Version AJF/6

EGT1
ENGINEERING TRIPOS PART IB

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Wednesday 7 June 2017 2 to 4

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**Paper 5**

**ELECTRICAL ENGINEERING**

*Answer not more than **four** questions.*

*Answer not more than **two** questions from any one section and not more than **one** question from each of the other two sections.*

*All questions carry the same number of marks.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Answers to questions in each section should be tied together and handed in separately.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Supplementary page: one extra copy of Fig. 3 (Question 2)
Engineering Data Book

**10 minutes reading time is allowed for this paper.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

## SECTION A

1 (a) Figure 1 shows the circuit diagram of a simple bipolar transistor amplifier circuit. The bipolar transistor has small-signal parameters of $h_{ie}$ = 1 kΩ, $h_{fe}$ = 400, and $h_{re}$ and $h_{oe}$ may be neglected.

(i) Explain how the combination of resistors $R_1$ and $R_2$ result in the circuit having a stable operating point. [4]

(ii) Draw the small-signal equivalent circuit for the bipolar transistor amplifier circuit shown in Fig. 1. [4]

(iii) Using your small-signal equivalent circuit, derive an expression for the small-signal voltage gain of the circuit. [6]

(b) Figure 2 shows the same bipolar transistor amplifier circuit being used to amplify an oscillating current being produced by a magnet that is vibrating inside a coil of wire, which can be modelled by an a.c. current source, $i_1$. The amplified current, $i_2$, is passed through an earphone to create an audible sound. The earphone behaves like a load resistance, $R_L$.

(i) If $R_{12} = R_1 R_2 / (R_1 + R_2)$ and $R_{3L} = R_3 R_L / (R_3 + R_L)$, show that the small-signal current gain of the circuit is given by

$$\frac{i_2}{i_1} = \frac{-h_{fe} R_{12} R_{3L}}{R_L \left[ R_{12} + h_{ie} + R_4 \left( 1 + h_{fe} \right) \right]}$$ [7]

(ii) Explain how the circuit of Fig. 2 could be changed to increase the current gain. [4]

