EGT0
ENGINEERING TRIPOS PART IA

Wednesday 3 June 2015 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 (**short**)

(a) A mass m_A of metal at a temperature T_A is put into thermal contact with another mass m_B of the same material at a temperature T_B. Derive an expression for the temperature of the masses T_F when thermal equilibrium is reached. [3]

(b) Alternatively, a heat engine could be connected between the masses (initially at their original temperatures). In this case show that, if the maximum possible work is to be delivered by the engine, then the following identity must hold:

$$m_A \frac{dT_H}{T_H} + m_B \frac{dT_C}{T_C} = 0$$

where T_H and T_C are the instantaneous temperatures of m_A and m_B respectively. Hence, or otherwise, derive an expression for the final temperature of the masses in terms of the initial temperatures T_A, T_B and r, where $r = m_A/(m_A + m_B)$. [7]

2 (**short**)

(a) Use the first law of thermodynamics, in the form $dq = du + dw$ to show that for a reversible process

$$T ds = du + p dv = dh - v dp$$

and explain why this expression may also be used between equilibrium states joined by irreversible processes. [3]

(b) On a $T - s$ diagram, draw, clearly label and justify the trends for the following processes involving a perfect gas:

(i) constant enthalpy;

(ii) constant pressure;

(iii) constant density;

(iv) reversible and adiabatic. [7]

3 (**long**) Figure 1 shows a horizontal axis piston and cylinder arrangement. The cylinder contains argon, initially at 1 bar and 300 K (p_1, T_1) in an environment also at 1 bar. The cylinder diameter is 0.1 m and the mass of argon is 0.01 kg. Initially, the spring is unloaded and has a length of 1 m. It has a spring constant k of 200 Nm^{-1}. The piston may be assumed to be frictionless and there is no heat transfer, except as described in (b) below. You may assume that argon is a perfect gas.

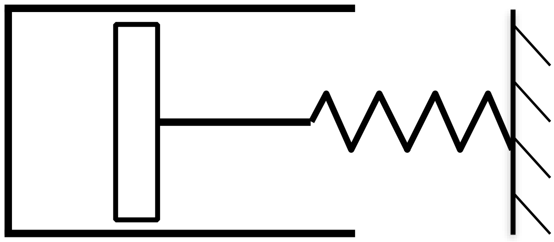

Fig. 1

(a) What is the initial volume V_1 of the trapped gas? [6]

(b) The gas is slowly heated until the spring length is 0.5 m. Calculate the final gas temperature. Also, calculate the work done by the gas and the heat transferred to the gas during the process. [12]

(c) If, instead of process (b), the piston is displaced from its initial rest position by a small distance and then released, show that the subsequent oscillations of the piston will be as a spring-mass system with an effective spring constant of $(k + \gamma p_1 A^2/V_1)$, where γ is the ratio of the gas specific heat capacities. It may be assumed that the expansion/compression oscillation process is isentropic. [12]

4 (**short**) The specific work output w produced when a perfect gas flows steadily through a given turbine depends only on the gas inlet velocity V_1, the gas inlet temperature T_1, the specific heat capacity at constant pressure c_p, the specific gas constant R, and the blade speed U within the turbine.

(a) What is the minimum number of non-dimensional groups according to Buckingham's rule? [4]

(b) Suggest a possible complete set of non-dimensional groups. [6]

5 (**short**) Figure 2 shows a device for controlling the water level in a small reservoir. The gate is rectangular, 0.5 m wide (into the page), and the other dimensions are as shown in the figure. The density of water may be assumed to be 1000 kg m^{-3}.

(a) Calculate the mass M such that the gate begins to open when the water level is 2 m. [5]

(b) It is suggested that, as an alternative method of controlling the water level at 2 m, a hole could be made in the gate, and a valve fitted to control the flow. The smallest possible size valve is to be used and the maximum expected water flow rate into the reservoir is 0.001 m^3s^{-1}. State, with your reasons, where the valve should be placed, and calculate its effective flow area. [5]

