1. (a) (i) Any four of the following:

Differential Input - measure both sides of the resistor;
Good CMRR - both inputs are at high voltages;
High gain for a small voltage measured - low loss in 10 Ω resistor;
Ideal inputs draw no current - accurate measurement;
Cheap - cheaper than a specific current measurment chip;
Good linearity with feedback - accurate measurements
Low output impedance - can drive a sample and hold*.

* Student answer

A wide bandwidth is not necessary as there will be decoupling capacitors after the battery and battery current measurement circuit. Few could explain why a high CMRR was needed.

(ii) Various methods, noting VDD = 3.7 - 0.08 x 10 = 2.9 V :

Superimposition: 3.7 x -10 +2.9 x 11 = - 5.1 V

Inverting Op-Amp with 2.9V Offset : 0.08 x 10 x -10 +2.9 = - 5.1 V

Safe ($V_- = V_+$) : (Vout - 2.9)/1Meg = (2.9 - 3.7)/100k => -5.1V

ANS: -5.1V

A bit unrealistic! 80mA was chosen to make the numbers bigger.

(iii) New $V_{DD}$ is 2.53:

Inverting Op-Amp with 2.9V Offset : 0.08 x 10 x -10 +2.53 = - 5.47 V

The ouput voltage drops by the same amount as the battery voltage. So to measure current then the battery voltage must also be measured. This makes sense as knowing the voltage and current in a battery one can estimate the 'State of Charge' - the charge left in the battery.

(b) (i) Include the finite gain and output resistance (Thevenin model).

$$\frac{A_o(v_i - v_o) - v_o}{R_o} = v_o j\omega C_o \text{ or } v_o = A_o(v_i - v_o)\frac{1/j\omega C_o}{1/j\omega C_o + R_o}$$

Rearrange: ANS: $$V_o = V_i \frac{A_o}{1 + A_o + j\omega C_o R_o} \qquad (1)$$

OR differential Equation:

$$\frac{A_o(v_i - v_o) - v_o}{R_o} = C_o \frac{dv_o}{dt}$$

Rearrange: $$\frac{dv_o}{dt} + \frac{(A_o+1)}{C_o R_o} v_o = \frac{A_o}{C_o R_o} v_o$$

For a step input: $$v_o = v_i \frac{A_o}{A_o+1}\left[1 - \exp(-\frac{A_o+1}{C_o R_o} t)\right]$$

The step input may be slew rate limited. The response to a step is not strictly a ratio, but was accepted when attempted, as using Laplace turns it into a ratio!

(ii) The finite bandwidth is represented with some kind of roll off:

From lectures: $$\frac{A_o}{1 + j\omega/\omega_o}$$

Substitute into (1) above in place of the flat gain $A_o$

$$V_o = V_i \frac{A_o}{A_o + (1 + j\omega C_o R_o)(1 + j\omega/\omega_o)}$$

ANS: $$\frac{A_o}{1 + A_o - \omega^2 \frac{C_o R_o}{\omega_o} + j\omega(C_o R_o + 1/\omega_o)}$$

The real term on the denominator can disappear at a certain frequency, leaving a high gain and a $90^{\circ}$ phase shift. i.e. It has gone resonant.

(On a step input is appears as an overshoot in the output voltage. With an ac input it can tune in on noise!)

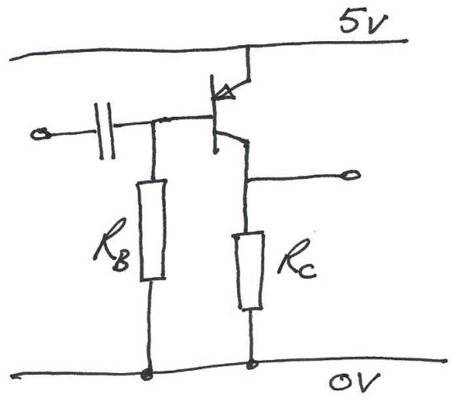
2. (a) (i) 2.5 V at the output with no signal, with a collector current of 100 mA.

