EGT1
ENGINEERING TRIPOS PART IB

Thursday 4 June 2015 2 to 4

Paper 6

INFORMATION ENGINEERING

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

Attachments: Additional copy of Fig. 2 and Fig. 4.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 A coupled mass-spring-damper system, with transfer function $G(s)$, is to be controlled in a standard negative feedback loop, as in Fig. 1, by a controller $K(s)$. Figure 2 shows the Bode diagram of $G(s)$.

(An *extra copy of Fig. 2* is available, which you should use and attach to your answer to this question.)

(a) If a proportional controller $K(s) = k_p$ is used, estimate the range of values of k_p for which the feedback system is stable. [6]

(b) $G(s)$ has the form

$$G(s) = ks^n \frac{(as^2 + bs + 1)}{(cs^2 + ds + 1)}$$

From Fig. 2 estimate the values of the constants a, b, c, d, k and n. [10]

(c) For the final design it is decided to use the controller

$$K(s) = \frac{s+1}{s+4}$$

Draw the Bode diagram of this controller on the extra copy of Fig. 2 and estimate the phase margin of the final design. [9]

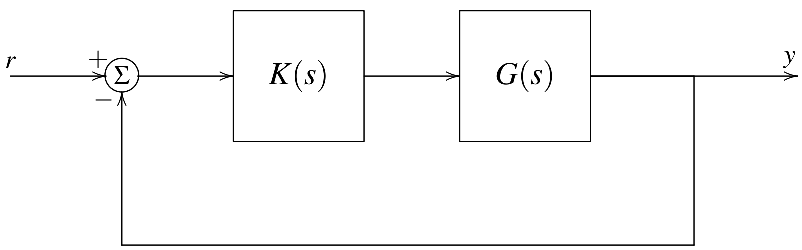

Fig. 1

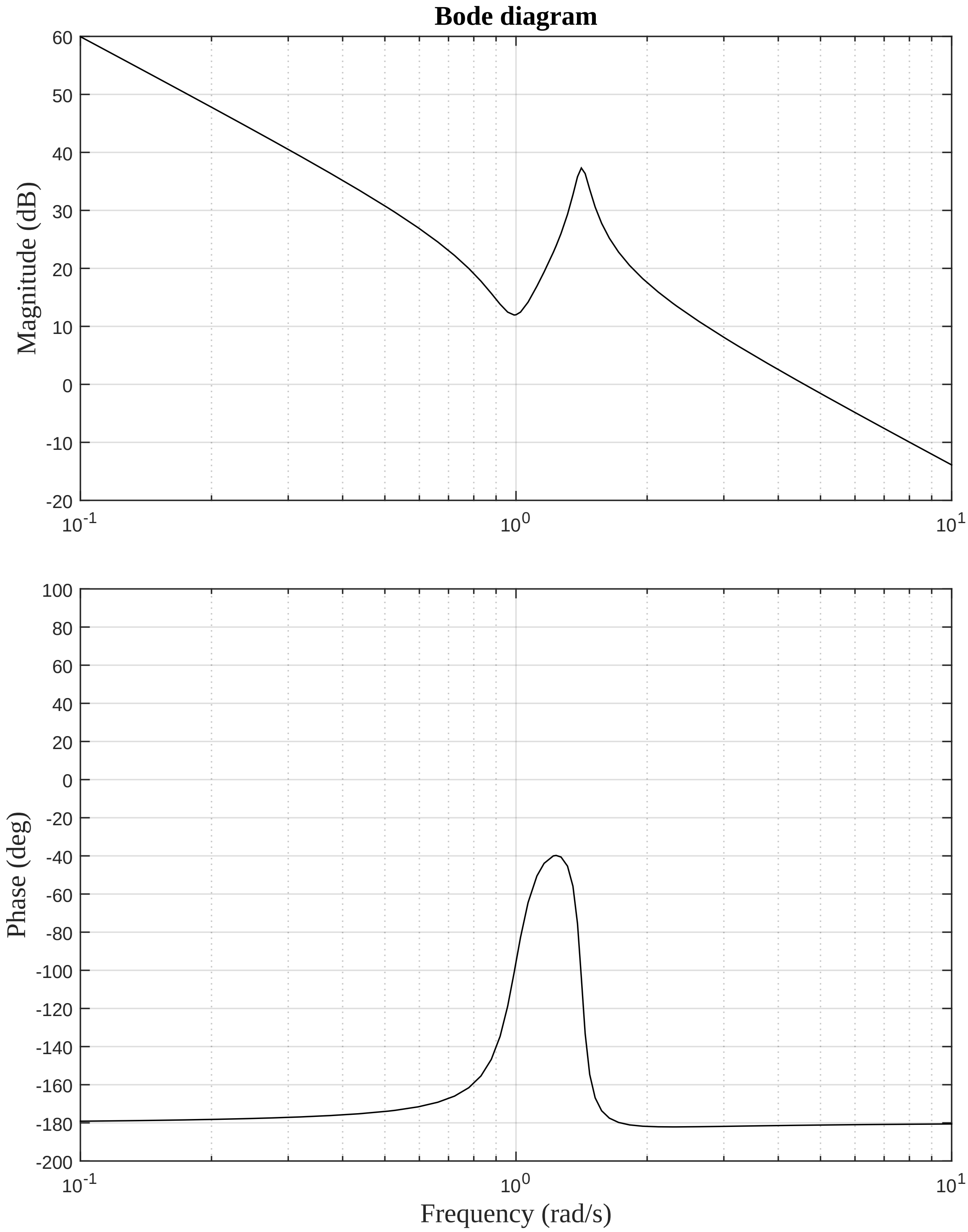
Bode diagram
Magnitude (dB)
60
50
40
30
20
10
0
-10
-20
10^{-1}
10^{0}
10^{1}
Phase (deg)
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
Frequency (rad/s)

