EGT0
ENGINEERING TRIPOS PART IA

Thursday 2 June 2016 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 **(short)** Figure 1 shows a portal frame subject to two external loads. Draw a copy of the figure and show on it the bending moment diagram, using the convention that the moment is plotted on the "tensile side" of the member. [10]

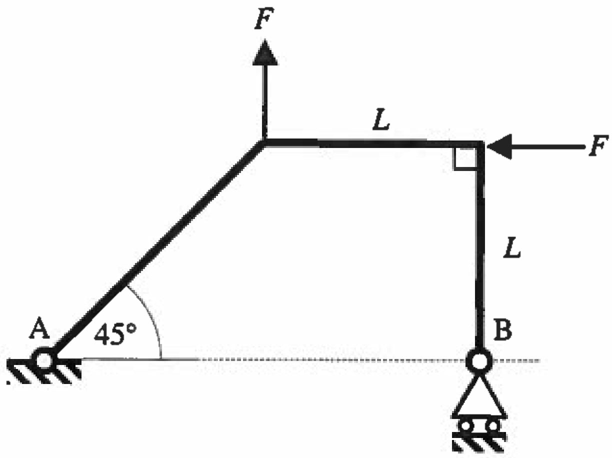

Fig. 1

2 (**short**) A cross-section of a floor beam is shown in Fig. 2. The beam is made from thin plates with uniform thickness t. Using the fact that $t \ll B$ to simplify your analysis, determine the second moment of area of the beam about its horizontal neutral axis. [10]

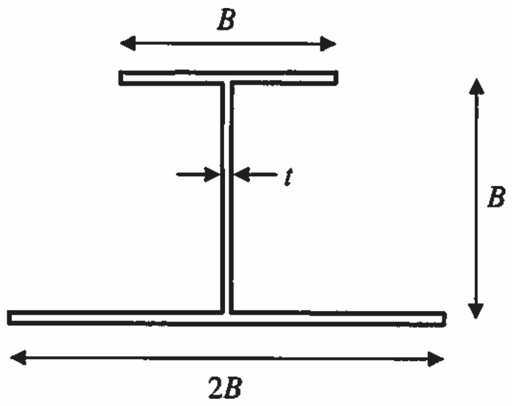

Fig. 2

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3 (short) The structure in Fig. 3 comprises two members with cross-sectional area A and Young's modulus E such that $L/AE = 10^{-6}$ m N^{-1}. When the applied force $F = 1$ kN, use GRAPHICAL METHODS ONLY to find the displacement of node A. [10]

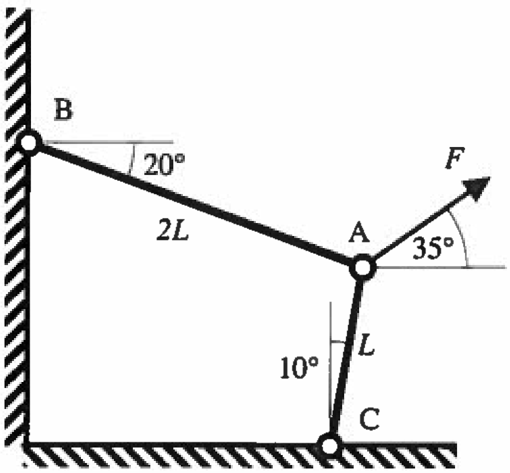

Fig. 3

4 (**short**) Figure 4 shows four rigid cylinders, of different weights but identical radii, arranged around a horizontal circular drum. The centre of D is vertically above the centre of the drum, and the centre of A is horizontally to the left of the centre of the drum. Cylinders A and D are welded securely to the drum which is stationary and the axes of all four cylinders and the drum are parallel.

