Version JMA/6

EGT0
ENGINEERING TRIPOS PART IA

Thursday 8 June 2017 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number <u>not</u> your name on the cover sheet.

STATIONERY REQUIREMENTS
Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 **(short)** Figure 1 shows the bending moment distribution $M(x)$ carried by a uniform simply supported beam, with second moment of area I and Young's Modulus E, acted on by a central load as shown. Derive a formula for the deflected shape of the beam using the co-ordinate system specified in the figure. [10]

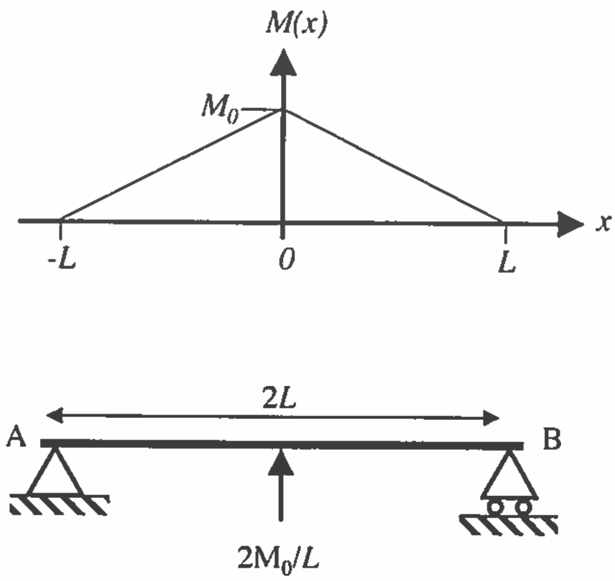

Fig. 1

2 **(short)** Figure 2 shows a representative cross-section through a wide sheet of corrugated cardboard. The board is made of three sheets of stiff paper with identical properties and thickness t. The upper and lower sheets are flat, and are firmly glued to the central sheet which has been corrugated into alternating semi-circles. Calculate the second moment of area per unit width of this design of cardboard. You should ignore the bending stiffness of the paper around its own axis and assume that the second moment of area can be averaged over the length λ. [10]

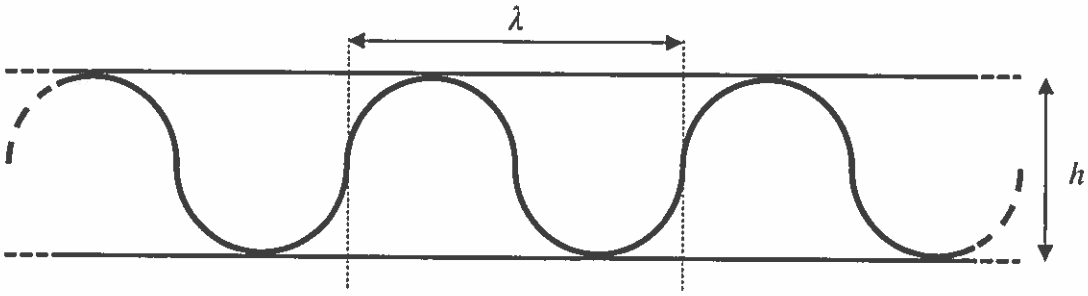

Fig. 2

3 (short) Figure 3 shows a steel cable submerged in water. The cable is supported on floats at a fixed depth below the surface and at a fixed separation L. The maximum dip of the cable d is small compared to length L. The cable which has density ρ and cross-sectional area A is under tension which is equal to T at the midpoint between each pair of floats. Derive a formula that could be used to show how d changes with the density of the cable. [10]