Fig. 2

2 (a) (i) Describe carefully what experiments might be carried out in order to determine the Nyquist diagram of a system and how the diagram could be obtained from the experimental data. [5]

(ii) Describe, with the aid of diagrams, how the gain and phase margins of a feedback system may be determined from the Nyquist diagram of the uncontrolled system. [5]

(b) A system with transfer function $G(s)$ is to be controlled by negative feedback of gain k_p as shown in Fig. 3. The experimentally determined Nyquist diagram of this system is given in Fig. 4, from which the answers to the following questions should be estimated. Angular frequencies, in rad s^{-1}, are shown on the diagram.

(An *extra copy of Fig. 4* is available, which you should use and attach to your answer to this question.)

(i) For what range of k_p will the feedback system be stable? [3]

(ii) For what value of k_p is the phase margin maximised? [2]

(iii) If $k_p = 2$ and if $r(t) = \cos(2.5t)$, what is the steady state response of $y(t)$? [5]

(iv) Keeping $k_p = 2$, estimate the range of frequencies over which the absolute value of the sensitivity function of the feedback system, $|1/(1 + \text{return ratio})|$, is less than 1, and also the frequency at which this function is maximised. [5]

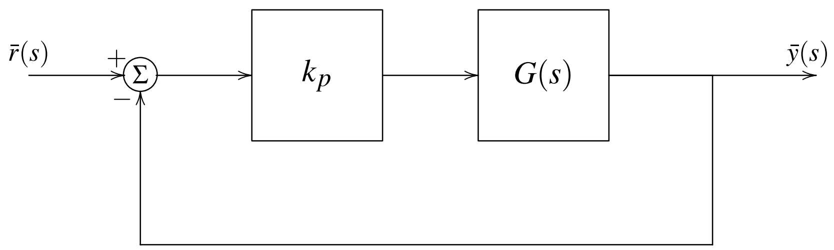

Fig. 3

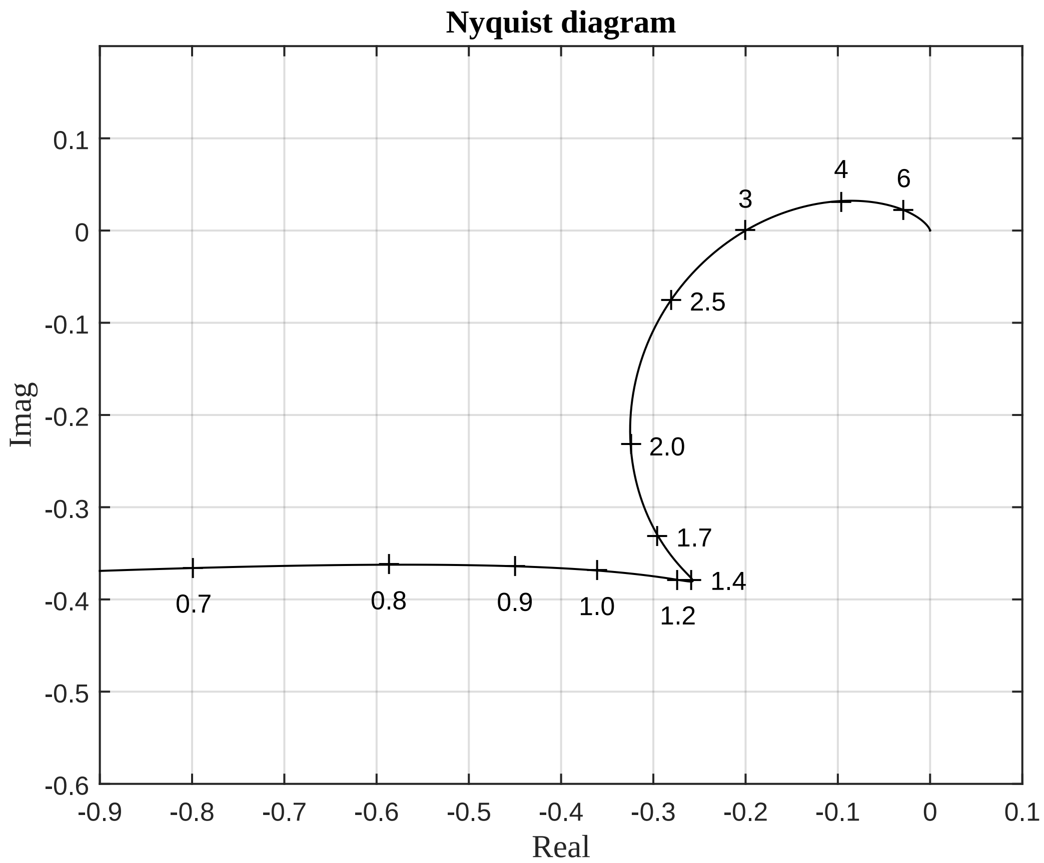
Nyquist diagram
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
Imag
Real
3
4
6
2.5
2.0
1.7
1.4
1.2
1.0
0.9
0.8
0.7

Fig. 4

3 A pedalled electrically assisted bicycle (Pedelec) produces a proportional torque T_e to assist the torque produced by the cyclist, T_r. There is, inevitably, a lag τ in this process, so the torque produced can be taken to follow the differential equation

$$\tau \dot{T}_e + T_e = T_d$$

where the demanded electrical torque T_d is given by

$$T_d = k T_r \qquad (1)$$

The dynamics of the bicycle are described by the differential equation