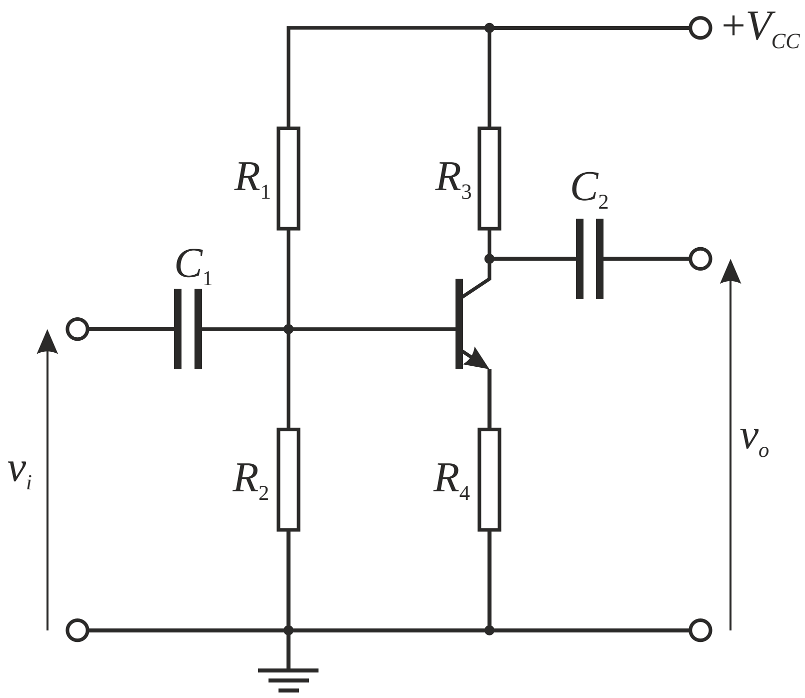



Fig. 1

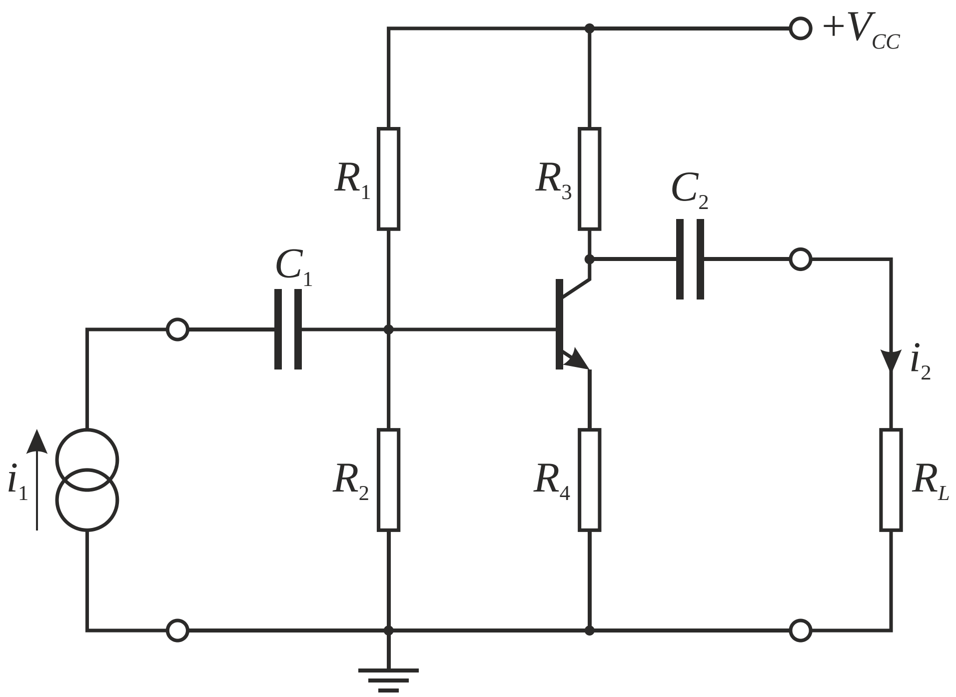



Fig. 2

2 (a) Figure 3 shows the circuit diagram of a 741 Operational Amplifier.

(i) By clearly labelling either loops or arrows as appropriate, annotate the following circuit elements on the additional copy of Fig. 3 which is attached to the question paper: all current mirrors, the output amplifier stage, the differential amplifier stage, the high gain amplifier stage, the inputs and the output. [6]

*An additional copy of Fig. 3 is attached to the back of this paper. It should be detached and handed in with your answers.*

(ii) Explain what each of the three amplifier stages does in the context of the way an operational amplifier is designed to behave. [6]

(iii) Explain the purpose of the 30 pF capacitor marked at position X in Fig. 3. [3]

(b) Figure 4 shows a circuit based on a 741 Operational Amplifier.

(i) If the Operational Amplifier is considered to be ideal, show that the relationship between the a.c. output voltage signal, $v_o$ , and the a.c. input voltage signal, $v_i$ , is given by

$$v_o = -RC\frac{dv_i}{dt}$$ [6]

(ii) Draw a diagram of a circuit based on another 741 Operational Amplifier which would have the following relationship between the input and output a.c. voltage signals

$$v_o = \frac{-1}{RC}\int v_i dt$$

The circuit should have a well-defined d.c. voltage gain. [4]

