EGT0
ENGINEERING TRIPOS PART IA

Thursday 4 June 2015 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 (**short**) Figure 1 shows a tall square closed can, with thin walls of uniform thickness t, subject to uniform external pressure P in excess of the internal pressure P_0. The faces of the can remain planar under load.

Derive formulae for the stresses σ_l and σ_c in the vertical wall shown in the figure, stating any assumptions you make. [10]

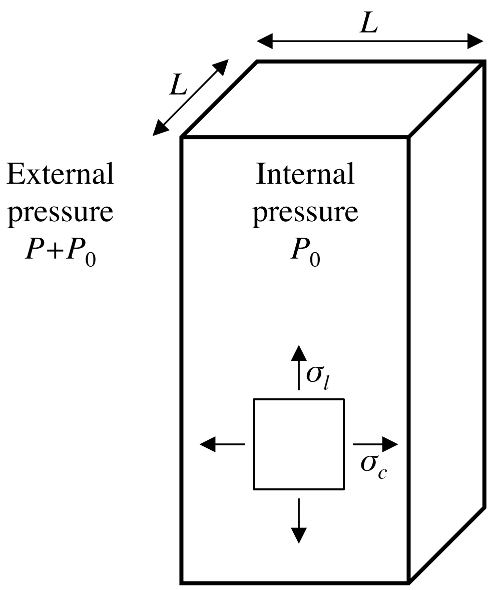

Fig. 1

2 (**short**) The semi-circular arch of radius R shown in Fig. 2 comprises three identical stiff, lightweight segments, pin-jointed at A, B and C, and built in at D. The arch is subject to a single applied force F acting vertically down at the centre of segment BC.

Derive a formula for the maximum bending moment in segment AB. [10]

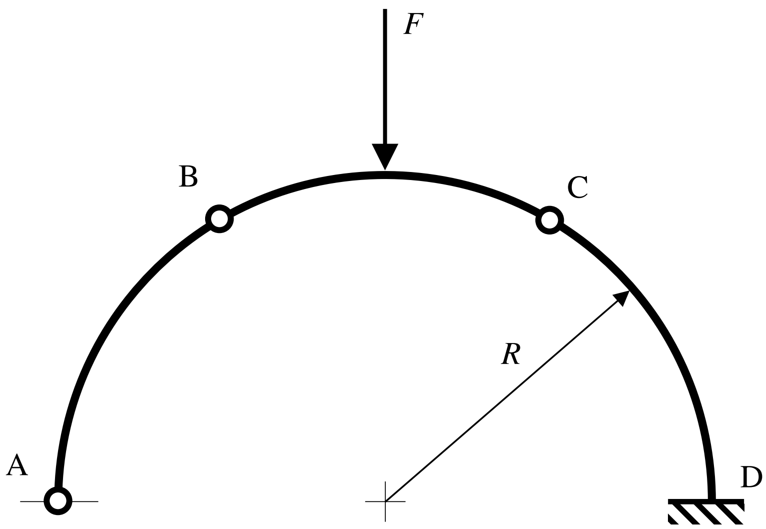

Fig. 2

3 (**short**) The planar pin-jointed truss structure in Fig. 3 comprises an outer regular hexagon with all members of length L and two diagonal members QT and RU which **do not touch** each other. Node S is pinned to the ground, and node P cannot move horizontally. All members in the structure have the same cross-sectional area A and Young's Modulus E.

Derive an expression for the displacement of node P caused by the vertical load F as shown. [10]