$$M\dot{v} + \lambda v = k_G(T_e + T_r)$$

where M is the combined mass of the bicycle and rider, v is their velocity and λv approximates the combination of the wind and rolling resistances. k_G represents the gearing between the rider and the road.

Finally, it is assumed that the rider is acting as a simple proportional controller in attempting to cycle at a constant speed v_d. That is

$$T_r = k_p(v_d - v)$$

(a) Draw a block diagram of the full system, and find the resulting transfer function from the desired velocity $\bar{v}_d(s)$ to the actual velocity $\bar{v}(s)$. [6]

(b) If $M = 80$ kg, $\lambda = 2.5$ N s m^{-1}, $\tau = 2$ s and the product $k_p k_G = 50$ N s m^{-1}, then what is the largest value for the power-assist gain k such that the closed loop damping ratio is no less than 70% of critical? If k is set to this value, what is the steady-state speed error when $v_d = 6$ m s^{-1} ? [8]

(c) It is proposed to modify the controller so that the demanded electrical torque, given above in equation (1), is instead given by

$$\bar{T}_d(s) = 2\left(\frac{1+s\tau}{1+s(\tau/4)}\right)\bar{T}_r(s)$$

What are the new values for closed loop damping ratio and steady-state error? [7]

(d) Comment on how different the bicycle will feel to the rider when the new controller replaces the old one. [4]

SECTION B

4 (a) If $P(\omega)$ is the Fourier transform of a pulse $p(t)$, calculate the Fourier transform of $p(t)e^{j\omega_0 t}$ from first principles, and compare this with the standard result in the Databook. [5]

(b) Hence (or otherwise) show that the Fourier transform of the raised cosine pulse

$$q(t) = p(t)\,[1+\cos(\omega_0 t)]\ , \quad \text{where } \ p(t) = \begin{cases} 1 & \text{for } |t| < \pi/\omega_0 \\ 0 & \text{otherwise,} \end{cases}$$

is given by

$$Q(\omega) = T\,\Big(\text{sinc}[(\omega-\omega_0)T] + 2\,\text{sinc}[\omega T] + \text{sinc}[(\omega+\omega_0)T]\Big)$$

where $T = \pi/\omega_0$. [8]

