EGT0
ENGINEERING TRIPOS PART IA

Monday 6 June $2016 \quad 9$ to 12

## Paper 3

## ELECTRICAL \& INFORMATION ENGINEERING

Answer all questions.

The approximate number of marks allocated to each part of a question is indicated in the right margin.

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number not your name on the cover sheet.

## STATIONERY REQUIREMENTS

Single-sided script paper
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed

10 minutes reading time is allowed for this paper.
You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

## 1 (short)

(a) State Thevenin's and Norton's theorems.
(b) Calculate the Thevenin and Norton equivalents of the circuit of Fig. 1.


Fig. 1
2 (short)
(a) State the assumptions made when an op-amp is described as being 'ideal'.
(b) The op-amp for the circuit in Fig. 2 may be assumed to be ideal. Derive an expression for the DC voltage gain of the circuit. Also find an expression for the smallsignal voltage gain including the effect of $C_{2}$, and hence derive the -3 dB frequency of the circuit.


Fig. 2

3 (short) Figure 3 shows a radio frequency tuned circuit connected to a $200 \mathrm{kHz}, 50 \mathrm{~V}$ (rms) RF power output. Determine the complex impedance of the circuit and hence find the magnitude and phase of the input current with respect to the voltage source.


Fig. 3

## 4 (long)

(a) Figure 4(a) shows the circuit for a general AC bridge, in which the voltage source is a sine wave with angular frequency $\omega$. Define what is meant by the balance condition for this circuit, and derive the condition for balance in terms of $Z_{1}$ to $Z_{4}$.
(b) Using the result of part (a), derive an expression for the frequency at which the AC bridge shown in Fig. 4(b) is balanced. Then derive the relationship between the other components.
(c) In a similar manner, derive the balance conditions for the AC bridge as shown in Fig. 4(c) in terms of its component values and $\omega$.


Fig. 4 (a)


Fig. 4(b)


Fig. 4(c)

5 (long) Figure 5 shows the circuit of an amplifier.
(a) Calculate the values of $R_{1}$ and $R_{2}$ required to set the transistor operating point, such that $V_{G S}=-2 \mathrm{~V}, V_{D S}=8 \mathrm{~V}$ and $I_{D}=0.2 \mathrm{~mA}$. It can be assumed that the transistor is ideal and that there is no gate current.
(b) Draw the small-signal model for the circuit, assuming that the impedances of the capacitors are negligible at small signal frequencies.
(c) Derive approximate expressions for the small-signal gain $v_{d} v_{i}$ and the output impedance of the circuit, assuming that the output circuit draws no current. Hence evaluate these expressions for the values calculated in (a), and taking $g_{m}=1 \mathrm{mS}$, and $r_{d}=200 \mathrm{k} \Omega$.
(d) What value of $C_{1}$ is required if the circuit is to have a -3 dB point of 15 Hz , with a $10 \mathrm{k} \Omega$ load connected to the output, and assuming that the reactance of $C_{2}$ is small compared to $R_{2}$ at this frequency?
(e) Under the conditions of part (d), find the maximum and minimum voltages at the drain of the transistor and across the $10 \mathrm{k} \Omega$ load resistor if the input voltage is a sinusoid of peak-peak amplitude 100 mV at a frequency of 1 kHz .


Fig. 5.

## SECTION B

6 (short)
(a) Prove de Morgan's theorems for two variables:

$$
\begin{equation*}
\overline{A \cdot B}=\bar{A}+\bar{B} \text { and } \overline{A+B}=\bar{A} \cdot \bar{B} \tag{4}
\end{equation*}
$$

(b) Consider the Boolean expression for Z:

$$
Z=B \cdot C+A \cdot \bar{B}+\bar{A} \cdot \bar{B}
$$

By constructing a Karnaugh map for $\bar{Z}$, or otherwise, find a simplified expression for $Z$ that can be implemented by two-input NAND gates only, and draw a circuit diagram that implements the expression.

7 (short) A certain memory chip has 13 address lines and 8 data lines. It is connected to a microprocessor with a 16 bit address bus and an 8 bit data bus.
(a) Determine the capacity of the memory chip in bytes, and compare its capacity to the maximum amount of memory that a modern microprocessor with a 32 bit address bus and an 8 bit data bus can address.
(b) Explain the roles of the $\overline{C S}$ and $R / \bar{W}$ lines in accessing data stored in the memory chip, and determine a Boolean expression for the $\overline{C S}$ input such that the memory chip appears in the microprocessor's memory map between locations $\mathrm{A} 000_{\mathrm{H}}$ and $\mathrm{BFFF}_{\mathrm{H}}$. Hence draw a circuit showing how the memory chip should be connected to the microprocessor.

