Reduce the height reduction $t_i - t_o$	This results in multiple rolling passes being needed.	None. In practice continuous mills with multiple stands of rolls are used.
Use front or back tension	Back tension can lead to slipping. Excessive front tension can cause tearing.	Use continuous process control to monitor and adjust rolls and tension accordingly.

[30%]

(d) (i) Assume all the work input to the rolls is dissipated as heat in the strip. Assume that none of the heat escapes the workpiece. Let the work rolls turn with angular velocity  $\omega \approx v/R$  where  $v$  is the foil production rate of  $900\text{m/min} = 15\text{m/s}$ . There are two work rolls, so the power input is  $2T\omega$ . The power used in heating the strip is its volume output rate  $\times$  its heat capacity  $\times$  temperature rise  $\Delta\theta$ . Thus

$$2T\omega = (vt_o D) \times (\rho c_p) \times \Delta\theta$$

$$\text{giving } \Delta\theta = \frac{2T\omega}{vt_o D \rho c_p} = \frac{2T}{Rt_o D \rho c_p} = \frac{2 \times 2.5\text{Nm}}{4\text{mm} \times 0.10\text{mm} \times 0.9\text{m} \times 2.6\text{MJm}^{-3}\text{K}^{-1}} = 5.3\text{K}$$

Note: nothing has been assumed about how the heat is distributed in the strip, as only the *mean* temperature rise was required. [15%]

(ii) Deformation heating will be uniformly distributed in the foil, so it is the flow of heat out to the roll, and heat generated due to friction at the interface with the roll, which generate a temperature gradient in the foil. The temperature profile in the roll and strip then needs to be modelled using a suitable unsteady heat flow equation, modelling generation of heat, conduction, heat transfer across the interface and cooling of the rolls by the lubricant. [10%]

[Good answers to (a) and very good answers to (c). Lack of care in (b) lost relatively easy marks. The more difficult thermal calculations were not well done, particularly grasping the physical mechanisms leading to a temperature gradient through the strip.]

MPFS

1/5/05

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Numerical Answers - 2004/5**

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$$3 \text{ (b) (i) } \frac{\partial T}{\partial t} = - (T - T_o) \left( \frac{\pi^2 a}{4l^2} \right), \text{ (ii) } 0.236, 106\text{mm}$$

$$4 \text{ (b) } T \approx \frac{DY}{2} \left( R(t_i - t_o) + \frac{\mu(R(t_i - t_o))^{3/2}}{(t_i + t_o)} \right), \text{ (d) (i) } 5.3\text{K}$$