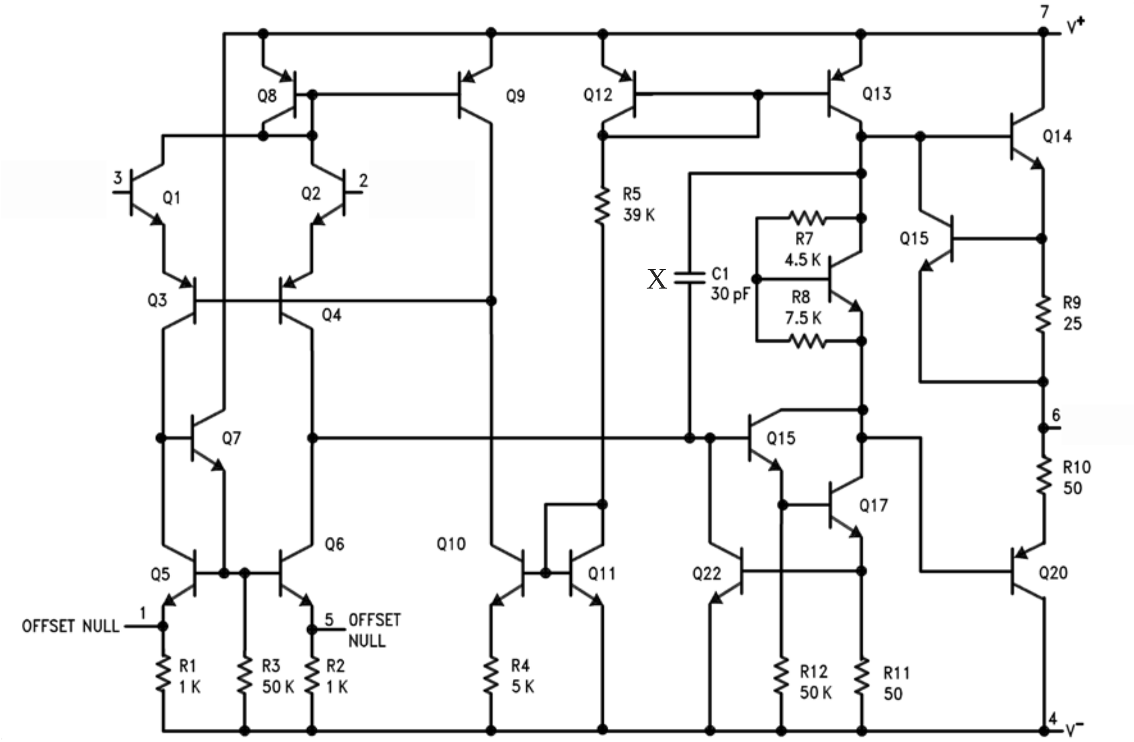

7
V+
Q8
Q9
Q12
Q13
Q14
3
Q1
Q2
2
R5
39 K
Q15
R7
4.5 K
X
C1
30 pF
R8
7.5 K
R9
25
Q3
Q4
6
Q7
Q15
R10
50
Q17
Q6
Q10
Q5
Q11
Q22
Q20
1
OFFSET NULL
5
OFFSET
NULL
R1
1 K
R3
50 K
R2
1 K
R4
5 K
R12
50 K
R11
50
4
V−

Fig. 3

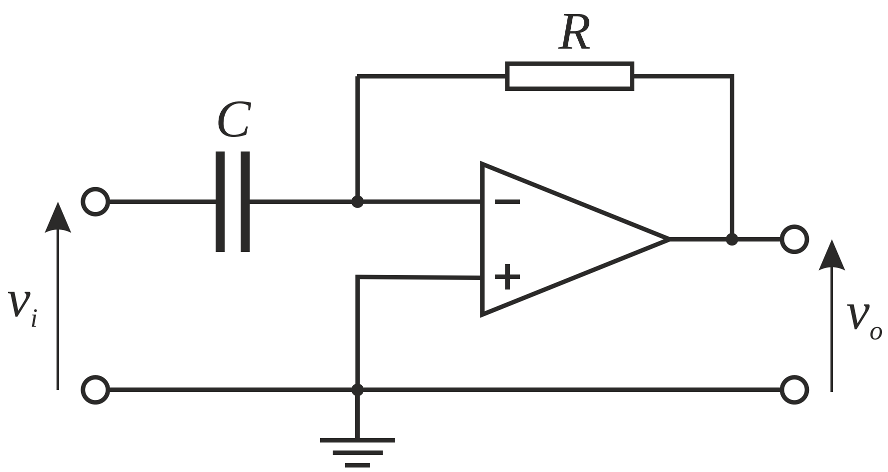

R
C
$v_i$
$v_o$

Fig. 4

## SECTION B

*Answer not more than two questions from this section*

3 (a) Explain what is meant by phase voltage and line voltage of a balanced star-connected three-phase voltage supply. [2]