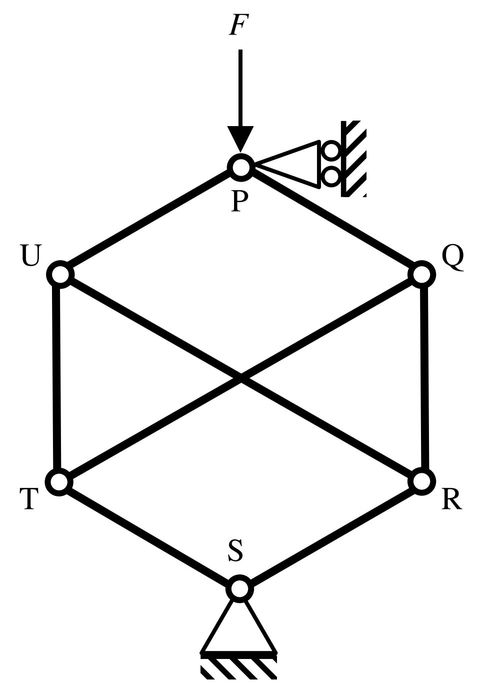

Fig. 3

4 (**short**) Figure 4 shows a test rig used to determine the angle of friction between a block and a test strip. The test strip moves around the conveyor belt as shown, at an angle of θ to the horizontal. The block, which has mass m, is restrained by a linear elastic spring of stiffness k. In the absence of friction, the spring would extend by an amount e_0. Therefore its extension Δe beyond this can be used to determine the angle of friction.

Draw a polygon of forces for the block, and derive a formula to calculate the angle of friction, ψ, in terms of θ and measurements of e_0 and Δe. [10]

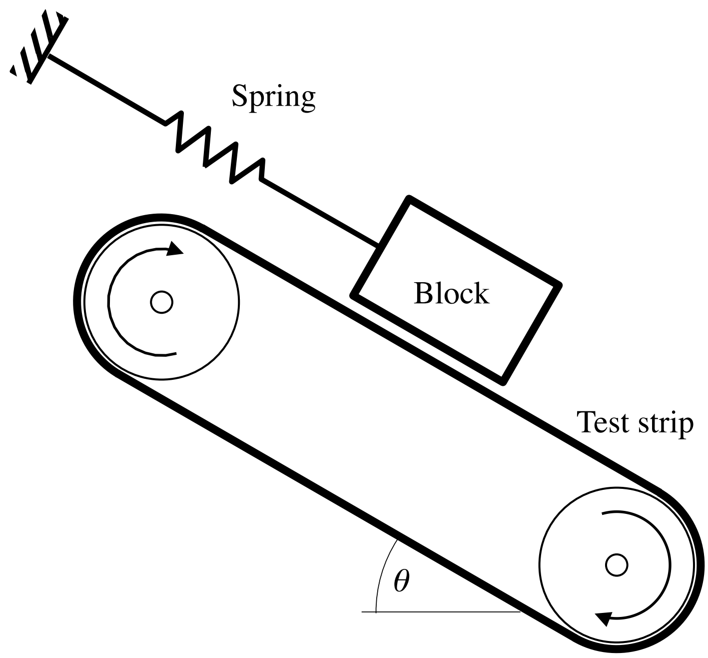

Fig. 4

5 (**long**) Figure 5 shows the design of a reinforced concrete beam, with a square section of side L and four steel reinforcing bars of circular cross section, diameter D, placed symmetrically with their centres a distance $2D$ from the adjacent sides. The dimensions of the beam and the material properties are as shown in the figure. It may be assumed that in tension the concrete has no strength.

(a) Show that when the beam is subjected to bending such that the top of the beam is in compression, the location of the neutral axis of the beam, indicated by αL in the figure, is given by $\alpha = 0.267$. [14]

(b) Calculate the second moment of area for the beam. [8]

(c) If the steel has a yield stress of 350 MPa calculate the maximum bending moment that the beam can sustain before the steel yields. [8]