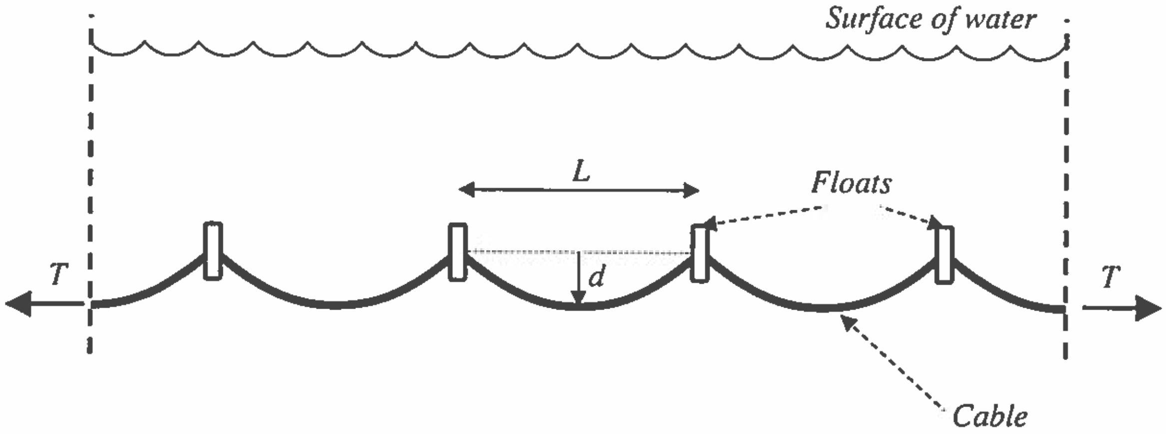

Fig. 3

4 (**short**) Figure 4 shows a cylinder of mass M on a slope at 30° to the horizontal. The cylinder supports a block also having mass M. The block is tethered horizontally by an inextensible cable so that its centre of mass is vertically above that of the cylinder. The contact between the block and the cylinder at A has angle of friction ψ_A. The contact between the cylinder and the slope at B has angle of friction ψ_B.

By taking moments about A and B of the forces acting on the cylinder, or otherwise, find the critical values of ψ_A and ψ_B. On axes of ψ_A against ψ_B show the four possible instantaneous responses of the cylinder as it is released from the position shown and as the two angles of friction vary independently between 0 and 45°. [10]

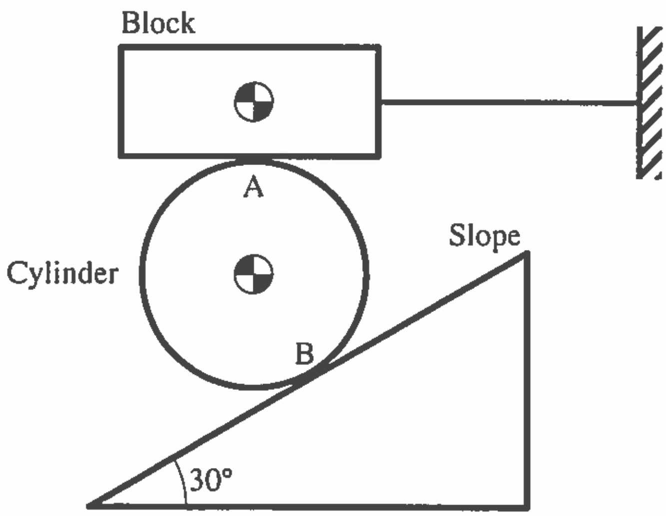

Fig. 4

(TURN OVER

5 (long) Figure 5 shows in plan view a square uniform bar of side b and length $2L$ mounted on a frictionless central support. The bar is subject to longitudinal and transverse distributed forces $r\alpha\delta m$ and $r\beta\delta m$ as shown where δm is the mass of an infinitesimal length of the bar at a distance r from the centre and α and β are constants.

(a) Assuming the bar has density ρ and Young's Modulus E, derive formulae for the bending moment and shear force in the bar as a function of r. You should ignore the effect of gravity. [10]

(b) Derive formulae for the maximum longitudinal and shear stresses along the length of the bar as a function of r. [10]

(c) The bar fails at $r=0$ when either the magnitude of the longitudinal stress exceeds σ_y or the magnitude of the shear stress exceeds $\sigma_y/2$. Sketch a graph of β against α, to show the limits to the safe use of the bar when $\rho L^2 = w_{ref}/4$ and $L/b = 10$ where w_{ref} is a reference distributed load. [10]