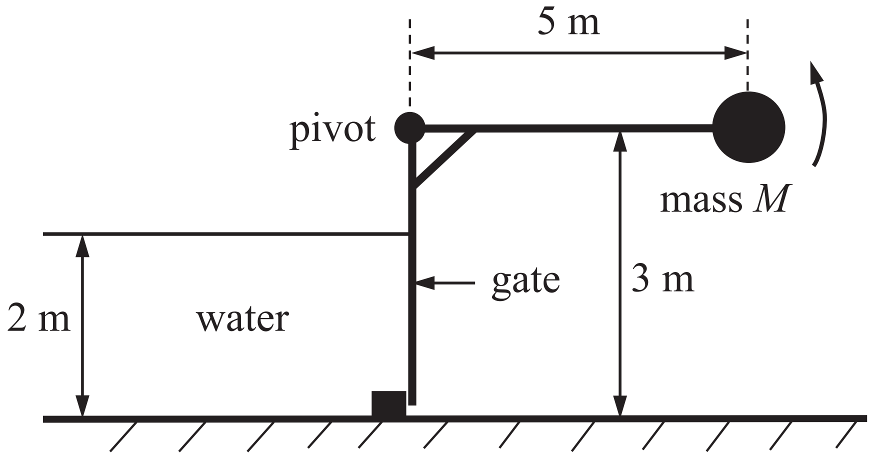

Fig. 2

6 (**long**) A circular jet of water impinges onto an axi-symmetric deflector as shown in Fig. 3. The velocity of the circular jet is V_1 and its cross-sectional area is A_1. After deflection through 180° to form an annular jet of cross-sectional area A_2, the velocity is V_2.

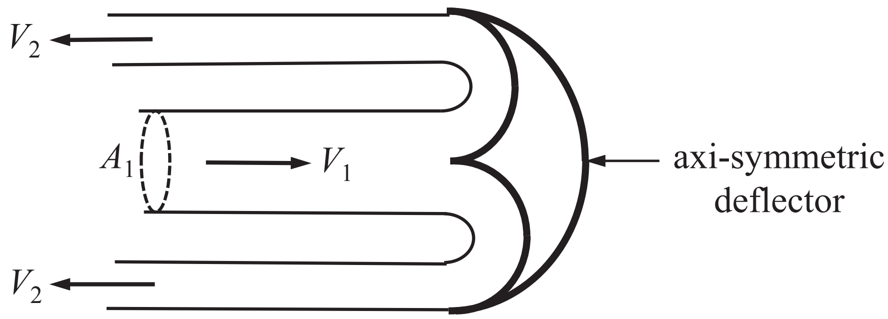

Fig. 3

(a) With the deflector stationary, and assuming the flow to be inviscid, give the relationship between the velocities V_1 and V_2 stating clearly the physical basis for your reasoning. [4]

(b) Calculate the relationship between the areas A_1 and A_2 stating clearly your assumptions. [5]

(c) Show that the force F on the deflector is given by

$$F = 2\rho Q V_1$$

where ρ and Q are the density and the volume flow rate of the water, respectively. [7]

(d) If now the deflector is moving to the right with velocity U (assuming $U < V_1$), what is the force exerted by the water on the deflector? [7]

(e) Show that the maximum power is extracted from the water jet when the deflector velocity U equals $V_1/3$. State clearly your assumptions. [7]

SECTION B

7 (**short**) The arm of a rotary mixer consists of four semi-circular plates attached to a thin light axle, as shown in Fig. 4. Each plate has mass M, diameter $2R$, and the mass and diameter of the axle and the thickness of the plates are negligible.

(a) Using an axis theorem, calculate the moment of inertia of the mixer arm around its axis. [5]

(b) If $M = 100$ g, and $R = 10$ cm, determine the average power required to spin up the mixer arm in vacuum in 1 second from rest to 600 rpm. [5]

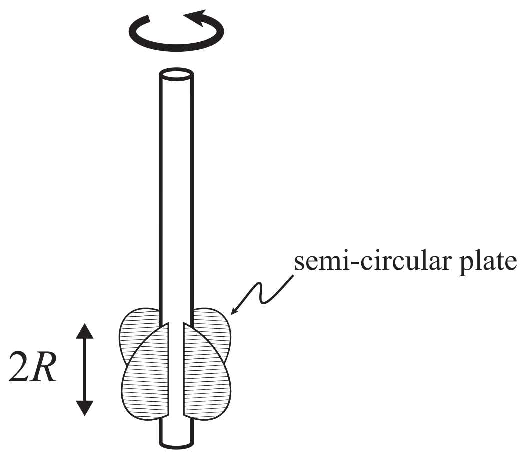

Fig. 4

8 (**short**) A small ball is rolling without friction with initial velocity v on a track which is initially horizontal and then goes around in a vertical loop of radius R, as shown in Fig. 5. You can assume that the rotational kinetic energy of the ball is negligible.