(b) Derive the relationship between the phase voltage and line voltage for the voltage supply in part (a). [5]

(c) For a delta-connected load derive an expression for the real power dissipated in terms of the line voltage and the line current. [3]

(d) A balanced three-phase supply provides a constant line voltage of 11 kV at 50 Hz. This is connected to a balanced star-connected load and two identical balanced delta-connected loads. Each phase of the star-connected load consists of a 400 Ω resistor in series with a 2 H inductor. Each phase of the delta-connected load consists of a 1.5 kΩ resistor.

(i) Find the line current drawn from the supply, and the overall real and reactive power consumed by the loads. [10]

(ii) Find the value of balanced star-connected capacitors that can lead to a power factor of 0.99. [5]

4 (a) Explain what is meant by an operating chart of a generator. Describe how this can be constructed from the phasor diagram of the generator and identify the various features this includes. [7]

(b) A generator is connected to a 11 kV at 50 Hz infinite bus. The synchronous reactance of the generator is $X_S = 0.3\ \Omega$.

(i) If the prime-mover power is set to 200 MW and the load has power factor 0.75 lagging, find the load angle, $\delta$, and the excitation voltage, $E$. [8]

(ii) Find the excitation voltage in order to have a unity power factor with the prime-mover power being the same as that in part (b)(i). [5]

(iii) Explain factors that could limit the maximum achievable power factor by the generator. [5]

5 (a) Explain what is meant by the per-unit system in power system analysis and explain its significance. [5]

(b) Explain what is meant by a symmetrical three-phase fault to earth and give three properties of a protection system designed to limit the impact of such faults. [5]

(c) A 200 MVA 11 kV generator with synchronous reactance of $X_S$ = 0.5 pu supplies, via a 132 kV transmission line, an industrial region connected to a 33 kV bus. The transmission line has a reactance of 20j Ω. The generator is connected to the transmission line via a 11 kV / 132 kV transformer with a rating of 100 MVA and a reactance of 0.1 pu. The 33 kV bus is connected to the transmission line via a 132 kV / 33 kV transformer with a rating of 100 MVA and a reactance of 0.05 pu.

(i) By drawing a circuit diagram of the interconnection in a per-unit representation, find the fault current (measured in Amps) in the 11 kV bus when a symmetrical three-phase fault to earth occurs at the 33 kV bus. [10]

(ii) Find the value of the reactance at the 33 kV bus that will reduce the fault current in the 11 kV bus to half its value calculated in part (c)(i). [5]

**SECTION C**

6 (a) What is the Poynting vector? [4]

(b) Figure 5 shows a waveguide.

(i) By using the integral form of the Ampère Circuital Law, derive an expression for the peak current, $I$, in the waveguide in terms of the peak magnetic field, $H_y$ . Derive a similar expression for the peak voltage, $V$, between the conductors in terms of the peak electric field $E_x$ . [4]

(ii) Show that the Poynting vector can be used to prove that the power being transmitted along the waveguide is $V_{rms}I_{rms}$ . [4]

(c) The air core toroid shown in Fig. 6 of mean radius, $R$, and cross-sectional area of $\pi r^2$ is wound with $N$ turns of wire, where $r << R$. A current with a time dependence $I(t) = Kt$ is turned on at time $t = 0$.