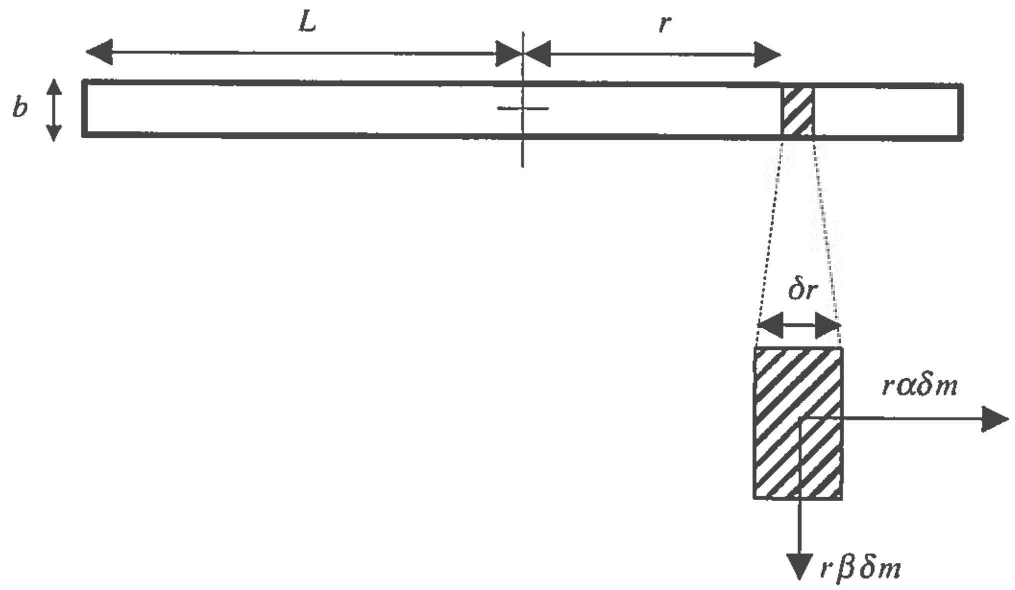

Fig. 5

6 (long) Figure 6 shows the design of a bicycle wheel. The rigid circular rim of the wheel is connected to the rigid triangular hub ABC by three identical pin-jointed metal spokes, aA, bB and cC such that aAB, bBC and cCA are straight lines. The wheel is subject to a vertical load F which is opposed by an equal and opposite force from the axle acting at O the wheel's centre. The angular position of the wheel is defined by the angle θ as shown in the figure.

(a) By using a free body diagram of the triangular hub ABC or otherwise, show that the force R_C in the spoke Cc varies with the angle θ as,

$$R_C = \frac{2}{3} F \sin\theta$$ [15]

(b) The spokes have length L, cross-sectional area A and Young's Modulus E. Derive a formula for the vertical deflection of the wheel's centre O as a function of θ when the wheel is subject to the load F and sketch this deflection as the wheel completes a single revolution. [15]

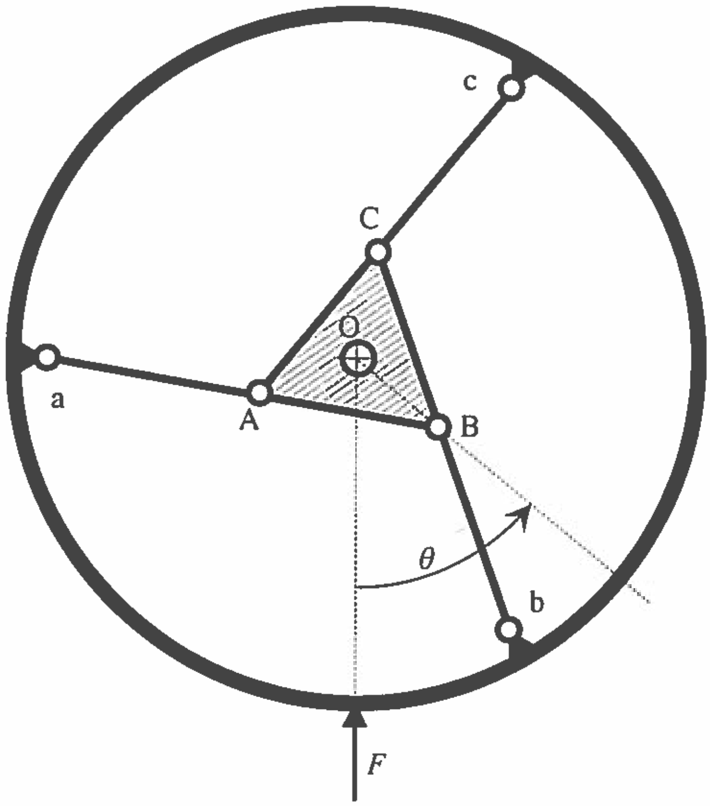

Fig. 6

SECTION B

7 **(short)**

(a) State the three main hardening mechanisms affecting the plastic deformation of metal alloys. In each case, identify the key microstructural parameters that control the hardening mechanism. [6]