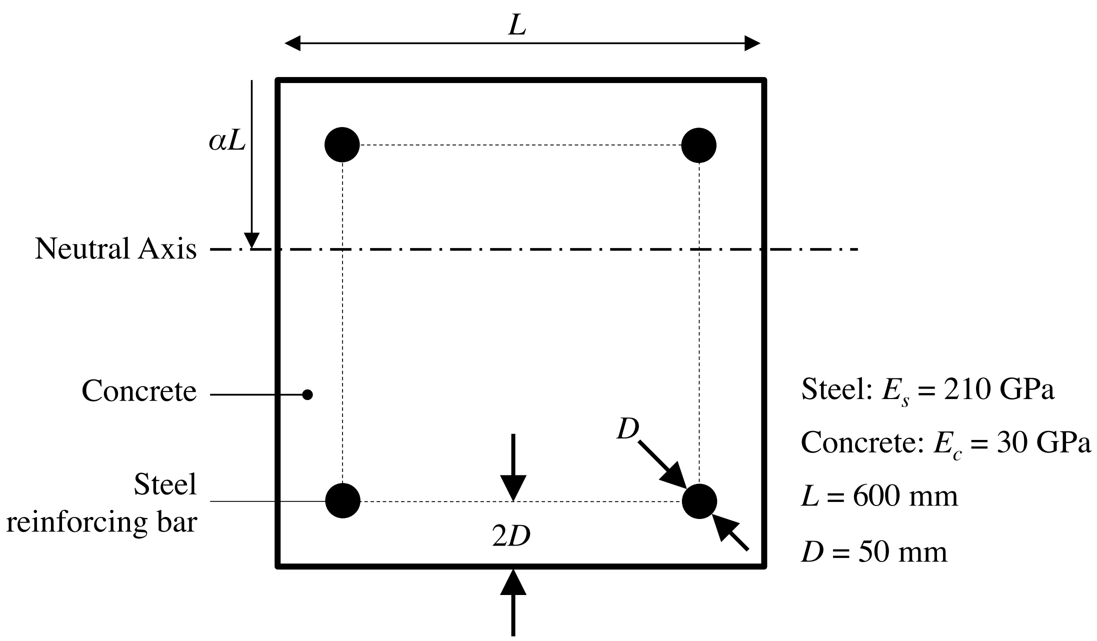

Fig. 5

6 (**long**) Figure 6 shows a simply supported steel beam of length $2L$ subject to uniform loading per unit length, w. The beam has a rectangular cross-section with constant width b and variable depth, described by the function $d(x)$ with x defined in the figure. The self-weight of the beam is **small** compared to the loading w.

(a) If the steel has yield stress σ_y and the beam is designed so that the greatest longitudinal stress due to bending is equal to the yield stress all along its length, show that the depth of the beam should be:

$$d(x) = \sqrt{\frac{3w}{b\sigma_y}\left(L^2 - x^2\right)}$$ [12]

(b) The design in (a) is infeasible because $d(x)$ is zero at the supports, so an alternative strategy is to design the beam so that it avoids failure due to shear. Assuming that the yield stress in pure shear is $\tau_y = \frac{1}{2}\sigma_y$, calculate an alternative beam design in which the maximum shear stress (which is at the neutral axis) is just equal to the yield stress in pure shear all along its length. [10]

(c) Use your answers from parts (a) and (b) to sketch a design for the beam that would just avoid yield due to either longitudinal or shear stress, marking salient points. Briefly discuss any other issues that should be taken into account in the design. [8]

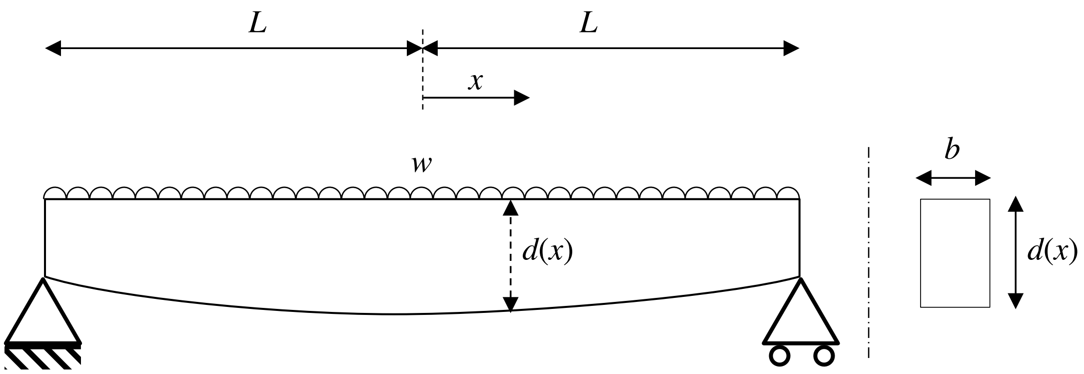

Fig. 6

SECTION B

7 **(short)** A metal on heating undergoes a change from a body-centred cubic (BCC) structure to a face-centred cubic (FCC) structure. At the change temperature, the lattice constant a has a value of 0.28 nm for the BCC structure and 0.35 nm for the FCC structure.