Assume: $V_{BE} = 0.6$ or $0.7$ V

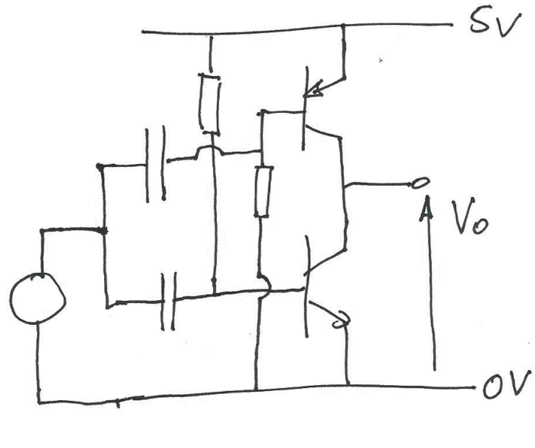
ANS: $R_C = 2.5/100\text{m} = 25\ \Omega$ ; $R_B = (5 - 0.7)h_{FE}/100\text{m} = 4300\ \Omega$

(ii) It is half way between the supply rails, so allows a maximum voltage swing (power) and is in the middle of the linear active region.

(iii) The pnp is the complement of the npn. Preserving the nodes of the circuit is the main concern: The pnp emitter is connected to the higher voltage supply; The output must remain at the collector; The base resistor must draw current from the base. See below:

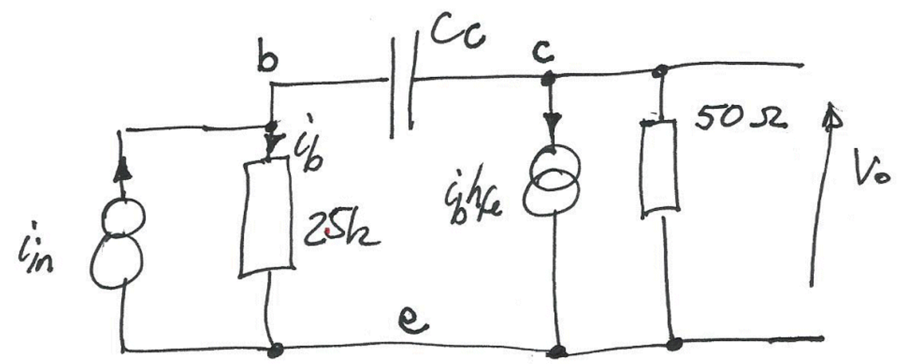



(iv) Adding the npn and pnp circuits eliminating $R_C$ (keeping the rest):




The base bias resistors would have a high value for Class B operation - so that the transistors are off until a signal turns them on. Current sources connected to each base can set the base bias currents accurately and allow Class AB operation, improving the linearity. This configuration is widely used in rail-to-rail op amps for low voltage circuits. Many students thought Class B meant emitter follower, confusing class with configuration.

(b) (i) $I_B$ is a bias current so does not appear in the Small Signal equivalent circuit:




(ii)

$$i_{in} - \frac{v_{be}}{h_{ie}} + \frac{v_o - v_{be}}{1/j\omega C_c} = 0$$

$$v_o = -h_{fe} i_b 50 = -h_{fe} \frac{v_{be}}{h_{ie}} 50 \qquad (2)$$

Equation (2) uses the Miller Effect approximation to ignore the very small current due to $C_c$ at the collector node. Eliminate $v_{be}$ and find the ratio $v_o$ to $i_{in}$.

Equate real and imaginary parts:

$$\frac{1}{50h_{fe}} = \omega C_c \left(1 + \frac{h_{ie}}{50h_{fe}}\right)$$

Hence

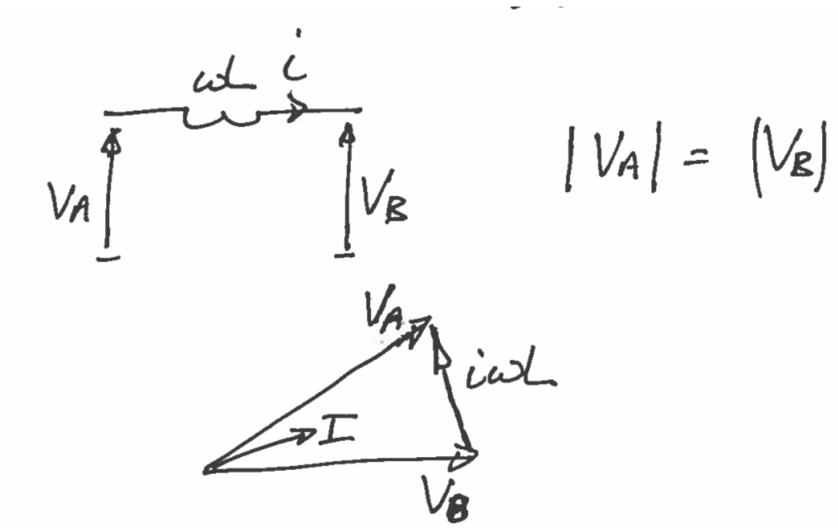
$$\omega = 2\pi f = \frac{1}{C_c(50h_{fe} + h_{ie})} \qquad (3)$$