(i) Find the energy stored in the magnetic field at time $t$ if the stored energy per unit volume in a magnetic field is $U_M = \mathbf{B}\cdot\mathbf{H}/2$. You may assume that all the magnetic field is confined within the toroid. [6]

(ii) Sketch a diagram of the toroid showing the direction of the magnetic and electric fields present. Hence, also show the direction of the Poynting vector and derive an expression that could be used to estimate its magnitude at time $t$. [7]

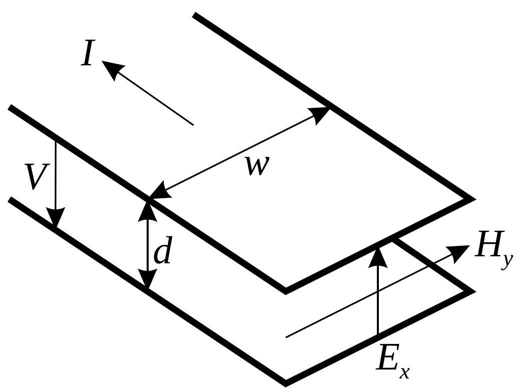



Fig. 5

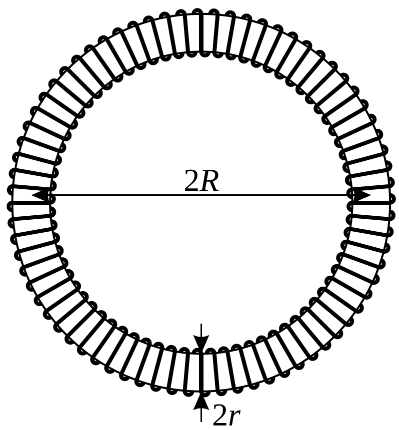



Fig. 6

7 We wish to analyse the transmission line properties of a coaxial cable. The cable is composed of an inner conductor of radius $\rho = a$, an outer conductor of internal radius $\rho = b$ and a slightly conducting dielectric between the inner and outer conductor which fills the gap entirely, as shown in Fig. 7. The dielectric has a permittivity of $\varepsilon = \varepsilon_0 \varepsilon_r$ and a permeability of $\mu = \mu_0$ . The start of the cable is taken to be at position $z = 0$.

(a) The expression for the radial electric field **E** in the coaxial cable is

$$\mathbf{E} = \frac{E_{0\rho}}{\rho} \mathbf{e}_\rho \exp(j[\omega t - \beta z])\exp(-\alpha z)$$

Using the Maxwell equations and the vector calculus expressions in cylindrical polar coordinates on page 14 of the Mathematics Data Book (2008 edition), show that the expression for the magnetic field **H** has the form

$$\mathbf{H} = \frac{H_{0\theta}}{\rho} \mathbf{e}_\theta \exp(j[\omega t - \beta z])\exp(-\alpha z)$$

and derive the expression for $H_{0\theta}$ in terms of $E_{0\rho}$ . [8]

(b) Derive an expression for $E_{0\rho}$ if the peak potential difference between the inner and outer conductors at the start of the cable is $V_0$. [6]

(c) The power transmitted by a cable of characteristic impedance, $Z$, is given by

$$P_t = \frac{1}{2Z}|V|^2 \text{ where } Z = \frac{\ln x}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}}$$

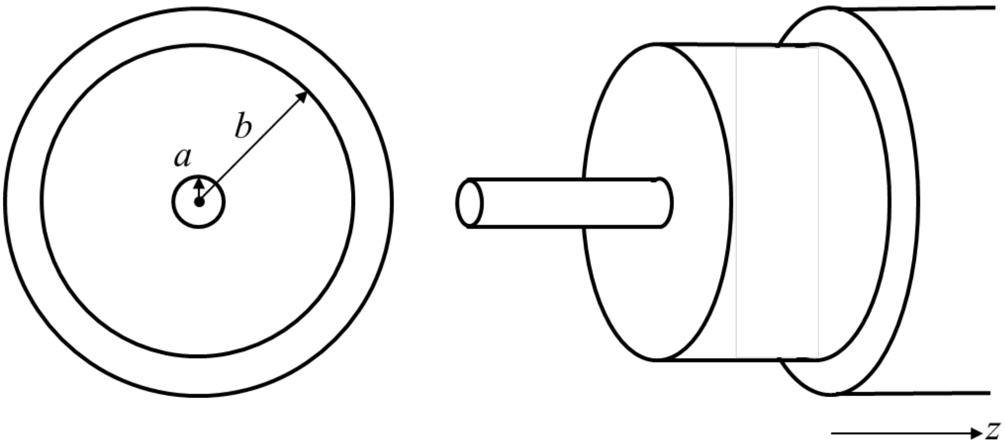
and the quantity $x$ is defined to be the ratio of the conductor radii so that $x = b/a$.