Cylinders B and C are in contact with their neighbours and the drum but not attached. If all interfaces are frictionless, what is the highest possible ratio of the weight of cylinder C, W_C, to the weight of cylinder B, W_B, before cylinder B loses contact with the drum? [10]

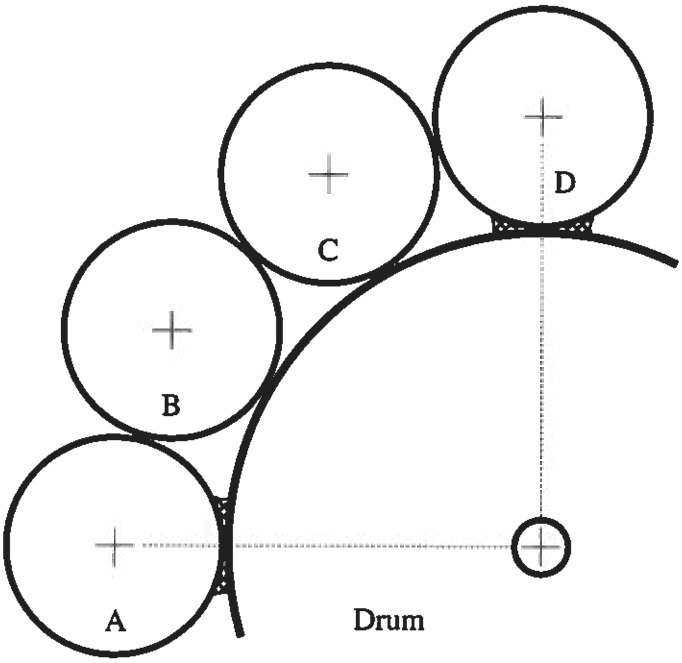

Fig. 4

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5 (**long**) Figure 5 shows two pin-jointed towers that support a heavy cable. All vertical and horizontal members in the tower have length $L = 10$ m, and all members have cross-sectional area $A = 0.001$ m^2 and Young's modulus $E = 200$ GPa. The cable can be assumed to have weight 400 N per horizontal metre, and at midspan has dipped 10 m.

(a) Calculate the vertical and horizontal loads exerted on the left hand tower by the cable. [5]

(b) Use virtual work to calculate the vertical deflection of the top of the left hand tower (node C) under this loading. [15]

(c) If the members in the tower are made from steel with a yield stress of 500 MPa, calculate the fraction of the material in the tower that could be saved if the cross-section of every member was designed to be at the point of yielding under the loading from the cable. [10]

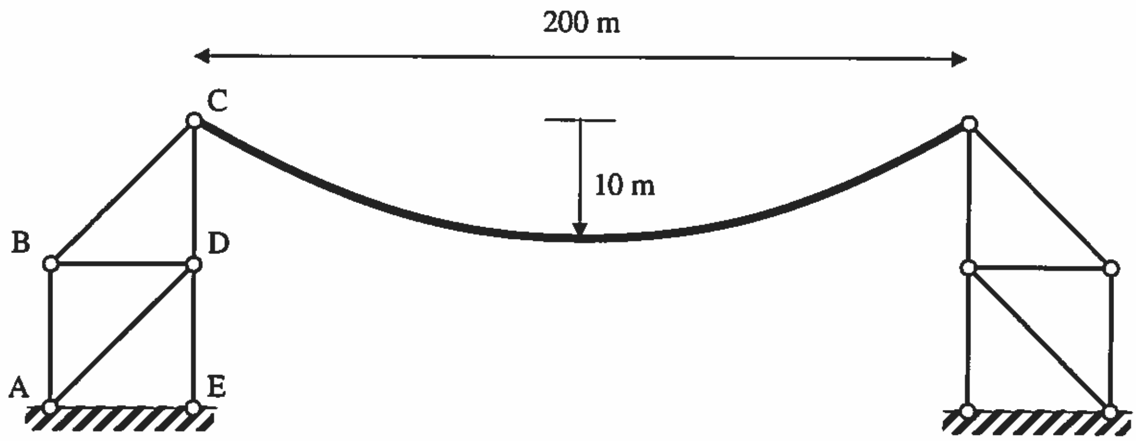

Fig. 5 NOT TO SCALE

6 (**long**) Figure 6 shows a tall chimney of height H subject to uniform lateral wind loading (force per unit height) of ω as shown. The chimney, which behaves as an elastic beam with bending stiffness EI, is supported by an inextensible cable at 45° to the horizontal, attached halfway up the chimney at P so that there is no lateral deflection at this point.