ANS: $f = 1\text{kHz}$

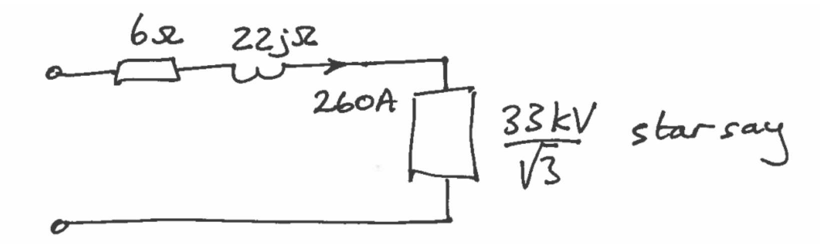
Note: Not making the approximation simply means that more care has to be taken with the 3dB analysis. Using the full Miller Approximation allows Equation 3 to be written down by inspection.

3. (a) Three pure sinusoidal phase voltages of the same magnitude displaced in time by +/- $120^{O}$ with a balanced three phase load gives rise to three sinusoidal currents with similar magnitudes. The three currents will then sum to zero. So three wires only are needed if properly balanced. A small fourth wire at around zero volts can often be seen.

(b) Note the magnitudes at each end are the same.




(c) (i) Max power at 260A and unity power factor.




ANS: Max P = $\sqrt{3} \times 260 \times 33000 =$ 14.9 MW

Losses in the line: Plines = 3 x $260^2$ x 6 = 1.22 MW
Qlines = 3 x $260^2$ x 22 = 4.46 MVAR

At the Windfarm: $S^2 = P^2 + Q^2 = (14.9 + 1.22)^2 + 4.46^2 =$ 279.7
S = 16.72 MVA

ANS: V = 16.72M/($\sqrt{3} \times 260$) = 37.1 kV

(ii) The capacitors are needed at the Windfarm to provide Q = 4.46 MVAR.

NB Star connected: $Q = 3V^2/Xc$ => $Xc = 3V^2/Q$ => $(37.1k)^2/4.46M$

The capacitive reactance required is 308.6 jΩ

At 50Hz, the capacitance is $1/\omega C$ => $C = 1/(100\pi \times 308.6)$.

ANS: C = 10.3 μF per phase.

(iii) This needs thought! The wind genarators and the town are at unity power factor, but the line needs to have its reactive power supplied.

Then the extra capacitance required at the town is given by

$$Xc = (33k)^2/4.46M$$

The capacitive reactance required is 244 jΩ

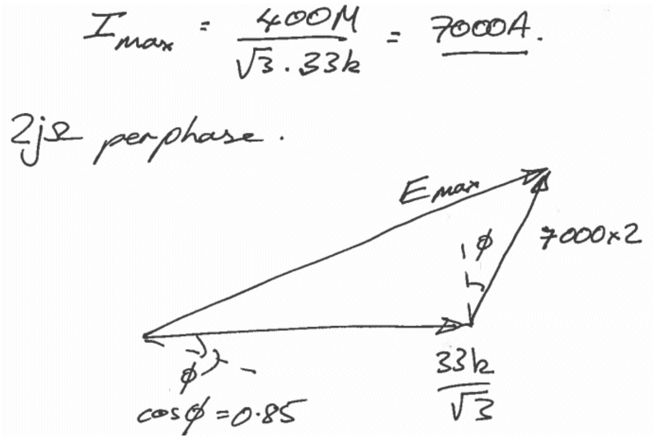
At 50Hz, the capacitance is $1/\omega C$ => $C = 1/(100\pi \times 244)$.