## 8 (short)

(a) Explain the difference between File Register Instructions and Literal Instructions in PIC programs.
(b) The PIC code below is executed.
movlw 15;
movwf 0x20;
movlw 0xF1;
addwf 0x20,W;
movlw 0xAA;
movwf $0 \times 20$;
movlw 255;
xorwf $0 \times 20$;
sleep;
Determine the contents of W, memory location 0x20 and the C, DC and Z flags after:
(i) the addwf $\mathbf{0 x 2 0}, \mathbf{W}$ instruction; [4]
(ii) the xorwf $0 \times 20$ instruction.

9 (long) Some decorative lights consist of banks of red $(R)$ and yellow $(Y)$ lights, with each bank of lights being turned on or off by a single output from a controller which is to be constructed from J-K bistables. When the lights are first switched on, they are to follow sequence 1 below:

$$
\bar{R} \cdot \bar{Y}, R . Y, \bar{R} \cdot \bar{Y} \ldots
$$

(sequence 1)
Pressing a button $P$ (so $P=1$ ) whilst in either state of sequence 1 will cause the lights to enter sequence 2 below, in the $R . \bar{Y}$ state:

$$
R . \bar{Y}, \bar{R} . Y, R . \bar{Y} \ldots
$$

(sequence 2)
Pressing button $P$ again (so $P=0$ ) whilst in either state of sequence 2 will cause the lights to enter sequence 1 again in the $\bar{R} \cdot \bar{Y}$ state.
(a) Draw a state diagram for the controller.
(b) Determine the number of J-K bistables required to implement the system.
(c) Find the corresponding state transition table and hence determine the J-K inputs of the bistables in terms of the state of the system outputs and push button P .
(d) Draw a circuit diagram that implements the controller. It is essential that on applying power to the controller the lights start off in the $\bar{R} \cdot \bar{Y}$ state.

## SECTION C

## 10 (short)

(a) For metallic conductors explain why it is assumed that any net charge is distributed evenly on the surface of the conductor.
(b) Two point charges of $5 \mu \mathrm{C}$ and $-7 \mu \mathrm{C}$ are placed in free space at positions $(1,0)$ and $(1,1)$ respectively on a two dimensional $(x, y)$ plane where coordinates are measured in centimetres.
(i) Find the electric field due to the two point charges at coordinate $(-1,-1)$, expressing the field in terms of magnitude and direction.
(ii) If a charge of $10 \mu \mathrm{C}$ is now inserted at coordinate $(-1,-1)$, determine the magnitude of the force acting upon it.

## 11 (short)

(a) Define the self inductance of a circuit.
(b) Determine the self inductance of a solenoid with $n$ turns per unit length, length $l$ and cross-sectional area $A$. Assume that the length $l$ is much greater than the diameter of the solenoid cross-section.
(c) Figure 6 shows a simple circuit containing an inductor of value $L$ and resistor $R$ connected through a switch at position $\mathrm{K}_{1}$ to a DC source. The current flowing through the circuit in the steady state is $I_{0}$. The switch is then moved to position $\mathrm{K}_{2}$. Derive an expression for the current in the circuit as a function of time.


Fig. 6

## 12 (long)

(a) A capacitor consists of two concentric spherical metal shells of inner and outer radii $r_{1}$ and $r_{2}$ respectively with an air gap between.
(i) Using Gauss's law derive an expression for the capacitance.
(ii) The electric field at which air breaks down is $3 \mathrm{MV} \mathrm{m}{ }^{-1}$. Assuming that $r_{1}=1 \mathrm{~cm}$ and $r_{2}=3 \mathrm{~cm}$ calculate the capacitor charge at which electrical breakdown of the air occurs.
(b) A hollow toroidal ferrite ring with inner radius 4 cm and outer radius 8 cm with cross section shown in Fig. 7 is evenly wound with a 200 turn coil through which 1 A of current flows. The relative permeability of the toroid material $\mu_{r}$ is 500 and the hollow within the toroid is air-filled.
(i) What is the magnetic flux density $B$ in the tangential direction through the ferrite in terms of $r$ ?
(ii) What is the magnetic flux density $B$ in the tangential direction through the air core in terms of $r$ ?
(iii) What is the magnetic flux $\Phi$ in the air core and in the ferrite?
(iv) What is the self-inductance of the coil?


Fig. 7

## END OF PAPER

## IA Paper 32016 Section B

## Short Answers

6 (b) $Z=\overline{B . \bar{C}}$
7 (a) $8 \mathrm{kB}, 4 \mathrm{~GB} \quad$ (b) $\overline{\mathrm{CS}}=\overline{\mathrm{A}_{15} \cdot \overline{\mathrm{~A}_{14}} \cdot \mathrm{~A}_{13} \cdot \text { Address Valid }}$
8 (b) (i) $\mathrm{C}=1, \mathrm{DC}=1, \mathrm{Z}=1,0 \times 20=0 \mathrm{~F}_{\mathrm{h}}, \mathrm{W}=00_{\mathrm{h}}$ (ii) $\mathrm{C}=0, \mathrm{DC}=0, \mathrm{Z}=0,0 \times 20=55_{\mathrm{h}}, \mathrm{W}=\mathrm{FF}_{\mathrm{h}}$
9 (b) 2 bistables