(a) Find the tension in the cable by appropriate superposition of standard "data-book" cases. [10]

(b) The chimney is constructed as a hollow circular cylinder, with outer radius R and wall thickness t, from masonry of density ρ. The chimney will fail if the longitudinal stress in its wall becomes tensile anywhere.

(i) For a design with $H = 100$ m, $R = 3$ m, $t = 0.1$ m, and with wind loading $\omega = 2000$ N m^{-1}, find the minimum value of ρ so that failure of the chimney at P is just avoided. [10]

(ii) For this design, calculate whether the chimney will fail at its base before it fails at P? [10]

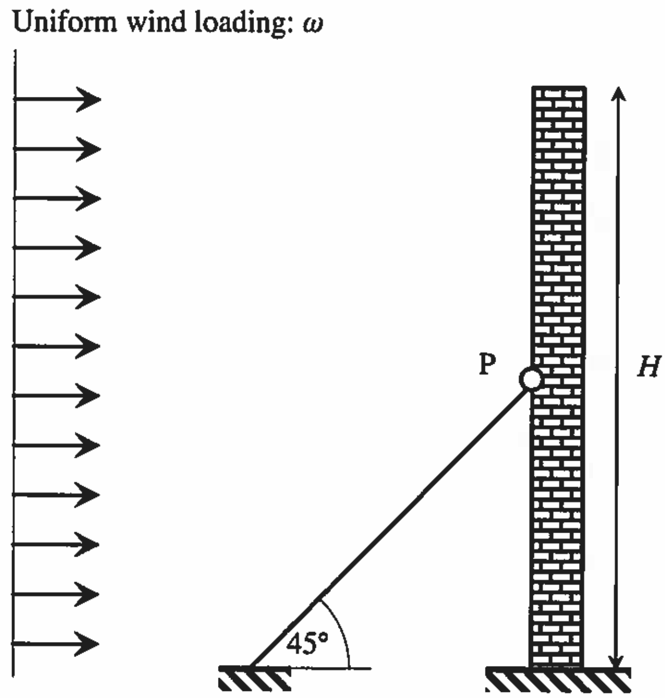

Fig. 6

SECTION B

7 **(short)**

(a) On the same axes, sketch both the nominal and true uniaxial tensile stress-strain curves for a strain hardening ductile metal, up to the point of fracture. Label on your sketches: the Young's modulus E; the yield stress σ_y; the tensile strength σ_{ts}; the permanent strain after fracture ε_f. [5]

(b) A cube of side length L of a rigid perfectly plastic material is loaded uniaxially to a nominal compressive strain of 10 %, and then unloaded. If this process is carried out n times to the same specimen, derive expressions for the final dimensions of the cube. [5]

8 **(short)** A plate loaded by a tensile stress σ_0 contains a hole of radius R, with a short edge crack of length $a = 10$ mm $\ll R$, as sketched in Fig. 7. The stress intensity factor K for the crack is given by: $K = 3.36\sigma_0\sqrt{\pi a}$.

(a) Explain what is meant by the *process zone* at the tip of a crack. How does the *process zone size* influence the use of the stress intensity factor K to calculate the value of σ_0 at failure? [3]

(b) Assuming failure is controlled by the fracture toughness K_{IC}, use relevant property data from the Materials Data Book to estimate the value of σ_0 at failure for the following material choices: magnesium alloy, with $\sigma_y = 400$ MPa; aluminium alloy, with $\sigma_y = 50$ MPa. In each case, comment on the validity of your estimate. [7]