ANS: C = 13 μF per phase.

The original wording was not as clear. From part (b) it is clear that the voltage at the wind farm will drop to something nearer to 33 kV if the line VARs are supplied at the town end. (The maximum power supplied to the town from the windfarm will drop, under these conditions, but the town will have another source of power in any case).

4. (a) (i) The rotor has an electromagnet created by the rotor current. The three phase stator creates a rotating magnetic field (it must have the same number of magnetic poles as the rotor). The two components of magnetic field interact to provide torque if there is a displacement angle between them. The rotor must rotate at the same speed as the stator magnetic field.

(ii) 400 MVA, 33 kV (star), 0.85 pf. Gives max E as follows:



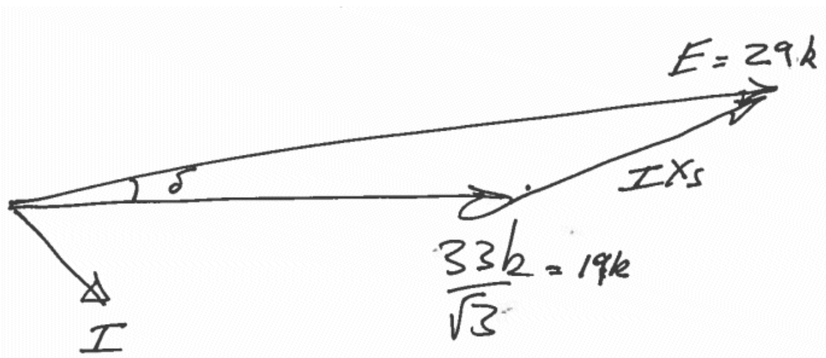

Power factor angle $\phi$ is 31.8°, so the angle opposite E is 90 + 31.8 = 121.8°. We want the bigger angle for the bigger E (rotor heating limit).

Cosine formula: $E^2 = 363 + 196 + 2x19x14x0.527$ in kV.

ANS: E = 29 kV under rated conditions.

(iii) Consider the operating chart. Max reactive power at Max E always. Four machines on for max reactive power and minimum power generated for max reactive power. At 60 MW each, the new version of the diagram is as follows:

. From the databook: $60 \text{ MW} = 3\frac{VE}{X_S}\sin\delta$ .




$$\sin\delta = \frac{60M\times 2}{3\times 19k\times 29k} = 0.073 \text{ .}$$

$\delta = 4.2°$

$(IX_S)^2 = 19^2 + 29^2 + 2x19x29\ \cos 4.2^O = 104.4$ , in $(kV)^2$

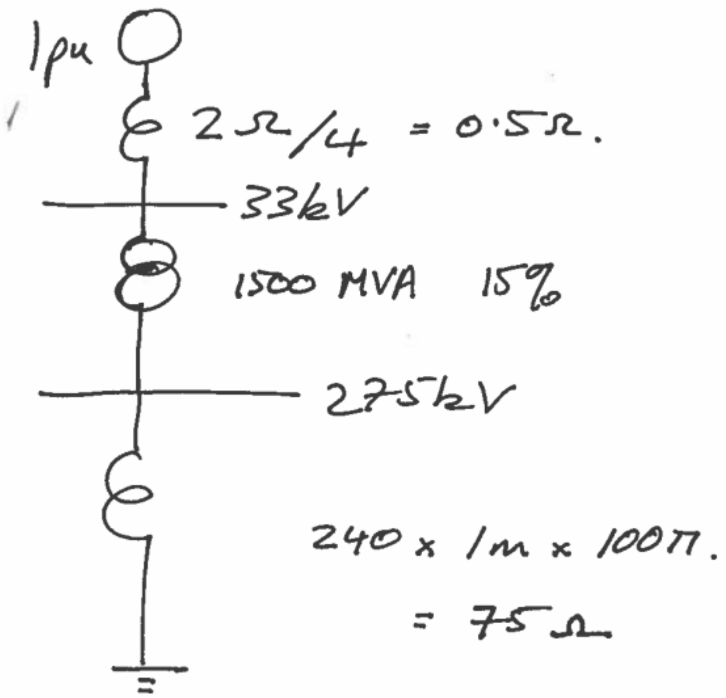
Reactive Power: $3VI\sin\phi$

From the diagram: $IX_S\sin\phi = E\cos\delta - V = 29\cos 4.2^O - 19 = 9.92$, in kV

Reactive Power 3x19x9.92/2 = 283 MVAR per machine.