(a) Calculate the speed of the ball as a function of the angle θ as it goes around the loop. [5]

(b) What is the minimum initial speed required for the ball not to lose contact with the track ? [5]

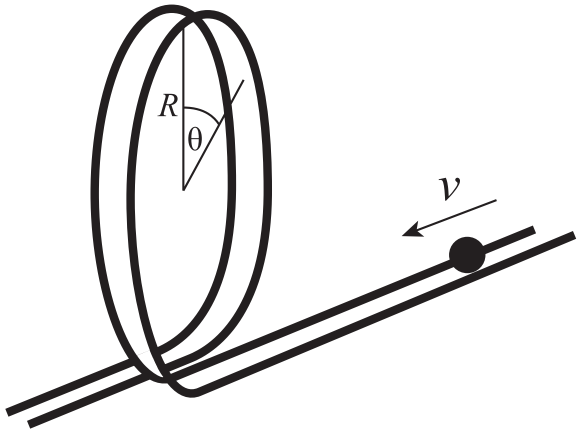

Fig. 5

9 (**long**) A probe is on a comet, made of porous ice, which is modelled as a sphere of radius R with uniform density ρ. The small probe drills a narrow hole of diameter a through the centre all the way across the comet as shown in Fig. 6.

(a) Ignoring the gravitational pull of the probe, derive an expression for the force on a particle of mass m at radius r in the bore hole. You may assume that the net force on the particle due to all material outside the sphere of radius r represented by the dashed line in the figure is zero, and that the amount of missing mass from the bore hole is negligible compared with the mass of the comet. [8]

(b) Obtain an expression for the work done in lifting to the surface a particle of mass m initially at a distance r from the centre of the comet, in terms of R, ρ, m, r and G (the universal gravitational constant). [6]

(c) Hence derive an expression for the total work done against gravity in lifting out all the material from the bore hole. (Assume that the work done by the drill to break down the material is negligible.) [8]

(d) Derive an expression for the frequency of oscillation of a particle that is released into the empty bore hole. [8]

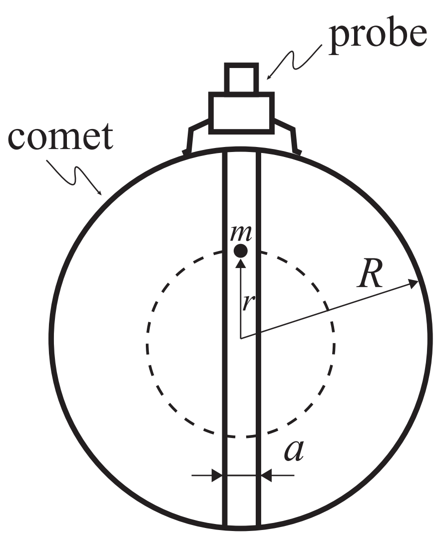

Fig. 6

10 (**short**) Figure 7 shows a radar station tracking an unidentified flying object at a range of 1 km due south moving in a horizontal plane at a speed of $v = 200$ m s^{-1} heading west and with an acceleration $\alpha = 10$ m s^{-2} as shown.

(a) Write down the velocity and acceleration in polar coordinates with the origin located at the radar station, and hence find the radial component of the acceleration. [6]

(b) Calculate the instantaneous radius of curvature of the path. [4]

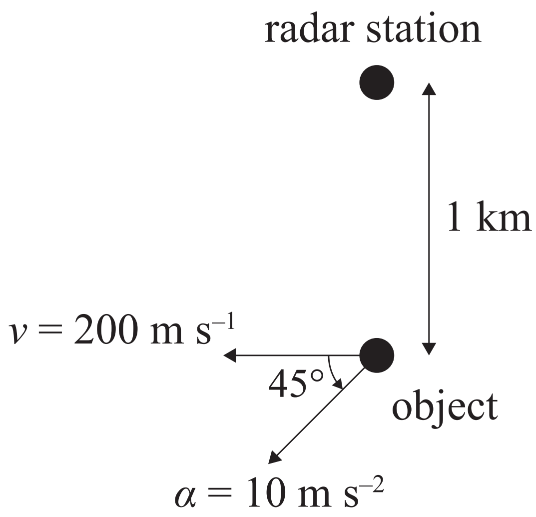

Fig. 7

11 (**short**) Figure 8 shows a schematic diagram of a particle of mass m tethered to fixed supports by springs of stiffness k and $8k$ as shown, allowing for motion only in the x-y plane.

(a) Determine by inspection the normal modes of the system and obtain expressions for the natural frequencies in terms of the system parameters. [4]

(b) The system is set in motion with the initial conditions $x = 0$, $y = 0$, $\dot{x} = 0$, $\dot{y} = 1$ at time $t = 0$. Derive an expression for the subsequent motion of the particle as a function of time. [6]