(c) Calculate the value of $Q(\omega)$ at $\omega = \frac{\pi}{T}\{0,\ 0.5,\ 1,\ 1.5,\ 2,\ 2.5,\ 3\}$, and sketch $Q(\omega)$ over the range $-\frac{3\pi}{T} \leq \omega \leq \frac{3\pi}{T}$. [6]

(d) If the pulse $q(t)$ is modulated onto a carrier wave of frequency ω_c to give a signal pulse

$$s(t) = q(t)\cos(\omega_c t)\ , \quad \text{where } \ \omega_c \gg \omega_0\ ,$$

calculate the bandwidth in Hz of $s(t)$, between the first zeros of its spectrum on either side of the carrier frequency, in terms of T. Also estimate (using results from part (c)) the ratio of the peak magnitude of the first sidelobe in the spectrum of $s(t)$ to the peak magnitude of the main lobe of its spectrum. [6]

5 (a) The Discrete Fourier Transform (DFT) of a sampled signal x_n, which is N samples long, is given by:

$$X_m = \sum_{n=0}^{N-1} x_n \exp\left(\frac{-j2\pi mn}{N}\right) \qquad \text{for } m = 0 \text{ to } N-1.$$

(i) If the sampling period is 100 μs, explain which frequency component of the signal x is measured by X_m, and show that the spectrum X is periodic. [6]

(ii) If $N = 6$ and $x = \{0,\ 1,\ 1,\ 0,\ -1,\ -1\}$, calculate the 6 frequency components, $X_0 \ldots X_5$, and sketch the magnitude of this spectrum. Explain why only some of these components are non-zero and why the spectrum is purely imaginary. [6]

(b) An analogue-to-digital converter converts a continuous audio signal within the range -1 V to $+1$ V into n-bit sampled values.

(i) Derive an expression for the rms quantising noise voltage as a function of n, listing your assumptions, and hence calculate the number of bits per sample needed in the converter to keep the noise below 1 mV rms. [6]

(ii) Giving reasons, estimate what transmission bit rate is then required if the input audio signal has a bandwidth of 20 kHz. You should allow for practical implementation of anti-aliasing filters. [4]

(iii) If additionally a block code, with 5 parity check bits for each block of 10 source data bits, is used to encode this audio data, what will be the new transmission bit rate and why is such coding likely to be beneficial on typical communication channels? [3]

6 (a) Compare the relative merits of amplitude modulation (AM, including its two main variants, DSB-SC and SSB-SC) and frequency modulation (FM) for transmitting signals over bandlimited channels. [6]

(b) Explain the concept of pulse amplitude modulation (PAM) for digital signals and outline the desirable properties for the modulating pulse $p(t)$ to produce signals that are suitable for bandlimited baseband channels and are also simple to demodulate and detect. [5]

(c) Show mathematically how symbols X_k, drawn from a complex constellation, may be modulated onto a pair of quadrature carrier wave components at frequency f_c Hz. Illustrate your answer using the square constellation for 16-symbol quadrature amplitude modulation (16-QAM). [5]

(d) Give the rule for optimum detection from the received complex signal samples at time k

$$Y_k = X_k + N_k$$

where the N_k are additive Gaussian noise samples. Assume that each X_k is chosen from a set of M valid constellation symbols by groups of equiprobable data bits. On a sketch of the 16-QAM constellation show the regions of Y_k which decode to each valid X_k. [4]

(e) For a given $p(t)$ and symbol rate, comment on how the data bit rate and the probability of bit error are likely to vary with the number of symbol states M, assuming that the mean energy per symbol remains constant. [5]