ANS: 1132 MVAR

b) Set up the circuit. Note: four generators and assume 1pu.




Choose 15 MVA base.

For the line, $Z_b = (275k)^2/1500 = 50.4\ \Omega$, Hence 75 Ω => 1.49 pu .

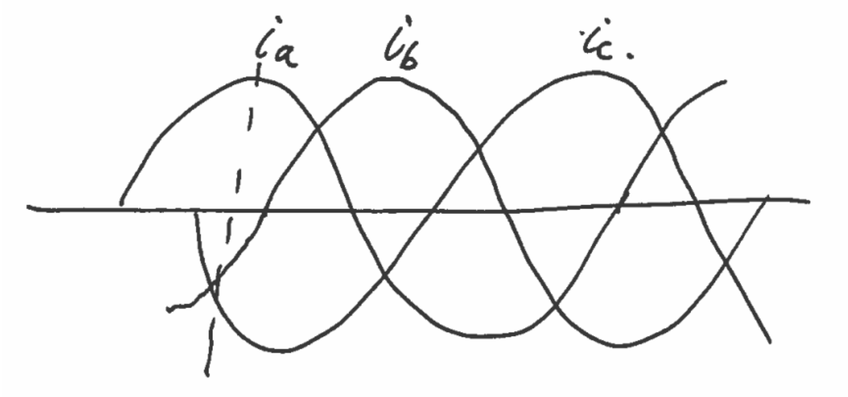
For the generators, $Z_b = (33k)^2/1500 = 0.726\ \Omega$ , So 0.5 Ω => 0.69 pu.

Then Ipu = 1/(0.69 + 0.15 + 1.49) = 0.429 pu.

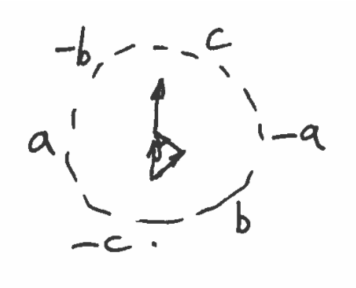
Ib = $1500M/\sqrt{3} \times 275k = 3149$

ANS: The fault current is 3149x0.429 = 1350 A

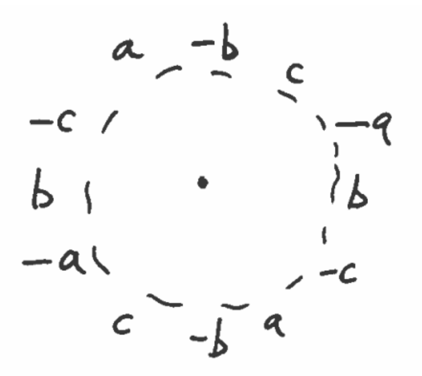
5. (a) (i)



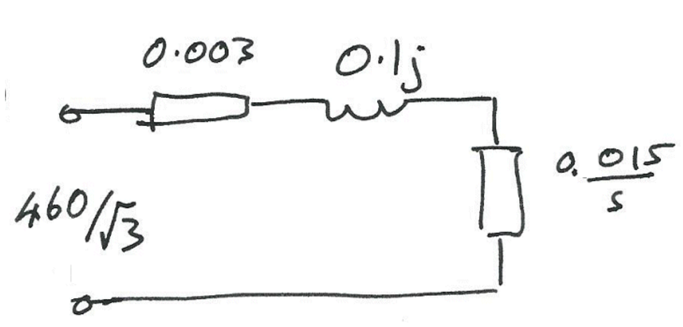

The currents create a magnetic field. For example at the time sampled the magnetic field is aligned with the a axis, with the b and c magnetic fields at half strength due to half peak current. At any time a constant magnitude of flux is found, rotated according to time. Hence constant torque.




(ii) One cycle of the supply current moves the flux from **a** back to **a**.
In the four pole winding below the angle from **a** back to **a** is only 180$^{O}$ so the motor goes half the speed of a two pole machine (at no load).