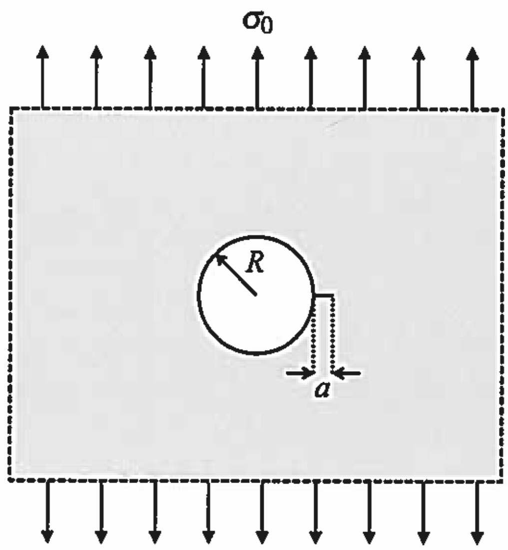

Fig. 7

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9 **(short)** The differential equation relating the total strain $\varepsilon(t)$ and the stress $\sigma(t)$ for the Maxwell model sketched in Fig. 8(a), with modulus E and viscosity η, is

$$\dot{\varepsilon}(t) = \frac{\sigma(t)}{\eta} + \frac{\dot{\sigma}(t)}{E} .$$

(a) Show that the corresponding stress response to a step strain $\varepsilon(t) = \begin{cases} 0 & t<0 \\ \varepsilon_0 & t \geq 0 \end{cases}$ is

$$\sigma(t) = \varepsilon_0 E \exp\left(-\frac{E}{\eta}t\right) \quad \text{for } t \geq 0 .$$ [6]

(b) A tendon may be modelled as a Maxwell material at small strains. The stress relaxation of a tendon was measured for a step strain $\varepsilon_0 = 0.04$, giving the curve shown in Fig. 8(b). Use this graph to determine the viscosity of the tendon, given its modulus $E = 0.1$ GPa. [4]

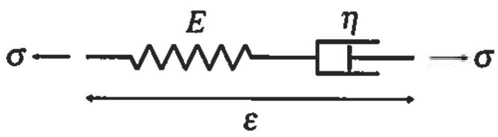

Fig. 8(a)

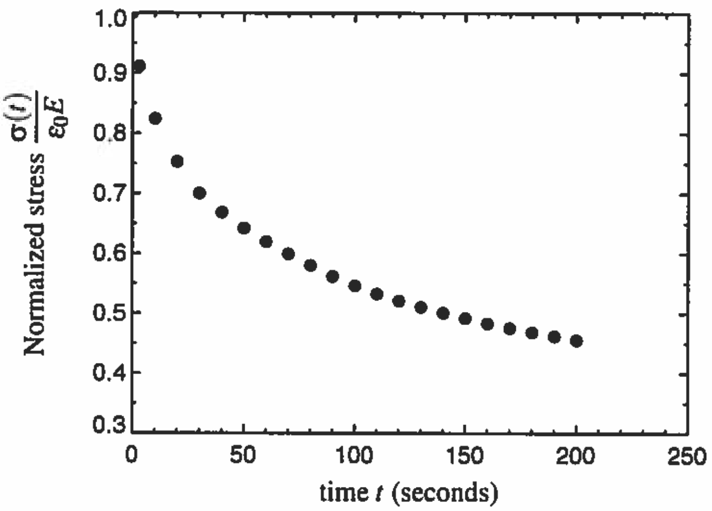

Fig. 8(b)

10 (**short**)

(a) A three-layer laminate is shown in Fig. 9(a). Each layer consists of an isotropic, linear elastic material with Young's moduli E_1 (outer layers) and E_2 (inner layer). Using the relevant equations in the Materials Data Book, give expressions for $E_{//}$ and $E_{\perp}$, the effective moduli of the laminate when it is loaded parallel and perpendicular to the layers, respectively. [3]

(b) The three layers are now replaced by a laminate to assemble the hybrid composite shown in Fig. 9(b). The laminate forming each of the three layers consists of alternating strips of the two materials with moduli E_1 and E_2 at equal volume fraction. Using the results of part (a), or otherwise, find expressions for E_x, E_y and E_z, the effective moduli in the x, y, and z directions of this hybrid construct. [7]