(b) For a polycrystalline pure metal, the yield strength σ_y is related to the grain size d by

$$\sigma_y = \sigma_0 + \frac{k}{\sqrt{d}}$$

where σ_0 and k are material constants. It is found that $\sigma_y = 33$ MPa when $d = 0.5$ mm, and $\sigma_y = 38$ MPa when $d = 0.1$ mm. Find the grain size needed to obtain a yield strength of 45 MPa. Comment on why this is a relatively weak hardening mechanism. [4]

8 **(short)**

(a) Calculate the volume of an FCC unit cell in terms of the atomic radius R. [4]

(b) Assuming an atomic radius of $R = 0.128$ nm and an atomic weight of 63.5 kg kmol^{-1}, calculate the theoretical density of FCC copper, and compare it with the Data Book value. Why might the theoretical and Data Book values differ? [6]

9 **(short)** A car is to be redesigned for reduced CO_2 emissions. It is proposed to replace 25 kg of steel parts with aluminium alloy parts with the same dimensions.

(a) Calculate the mass saving from this switch, and the change in embodied CO_2 of the car. The embodied CO_2 of steel is 1.37 kg CO_2 kg^{-1} and that of aluminium alloy is 8.24 kg CO_2 kg^{-1}. [4]

(b) A 10 kg reduction in weight leads to a reduction of 1×10^{-3} kg of CO_2 per km travelled. If the distance travelled each year is 10 000 km, calculate how many years it would take to make up for the change in embodied CO_2. Comment on the success of the weight reduction strategy. [6]

10 **(short)** An aluminium alloy has Young's modulus $E = 70$ GPa, Poisson's ratio $\nu = 0.3$, yield strength $\sigma_y = 200$ MPa and coefficient of thermal expansion $\alpha = 24 \times 10^{-6}$ K^{-1}. These properties can be assumed to be temperature independent.

(a) A strip of the aluminium alloy is held at constant length while being cooled down until it yields in uniaxial tension. Calculate the required temperature change ΔT. [4]

(b) A thin layer of the aluminium alloy is perfectly bonded to the surface of a large block with coefficient of thermal expansion approximately equal to zero. Both materials are then heated by $\Delta T = 5$ K. It can be assumed that the large block remains stress free. Calculate the stress in the aluminium alloy layer. Is it in tension or compression? [6]

(TURN OVER

11 (long)

(a) A component consists of two thin-walled cylinders (A and B), which are to be joined as shown in Fig. 7 using friction welding. Cylinder A is made from a low hardness mild steel (hardness H_A), and cylinder B is made from a hardened steel (hardness H_B). Both cylinders have radius R and wall thickness h.

(i) The cylinders are first pressed into contact by applying a force F, as shown in Fig. 7. If $H_B \gg H_A$, explain why the true area of contact A_t satisfies

$$A_t \approx \frac{F}{H_A}$$ [4]

(ii) Next, with cylinder A held fixed, cylinder B is rotated about its axis with angular velocity ω, so that there is sliding at the interface. For a short period of time, abrasive contact occurs between the two cylinders. If $H_B \gg H_A$, the wear rate Q (defined as the volume of material removed per unit sliding distance) is given by

$$Q = \frac{KF}{H_A}$$

where K is the wear coefficient. Derive an expression for the rate of shortening of cylinder A due to abrasive wear. [4]

(b) In service, the component is observed to fail in fatigue. Laboratory fatigue tests are performed, subjecting it to a sinusoidal loading with stress range $\Delta\sigma$ at zero mean stress. The number of cycles to failure N_f is plotted in Fig. 8.

(i) On a sketch of the figure, indicate the ranges of N_f corresponding to low cycle fatigue and high cycle fatigue. What is the stress range at the endurance limit? [6]

(ii) Using the definition of Basquin's law in the Materials Data Book, estimate the values of constants C_1 and α for the high cycle fatigue regime. [6]

(iii) In service, the component experiences oscillating loading at frequency Ω and stress range $\Delta\sigma$ (again, at zero mean stress). The values of Ω and $\Delta\sigma$ change during service, as shown in Table 1. Predict the total lifetime of the component in days. Comment on the likely accuracy of this prediction. [10]

Proportion of run time	Frequency, Ω (Hz)	Stress range, $\Delta\sigma$ (MPa)
98%	40	50
1.7%	12	300
0.3%	6	400