(a) Sketch the unit cells of the BCC and FCC structures, showing the atomic sites. Determine the atomic density of the metal in atoms per m^3 for each structure at the change temperature. [6]

(b) Estimate the fractional change in length $\Delta L / L$ of a bar of the metal as it is heated through the change temperature. It may be useful to note that

$$\frac{\Delta L}{L} \simeq \frac{1}{3}\frac{\Delta V}{V}$$

where $\Delta V / V$ is the fractional change in volume. [4]

8 **(short)**

(a) Define the terms *hardness* and *yield stress*. A metal deforms in a standard tensile test according to the following relationship between true stress σ_t and true strain ε_t :

$$\sigma_t = 450\sqrt{\varepsilon_t} \quad \text{MPa.}$$

In a forming operation a cylinder of this material is stretched in simple tension such that its length increases by 25%. Estimate the yield stress and Vickers hardness of the metal after forming. [6]

(b) An identical cylinder of the same metal is deformed in simple compression. What percentage change in cylinder length will produce the same hardness as that found in the stretched cylinder? Find the ratio of the loads at the end of the two forming operations. [4]

9 **(short)** A fibre-reinforced composite consists of aligned carbon fibres in an epoxy resin matrix. The fibres have a Young's modulus n times that of the epoxy matrix E_m. The volume fraction of the fibres is V_f. Using the Materials Data Book expressions for the longitudinal and transverse Young's moduli of a unidirectional fibre-reinforced composite show that:

(a) the longitudinal Young's modulus E_L when the composite is loaded along the fibre direction is given by

$$E_L = E_m\left(1+(n-1)V_f\right)$$ [2]

(b) the transverse Young's modulus E_T when the composite is loaded in a direction perpendicular to the fibres is given by

$$E_T = \frac{nE_m}{\left(n-(n-1)V_f\right)}$$ [2]

(c) the maximum ratio E_L / E_T occurs for $V_f = 0.5$ and is independent of n. [6]

10 **(short)** A design for a strut requires that it shall carry an axial load without buckling (the Euler buckling load is $F = \pi^2 EI / L^2$ where E is Young's modulus, I is the second moment of area and L is the length). The strut has a fixed length and a hollow, thick-walled circular section.

(a) The thick-walled circular tube has an outer radius of $5t$ and a wall thickness of $2t$. Show that the shape efficiency factor ϕ_B^e for such a strut is approximately equal to 2 using

$$\phi_B^e = \frac{I}{I_{ref}} = \frac{12I}{A^2}$$

where A is the cross-sectional area and I_{ref} is a reference second moment of area of a solid square of the same cross-sectional area. [5]

(b) Taking the cross-sectional area to be a free variable, show that, in order to minimise mass, the following shape-dependent performance index M should be maximised

$$M = \frac{\sqrt{E\phi_B^e}}{\rho}$$

where ρ is the density. Using the data in Table 1, choose (without detailed calculation) between a steel circular tube and a wooden square, based on this performance index.

Material	Young's modulus (GPa)	Density ($\mathrm{Mg\ m^{-3}}$)
Steel	210	7.8
Wood	20	0.8

Table 1

[5]

11 **(long)**

(a) Under fatigue loading, a crack in a gas turbine blade made of titanium alloy grows at a rate

$$\frac{da}{dN} = 4 \times 10^{-11} \Delta K^5 \text{ (m cycle}^{-1}\text{)}$$

where a is the crack length, N is the number of fatigue cycles and ΔK is the stress intensity factor range (measured in units of MPa m$^{1/2}$). The fracture toughness of the titanium alloy is $K_{IC} = 30$ MPa m$^{1/2}$. In service, the blade experiences a constant stress range with a $\sigma_{\min} / \sigma_{\max}$ ratio of 0.1. During operation, the blade failed after 10 000 loading cycles. Analysis revealed that failure resulted from the growth of an internal circular fatigue crack. The crack had grown to a radius of 5 mm, before the component failed by fast fracture. The stress intensity factor K for an internal circular crack of radius a is given by $K = (2/\pi)\sigma\sqrt{\pi a}$ where σ is the remote stress.

(i) Determine the maximum stress normal to the plane of the crack during a loading cycle, and the stress range of the cycle. [8]

(ii) What was the initial crack length? [10]

(b) The blades of a gas turbine are to be manufactured from silicon nitride. A blade is 30 mm long, it has a cross-sectional area of 20×10^{-6} m^2 and is attached to a rotor of radius R = 200 mm. When the rotor rotates at an angular velocity of ω rad s^{-1}, the axial stress $\sigma(x)$ in the blade may be approximated by

$$\sigma(x) = \rho\omega^2 Rx$$

where x is the distance from the tip and $\rho = 3200$ kg m^{-3} is the density of the material.