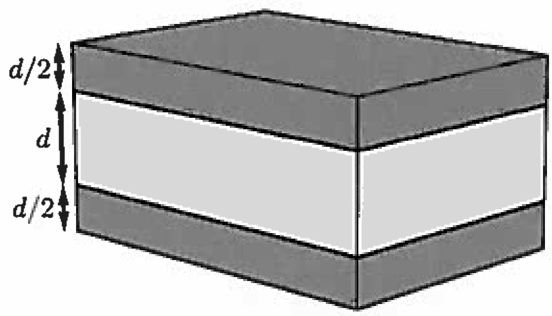

Fig. 9(a)

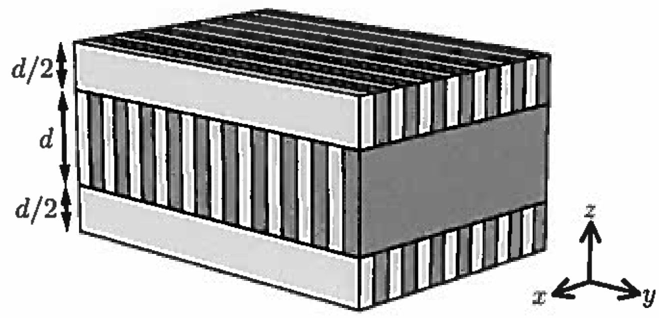

Fig. 9(b)

11 **(long)** A model for part of a micro-fabricated (i.e. dimensions ~ μm) gyroscope for a mobile phone is shown in Fig. 10(a). It consists of a rigid body supported by two elastic beams of length L, depth d, width b and Young's modulus E. The dimension d can be varied, but the width b and length L are both fixed. The rigid body is loaded by a force F which has a maximum value F_0 (Fig. 10).

(a) A free body diagram showing a beam bending under load is shown in Fig. 10(b). Using the results in Section 4.5.2 of the Structures Data Book, show that the maximum displacement of the rigid body, δ_0, and the maximum tensile stress in the beam, σ_0, are given by:

$$\delta_0 = \frac{F_0 L^3}{2Ebd^3} \qquad \text{and} \qquad \sigma_0 = \frac{3F_0 L}{2bd^2}\,. \qquad [6]$$

(b) As a first design iteration, a material is to be selected for the beams that minimises δ_0, without the maximum stress σ_0 exceeding the failure strength σ_f.

(i) State the objective, functional constraints and geometric constraints for this problem. [3]

(ii) Derive a material performance index that must be maximised to obtain the best material choice. [5]

(iii) Hence, using a suitable selection chart from the Materials Data Book, identify the three best *metal alloys* for this application. Why is it reasonable to disregard natural materials and composites? [4]

(c) For a more refined design, a material is to be chosen that minimises the *mass* of each beam, subject to the following constraints: when $F_0 = 50$ μN the deflection amplitude $\delta_0 \leq 2$ μm, and the maximum stress $\sigma_0 \leq \sigma_f$ (the failure strength). Processing constraints limit the material options to those given in Table 1. The width $b = 2$ μm and the length $L = 200$ μm.

(i) For each material, calculate the required value of the depth d. In each case, identify the active constraint. [8]

(ii) Hence, rank the material choices from best to worst. [4]

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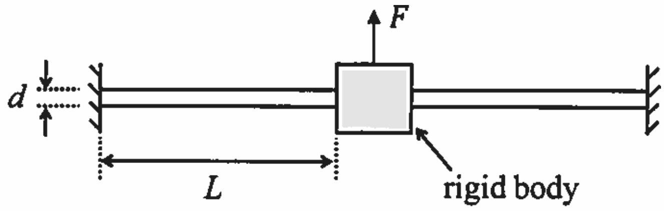

Fig. 10(a)

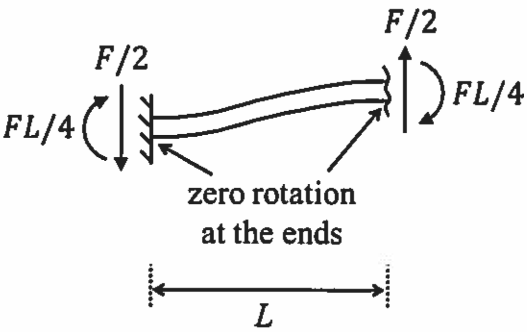

Fig. 10(b)