(i) If the maximum electric field in the coaxial cable is limited by the breakdown strength of the dielectric, $E_{max}$, use your answer to part (b) to calculate the optimum value for $x$ to achieve maximum power transmission. (Hint: you should consider the optimum value for $a$ for a fixed value of $b$). [8]

(ii) What is the characteristic impedance of such an optimised coaxial cable if the dielectric has a relative permittivity $\varepsilon_r = 2.25$? [3]



(cont.




Cross Section Side View

Fig. 7

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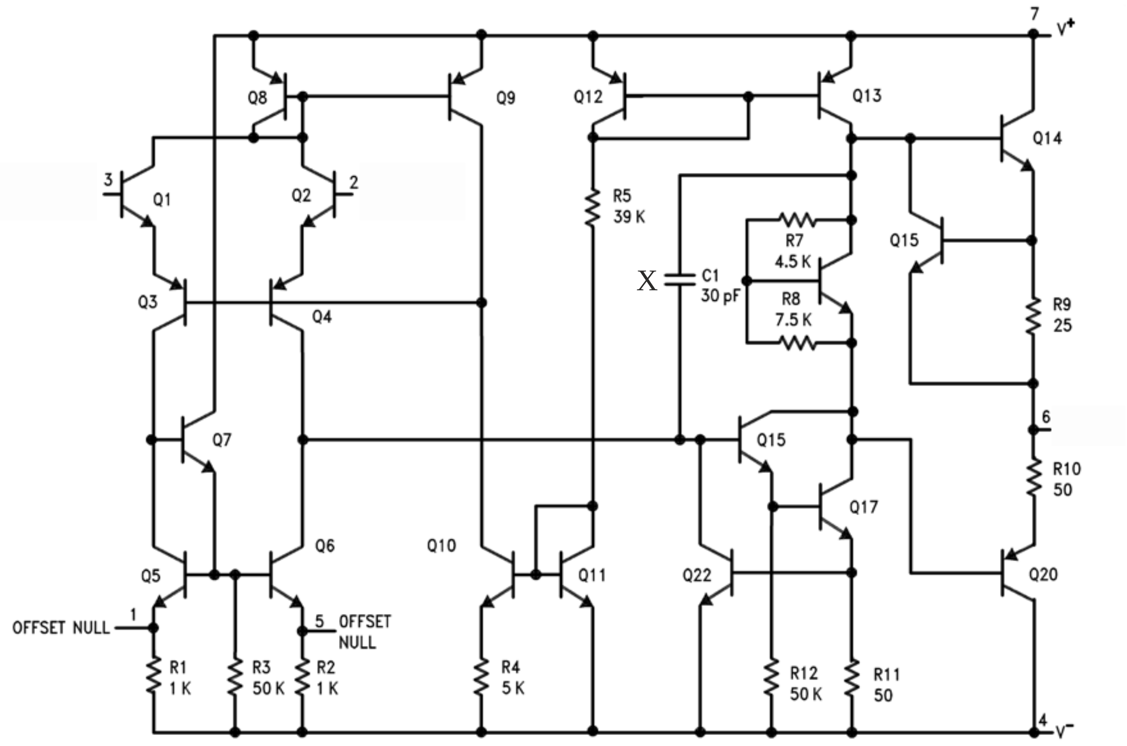
ENGINEERING TRIPOS PART IB

Wednesday 7 June 2017, Paper 5, Question 2




Extra copy of Fig. 3: Circuit diagram of a 741 operational amplifier for Question 2.

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