(b) (i) Rated: slip of 0.03 ; 60 Hz ; 460 V star ;




I = 266/0.512 = 520 A

T = 3 x 520$^2$ x 0.5/120π

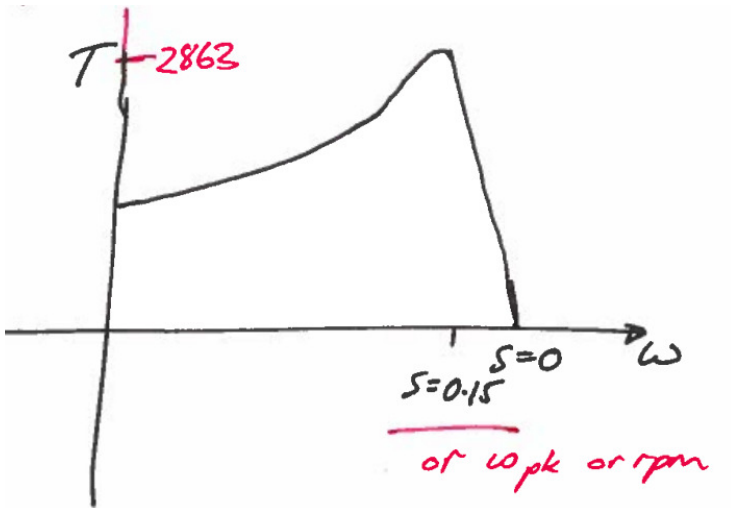
ANS: 1080 Nm

(ii) Peak Torque when $R_2/s$ is equal to series terms. Since 0.1j dominates,

$$R_2/s = 0.1 \;\Rightarrow\; s = 0.15$$

Current at peak torque = 266/(0.1+0.1j) => 1881 A

$$T = 3 \times 1881^2 \times 0.1/120\pi = 2816 \text{ Nm}$$




(iii) $I^2R$ Losses in the stator and rotor:

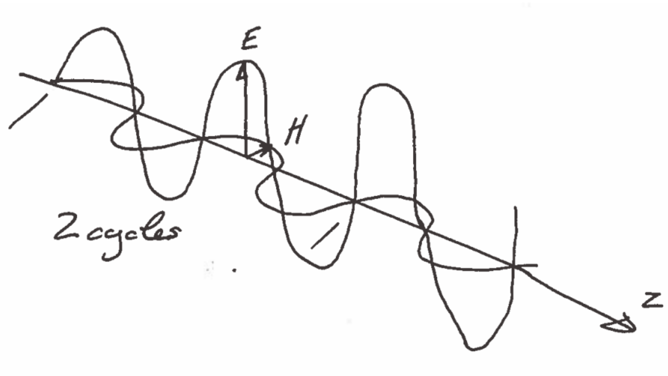
$$3 \times 1881^2 \times (0.003 + 0.015) = 191 \text{ kW}$$

9 kW Iron Losses

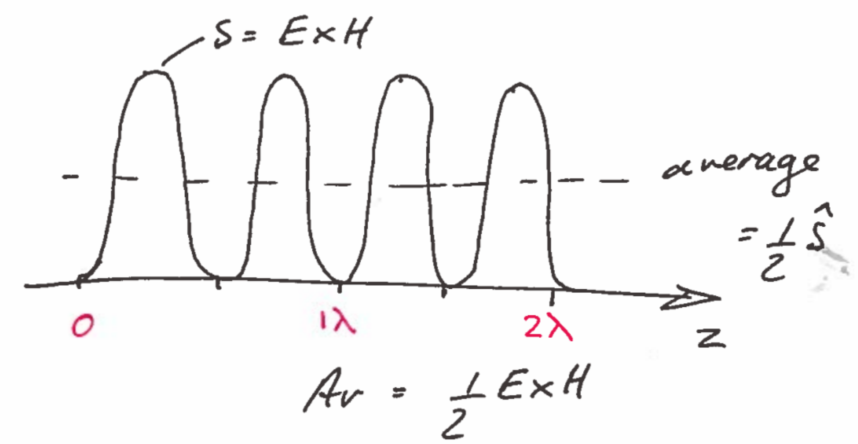
Mechanical output: $3 \times 1881^2 \times 0.1 \times (1\text{-}s) \Rightarrow 902$ kW