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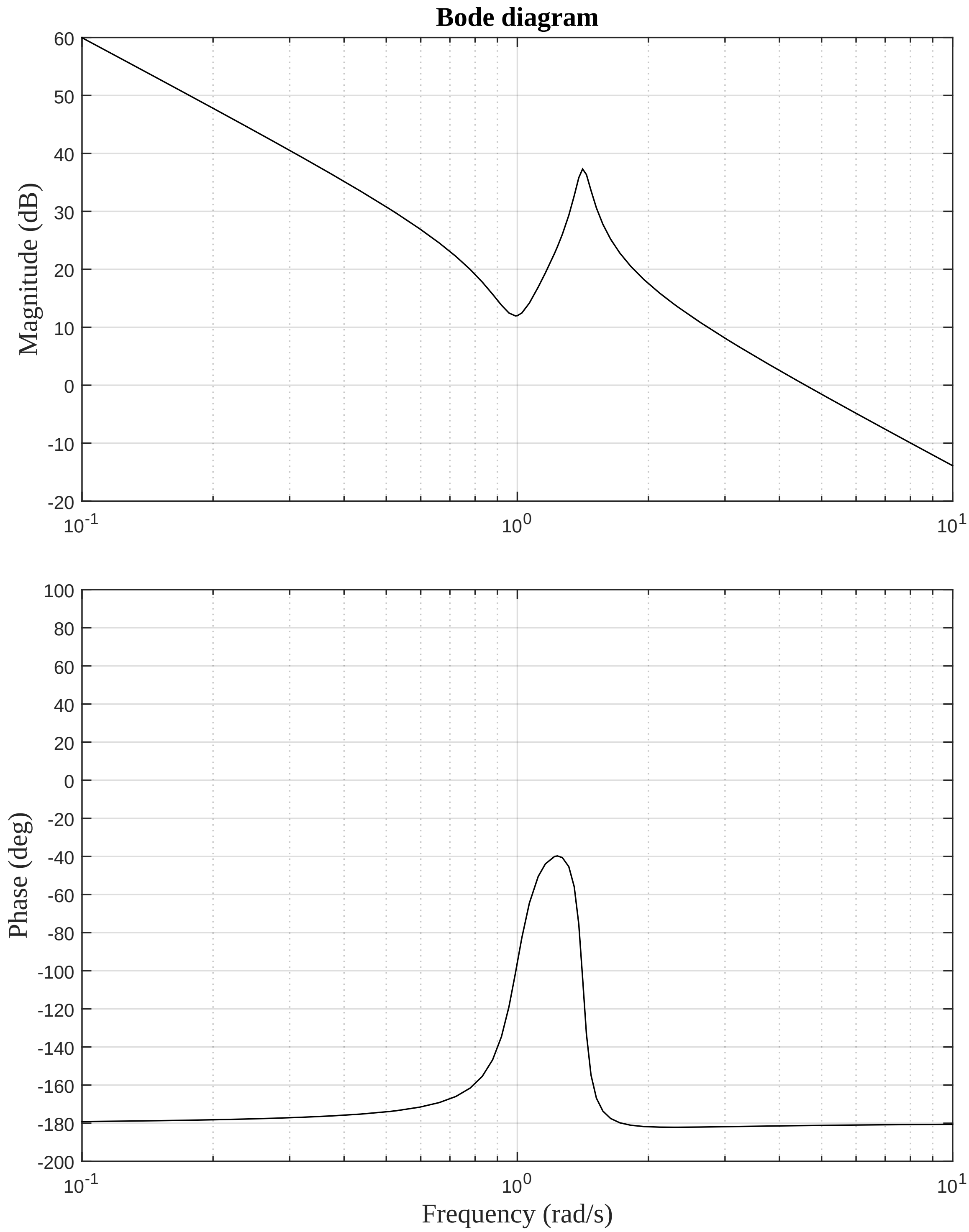

Extra copy of Fig. 2: Bode diagram for Q1.