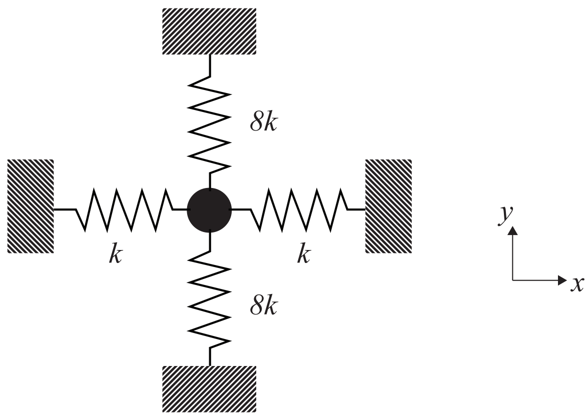

Fig. 8

12 (**long**) Figure 9 shows a schematic diagram of a vibration transducer consisting of a body of mass m connected to an outer casing through a spring-dashpot arrangement. The absolute displacements of the body and the outer casing are x and y respectively and x is measured with respect to the equilibrium position of the body. The relative displacement of the body with respect to the casing is given by $z = x - y$. An initial choice of device parameters is specified as $m = 1$ kg, $k = 100$ N m^{-1}, $\lambda = 5$ N s m^{-1}.

(a) Show that the equation of motion for this system can be written in the form

$$m\frac{d^2z}{dt^2} + \alpha\lambda\frac{dz}{dt} + \beta kz = -m\frac{d^2y}{dt^2}$$

and obtain values for α and β. [6]

(b) Steady sinusoidal motion of the base $y = Y\cos\omega t$ gives rise to a sinusoidal response of the body given by $z = Z\cos(\omega t - \phi)$. Starting from the equation of motion, derive a relationship between Z and Y as a function of ω. Calculate Z and ϕ when $Y = 10$ mm and $\omega = 5$ rad s^{-1}. [8]

(c) The device is now configured as a seismic vibration transducer so that the displacement of the body tracks that of the casing. Specify a new value for λ so as to limit the response at resonance to within 40% of the nominal value while maximising the working range of frequencies for device operation. Estimate the damped natural frequency for this value of λ. [6]

(d) The device is dropped on the floor. The resulting acceleration can be modelled as an impulse imparted to the outer casing of unit magnitude $\delta(t)$. Calculate the maximum resulting displacement of the body for the choice of parameters in (c) above. How long does it take for the displacement of the body to settle to within 0.1% of its maximum value? [10]

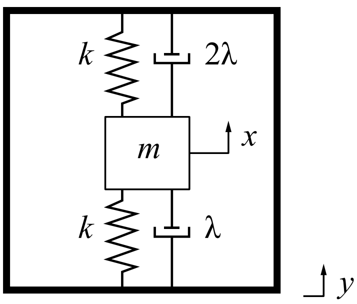

Fig. 9

END OF PAPER

Answers

1 (a) $\frac{m_A T_A + m_B T_B}{m_A + m_B}$ (b) $T_F = (T_A)^r (T_B)^{1-r}$

2 (b)

- (i) Constant h is also constant T
- (ii) $(\partial T/\partial s)_p = T/c_p$
- (iii) $(\partial T/\partial s)_v = T/c_v$
- (iv) Reversible and adiabatic is also constant s

3 (a) 0.00624 m^3 (b) 552.7 K, 417.5 J, 1200.8 J

4 (a) 3 (b) $\frac{w}{U^2} = f\left(\frac{V_1}{U}, \frac{U}{\sqrt{c_p T_1}}, \frac{c_p}{R}\right)$ (These are the conventional groups)

5 (a) 467 kg (b) Bottom of the gate. 1.6×10^{-4} m^2

6 (a) $V_1 = V_1$ (Bernoulli) (b) $A_2 = A_1$ (mass continuity) (d) $F = 2\rho A_1 (V_1 - U)^2$

7 (a) MR^2 (b) ≈ 2 W

8 (a) $u(\theta) = \sqrt{v^2 - 2Rg(1 + \cos\theta)}$ (b) $\sqrt{5Rg}$

9 (a) $\frac{4}{3}\rho\pi Gmr$ (b)$\frac{2}{3}\rho\pi Gm(R^2 - r^2)$ (c) $\frac{2}{9}(\rho\pi a)^2 GR^3$ (d) $\sqrt{4\rho\pi G/3}$

10 (a) $\vec{v} = -200\hat{e}_\theta, \vec{a} = \frac{10}{\sqrt{2}}\hat{e}_r - \frac{10}{\sqrt{2}}\hat{e}_\theta, \ddot{r} = 47.07$ ms^{-1} (b) 5.66 km

11 (a) Mode shapes: $[0,1], [0,1]$, natural frequencies $\omega_x = \sqrt{2k/m}, \omega_y = 4\sqrt{k/m}$
(b) $x = 0$, $y = \frac{1}{4}\sqrt{m/k}\sin\omega_y t$

12 (a) $\alpha = 3$, $\beta = 2$ (b) 1.31 mm, 0.4 rad (c) 3.67 N s m^{-1}, 16.9 rad s^{-1}
(d) 43 mm, 1.25 s