ANS: Efficiency 82 %

6. (a) (i)




(ii)



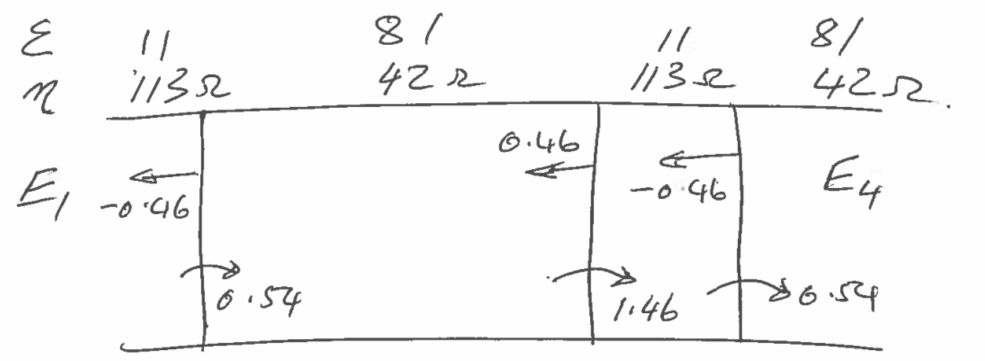

(b) Pulse is 60J, 50ms, 1cm$^2$ : Power = 60x100$^2$/(50x10$^{-3}$) = 12 MWm$^{-2}$

Reflection Coef. (Databook) : $\frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\frac{1}{\sqrt{81}}-\frac{1}{\sqrt{11}}}{\frac{1}{\sqrt{81}}+\frac{1}{\sqrt{11}}}$ = -0.46 (for 11 to 81).

(i) The energy reflected back from the first reflection is 0.46$^2$ x 60J

ANS: 12.7 J

(ii) Transmitted E field is (1+ $\rho_L$). Putting the numbers:



For the first pass:

$$E_4 = 0.54 \text{ x } 1.46 \text{ x } 0.54\ E1 = 0.42\ E_1$$

$$S_1 = E_1{}^2/42 = 12\text{x}10^6 \quad , \quad S_4 = E_4{}^2/42$$

$$E_4 = 15.5 \text{ x } 10^3 \text{ (RMS)}$$

We need peak

ANS: $21.9 \text{ x } 10^3 \text{ Vm}^{-1}$

This is very high, but around 200V/cm. It's a very short pulse, and intended to put a lot of energy into the hairs. "This may feel like an elastic band snapping at your skin" and will cause redness lasting 24 hours (www.nhs.uk).

Reflections can be included if we look at the figure, but these are somewhat smaller (the original question asked for this calculation).

*E.g.* $E_4 = 0.54 \text{ x } 0.46 \text{ x } 0.46 \text{ x } 1.46 \text{ x } 0.54\ E_1 = 0.09\ E_1$

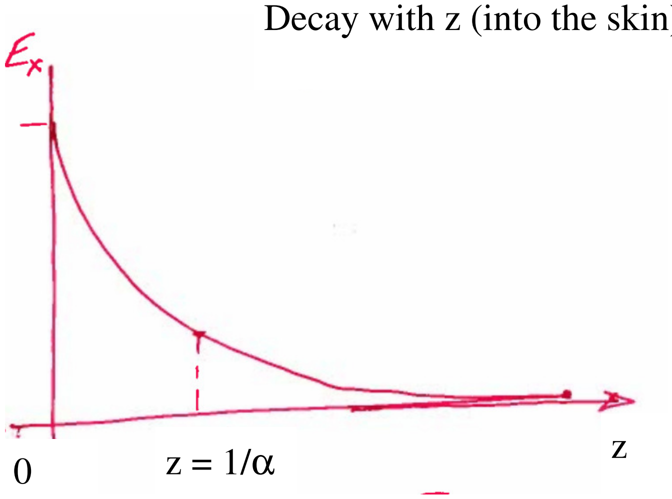
And so on. Note this is less than 5% of the power in the first ray.

(ii) Databook:

$$E_x = |E_x| e^{-\alpha z} e^{-j\beta z} e^{j\omega t}$$

Where the term $e^{-\alpha z}$ term is the decay in magnitude with distance.