Table 1

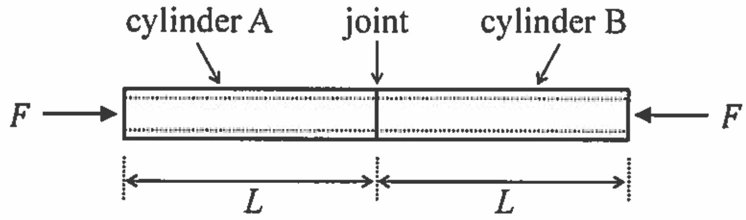

Fig. 7

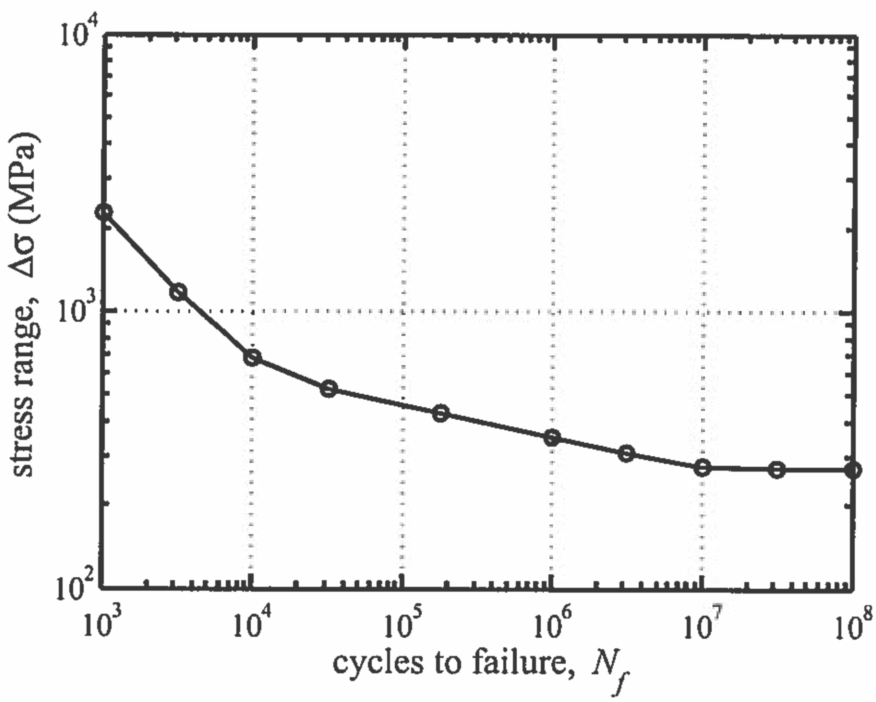

Fig. 8

(TURN OVER

12 (long)

(a) In the stiffness-limited design of a beam in bending, the 'shape factor' ϕ is

$$\phi = \frac{I}{I_{\text{ref}}}$$

where I is the second moment of area of the beam's cross section, and I_{ref} is the second moment of area of a reference cross-sectional shape that has the same area, A.

(i) The weight of a beam in bending is to be minimised, subject to a maximum deflection constraint. Explain briefly why it is desirable to maximise ϕ. [2]

(ii) Describe two factors that might limit the maximum value of ϕ achievable in practice. [2]

(b) A beam of fixed length L is simply supported at its ends. The beam has a solid circular cross section with radius R, which is a free variable. The objective is to choose a material that minimises the deflection of the beam under self-weight. No other loads act on the beam. There is a constraint on the cost of the beam, which must not exceed a budgeted value C.

(i) Derive an expression for the mid-span deflection of the beam. [4]

(ii) Hence, show that the following material performance index M should be maximised

$$M = \frac{E}{C_m \rho^2}$$

where E is the Young's modulus, ρ is the density and C_m is the material cost per unit mass. [5]

(iii) Rank the materials listed in Table 2 from best to worst. For a cost $C = £500$ and a length $L = 5$ m, identify any material choices that would deflect by more than 10 mm. [8]

(c) The same beam described in part (b) is now to be designed with a more efficiently shaped cross-section: a thin-walled tube. The second moment of area is I.

(i) Taking the reference cross-section to be a solid circle, derive a relationship between I, the area A and the shape factor ϕ for the thin-walled tube. [3]

(ii) Treating the cross-sectional area A as the free variable, derive a material- and shape-dependent performance index that should be maximised if the objective and constraint are as defined in part (b). [6]

(cont.

Material	E (GPa)	ρ (kg m^{-3})	C_m (£ kg^{-1})
CFRP	120	1500	60
Al alloy	70	2700	2
Nylon	3	1100	4
Wood	12	600	2

Table 2

END OF PAPER

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