Material	E (GPa)	ρ (kg m^{-3})	σ_f (MPa)
silicon nitride	290	3000	750
aluminium	70	2700	50
nickel	200	8900	150

Table 1

12 **(long)** A wall standing on rigid ground has a height H and a variable thickness $D(z)$ that depends on z, the distance from the ground (see Fig. 11(a)). The wall is infinitely long in the x direction and is made of concrete, which can be considered to be an elastic material with Poisson's ratio ν, Young's modulus E and mass density ρ. A pressure P is applied vertically on the top of the wall. The convention for this problem is to take **compressive stresses and strains to be positive.**

(a) The thickness of the wall follows $D(z) = D_0 \exp(-\rho g z/P)$, with D_0 the thickness at the base, which is a constant, and g the acceleration due to gravity.

(i) Calculate the mass of the upper portion of the wall between an arbitrary position z and the top of the wall $z = H$. [5]

(ii) Hence, show that the stress acting in the z direction in the wall is $\sigma_z(z) = P$ for all z. [3]

(iii) Use Hooke's Law in 3D to derive an expression for the vertical strain component ε_z in the wall. The wall is slender, so that its stress in the y direction is zero for all z. [7]

(iv) Hence, derive an expression for the change in height of the wall, ΔH. The material is concrete with the properties given in Table 2. Evaluate $\Delta H/H$ for $P = 0.8\sigma_f$, where σ_f is the concrete's compressive failure strength. [3]

(b) The wall is glued to the ground with an adhesive cement (see Fig. 11(b)). In this process, a very thin layer of adhesive (thickness h much smaller than the dimensions of the wall) is compressed between the wall and the ground by the pressure P. The adhesive film remains perfectly bonded to the ground and the wall's base, so that the horizontal components of its strain are zero.

(i) Modelling the adhesive as an elastic solid with Young's Modulus E_a and Poisson's ratio ν_a, find the horizontal stress components $\sigma_{a,x}$ and $\sigma_{a,y}$ in the adhesive layer. Assume $E_a \ll E$, so that the wall may now be regarded as rigid. [7]

(ii) The adhesive has a coefficient of thermal expansion α. Neglecting thermal expansion of the wall and the ground, how do the expressions for $\sigma_{a,x}$ and $\sigma_{a,y}$ change if the adhesive is cooled by ΔT? [5]

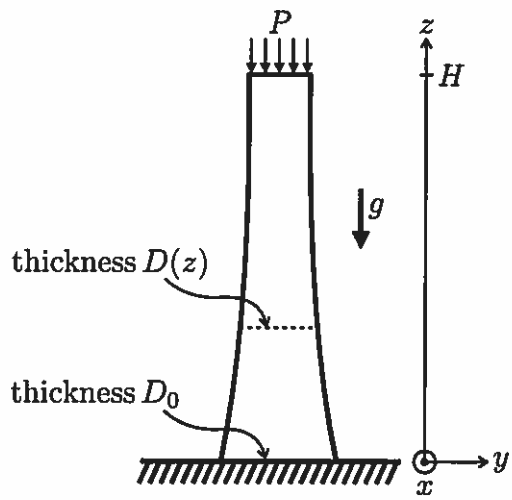

Fig. 11(a)

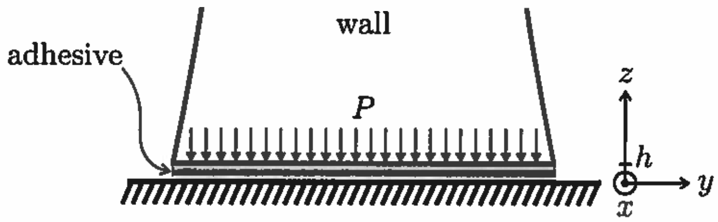

Fig. 11(b)

Properties of concrete	
Mass density	$\rho = 2400\,\mathrm{kg\,m^{-3}}$
Young's modulus	$E = 17$ GPa
Poisson's ratio	$\nu = 0.2$
Compressive failure strength	$\sigma_f = 20$ MPa

Table 2

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