In service, the rotor operates at a maximum speed of $\omega = 3000$ rad s^{-1}. The design allows for a maximum failure probability of 10^{-7}. Assuming a Weibull modulus $m = 10$, specify a target uniaxial tensile strength which corresponds to a failure probability of 1% in tests conducted on samples of cross-sectional area 16×10^{-6} m^2 and length 40 mm. Use the Weibull equation for the survival probability P_s from the Materials Data Book. [12]

12 **(long)**

(a) Distinguish a thermoplastic from a thermoset. Give two examples of each. Sketch the variation of Young's modulus with temperature for

(i) an amorphous thermoplastic;

(ii) a thermoset.

Label critical temperatures and account for transitions in behaviour. [10]

(b) The viscoelastic response of polymers can be modelled by a system of linear elastic springs and viscous dashpots. Write down the governing equations for the stress for each of the two components. Without carrying out any calculations, sketch strain versus time for the case of an imposed step in stress when a spring is connected with a dashpot (i) in series and (ii) in parallel. Explain why the plots take the forms you have sketched. [8]

(c) The deformation response of a polymer is to be represented by a linear dashpot of viscosity η in series with a linear spring of Young's modulus E_1, as shown in Fig. 7(a). Derive the governing differential equation relating the stress to the strain. Hence find the harmonic response. [8]

(d) To simulate the deformation response of another polymer, an additional spring of Young's modulus E_2 is now loaded in parallel with the system from part (c), as shown in Fig. 7(b). Use the "electrical circuit method" to write down the harmonic response. [4]

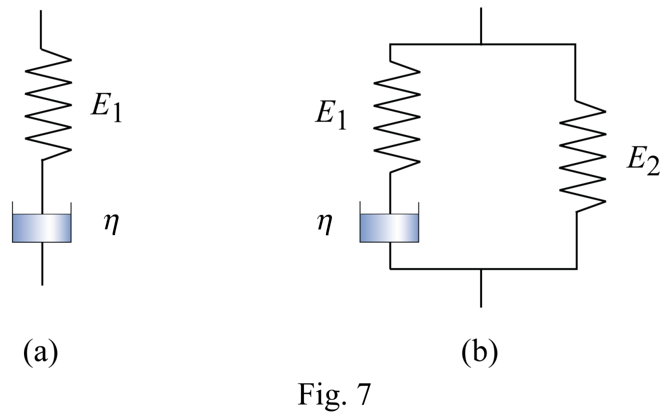

Fig. 7

END OF PAPER

Answers

1. $\sigma_l = \frac{-PL}{4t}, \quad \sigma_c = \frac{-PL}{2t}$

2. $FR\left(\frac{1}{\sqrt{3}} - \frac{1}{2}\right)$

3. $\frac{10FL}{AE}$ downwards.

4. $\tan\psi = \frac{\Delta e}{e_0}\tan\theta$

5. (b) $4.1\text{x}10^9$ mm^4 (c) 600kNm

6. (b) $d(x) = \frac{3\omega|x|}{b\sigma_y}$

   (c) the two designs intersect at $x = \frac{L}{\sqrt{1+k}}, d(x) = \frac{kL}{\sqrt{1+k}}$ where $k = \frac{3\omega}{b\sigma_y}$

7. (a) Atomic density BCC: 91.1×10^{27} atoms m^{-3}, FCC: 93.3×10^{27} atoms m^{-3}

   (b) $\frac{\Delta L}{L} \simeq -0.79\%$

8. (a) $\sigma_y \simeq 213$ MPa, $H = 639$ MPa. (b) $\varepsilon_n = -0.20$ i.e. 20% reduction, $\frac{F_c}{F_t} = 1.56$.

11. (a) (i) $\sigma_{\max} = 376$ MPa, $\Delta\sigma = 338.4$ MPa;

   (ii) $a_o = 35$ µm, initial crack length $2\times a_o$;

   (b) $\sigma = 427$ MPa

12. (c) $\frac{\hat{\varepsilon}}{\hat{\sigma}} = \frac{\left(i\omega + \frac{E_1}{\eta}\right)}{i\omega E_1}$ (d) $\frac{\hat{\varepsilon}}{\hat{\sigma}} = \frac{E_1 + i\omega\eta}{E_2(E_1 + i\omega\eta) + E_1 i\omega\eta}$