Decay with z (into the skin)




Penetration depth is 1/57. ANS: 0.0175 m

Getting good numbers is hard, but the decay in ‘skin’ is reasonable (e.g. these days heart rate is taken using a finger monitor). In reality the problem is more that the dry top layer of the skin cuts the intensity down, so a cooling moisturiser is often used.  The light gets to the dark hair easily, energy is absorbed by the hair, and significant damage is done to the hair folicule. Red marks are left on the skin for about two days. Much higher intensities are used in opthalmology for their destructive effects.

7. (a) (i) The Characteristic Impedance is the ratio between the voltage and current of a unidirectional wave at any point on a transmission line (Lecture 2). It is related to the capacitance and inductance per unit length, and therefore it does not dissipate power.

(ii) $$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{525n}{52p}}$$ ANS: 100.47 Ω

$$v = \frac{1}{\sqrt{LC}}$$ ANS: $191 \times 10^6$ $ms^{-1}$

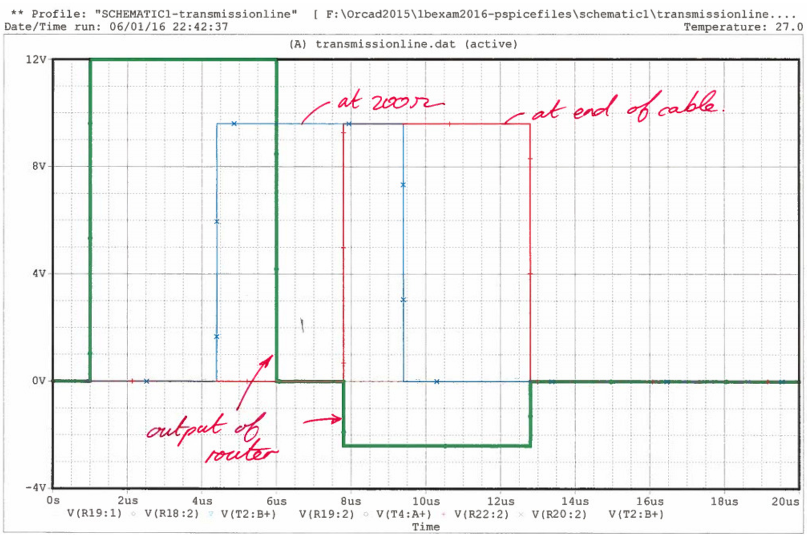
(iii) The co-axial cable is studied in 1A. Similar ideas work for twisted pair too, particularly the ln form that appears in both. Then the inductance increases with separation, the capacitance reduces. The inductance reduces with inner conductor diameter the capacitance increases. Looking at the equation for $Z_0$, the characteristic impedance depends on separation, inner wire diameter and relative permittivity.

However, Wave velocity is given by $\frac{1}{\sqrt{LC}}$ . Therefore the separation and inner conductor diameter aspects cancel, so it depends on relative permittivity only.

(b) The fault is halfway, so the cable at the fault has its characteristic impedance. Thus the impedance at the fault is the parallel combination of the characteristic 100Ω and the 200Ω fault. This gives 66.7Ω at the fault and therefore the test pulse will be reflected back to the router.

Databook: $\rho_L = (66.7-100)/(66.7+100) = -0.2$

The pulse at the output of the router (0.5 x 24V) is reflected back as a - 2.4 V pulse. The delay time for travelling 650 m out and back is $1300/191 \times 10^6$ = 6.8 μs. Pulse is 5μs, so no overlap. See PSPICE below:



(ii) Databook Phase constant $\beta = \omega/v = 2\pi \times 2\text{x}10^6/191\text{x}10^6 = 65.8\text{x}10^{-3}$

Length to fault, $l = 650$m , Zo = 100.5 , $Z_L = 66.7$ , $\tan \beta l = -2.7$

Databook Input Impedance

ANS: $Z_{in} = 136\ \Omega$ , - 0.26 rad

This answer varies quite a lot with rounding errors!

In (i) and (ii) some students forgot that the remaining 650 m of cable is still attached so they worked with 200 Ω not 66.7 Ω. This error only incurred a small penalty.

Dr Patrick R. Palmer,
Examiner
19 October 2016
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