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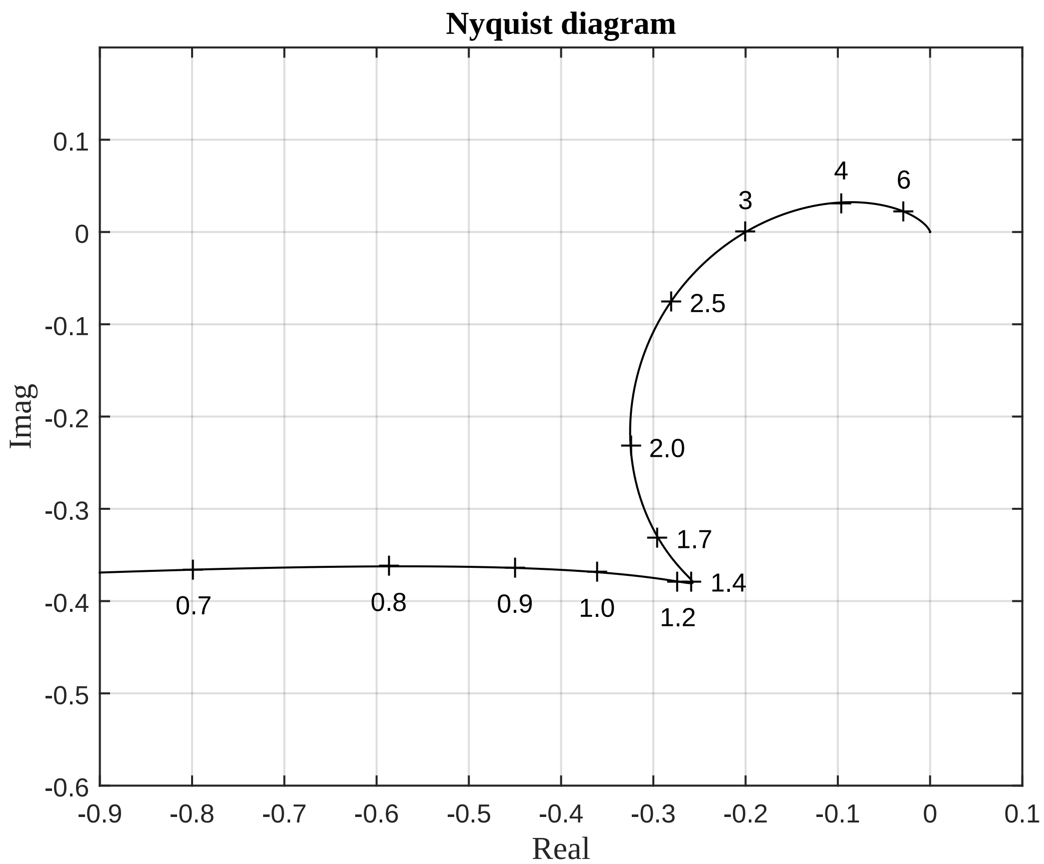
Thursday 4 June 2015, Paper 6, Question 2.



Extra copy of Fig. 4: Nyquist diagram for question 2.

Part IB Paper 6 2015 Answers:

1 (a) $0 \le k_p < 0.07$

(b) $a = 1, b = 0.4, c = 0.5, d = 0.1, k = 10, n = -2$

(c) Phase margin $\simeq 29°$ (at $\omega \simeq 4$ rad s^{-1})

2 (a) –

(b) (i) $0 \le k_p < 5$

(ii) $k_p = 2.17$

(iii) $y(t) = 1.25\cos(2.5t - 2.58)$

(iv) $\omega < 0.68$ rad s^{-1}; $\omega \simeq 2.4$ rad s^{-1}

3 (a) $\dfrac{\bar{v}(s)}{\bar{v}_d(s)} = \dfrac{k_p k_G (1 + s\tau + k)}{(1 + s\tau)(\lambda + sM) + k_p k_G (1 + s\tau + k)}$

(b) $k \le 1.09$; steady state speed error $= 0.14$ m s^{-1}

(c) Closed-loop damping ratio $= 0.68$; steady state speed error $= 0.098$ m s^{-1}

(d) –

4 (a) $P(\omega - \omega_0)$; standard frequency shift result

(b) –

(c) $Q(\omega) = T\{2,\ \frac{16}{3\pi},\ 1,\ \frac{16}{15\pi},\ 0,\ \frac{-16}{105\pi},\ 0\}$

(d) $\frac{2}{T}$ Hz; 0.0243 (or -32.3 dB)

5 (a) (i) $\frac{m}{N} \,.\, 10^4$ Hz; $X_{m+N} = X_m$

(ii) $X = \{0,\ -2j\sqrt{3},\ 0,\ 0,\ 0,\ 2j\sqrt{3}\}$

(b) (i) RMS noise $= \frac{2^{-n}}{\sqrt{3}}$ volt; $n = 10$ bits per sample

(ii) 440 kbit s^{-1}, allowing $+10\%$ for practical filters

(iii) 660 kbit s^{-1}

6 (a) –

(b) –

(c) $x(t) = \sum_k p(t - kT) \{ \mathrm{Re}[X_k] \cos(2\pi f_c t) - \mathrm{Im}[X_k] \sin(2\pi f_c t) \}$

(d) –

(e) Bit rate $\propto m = \log_2 M$ (slowly gets better as M increases)
Distance between samples $\propto \frac{1}{\sqrt{M}}$
(The voltage SNR is proportional to this, so the error probability rapidly gets worse as M increases.)