EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 22 April 2015 9.30 to 12.30

Module 3A1

FLUID MECHANICS I

Answer not more than ***five*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachments: 3A1 Data Sheet for Applications to External Flows (2 pages); Boundary Layer Data Card (1 page); Incompressible Flow Data Card (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) A two-dimensional point vortex has circulation Γ and is located at $x = x_0$ and $y = y_0$. What are the Cartesian components of the flow velocity at the general position (x,y) ?

[20%]

(b) The vortex now moves with constant speed U in the negative x direction.

(i) By considering the flow in a frame of reference fixed relative to the vortex, derive an expression for the magnitude of the external force that must be applied to the vortex to maintain its motion. Define any additional variables that you introduce.

(ii) A free-vortex experiences no external force. What value of U does this imply?

[30%]

(c) The two-dimensional "vortex street" wake formed behind a bluff body can be modelled by two rows of free point vortices as shown in Fig.1. Each vortex has circulation of magnitude Γ ; its direction is anti-clockwise in the lower row and clockwise in the upper. The fluid far from the vortex street is at rest.

(i) Without performing any calculations, state the direction in which the vortices move.

(ii) Using the result

$$\sum_{k=-\infty}^{\infty} \frac{1}{(k-a)^2+b^2} = \frac{\pi}{b} \frac{\sinh(2\pi b)}{\cosh(2\pi b)-\cos(2\pi a)}$$

or otherwise, find the speed of one of the vortex rows. Why does this represent a lower bound for the speed of the body producing the wake?

(iii) With reference to another part of the flowfield, discuss the likely true value of the body's speed – and compare it with your answer from Part (c,ii).

[50%]

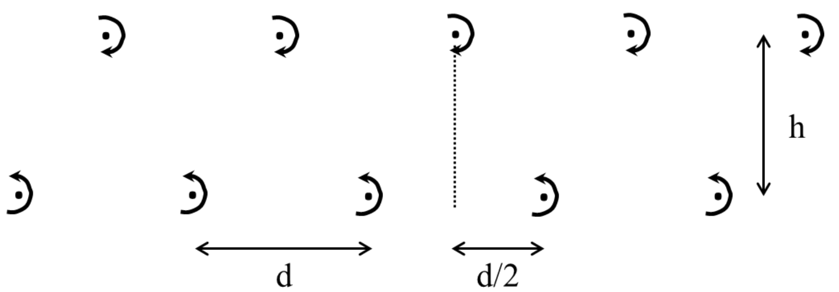

Fig.1

2 (a) Figure 2 shows three possible potential flows around a two-dimensional airfoil. Which one represents the flow that would be observed in practice? Explain your answer.

[25%]

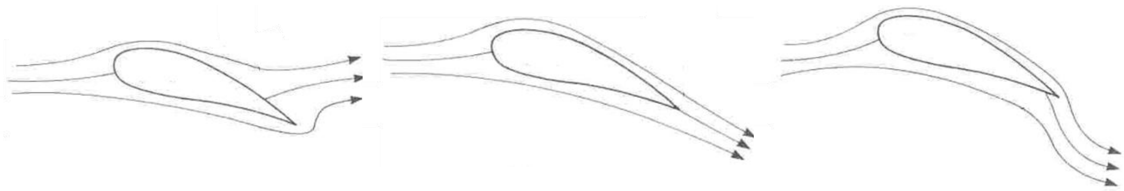

Fig.2

(b) The relation

$$\zeta = \frac{(z-a)^k}{(z-a\varepsilon)^{k-1}}$$

with $k > 1$ and $0 < \varepsilon < 1$, defines a mapping from the complex z-plane to the complex ζ-plane. In particular, a circle of radius a centred at the origin of the z-plane is transformed into a symmetrical airfoil in the ζ-plane, with its chord on the Re (ζ) axis. What is the chord length?

[25%]

(c) A flow around the z-plane cylinder has free stream velocity U, incidence α and circulation Γ. Its complex potential is given by

$$F(z) = Uze^{-i\alpha} + U\frac{a^2}{z}e^{i\alpha} - \frac{i\Gamma}{2\pi}\ln z$$

(i) Explain how this result can be used to deduce the complex potential for the flow at incidence α around the airfoil in the ζ–plane.

(ii) What value of Γ must be chosen to produce a ζ–plane flow matching your choice from Part (a).

[25%]

(d) What is the lift coefficient of the airfoil at incidence α?

[25%]

(TURN OVER

3 (a) (i) State Kelvin's theorem; take care to specify the conditions under which it applies.

(ii) Explain why the theorem implies that the component of vorticity parallel to a fluid filament of length l is proportional to l .

[20%]

(b) A pipe flow undergoes a smooth contraction from an initial radius R_1 to a final radius R_2 as shown in Fig.3. The fluid is incompressible and may be assumed inviscid over the length of the contraction. The velocity field upstream is axisymmetric with profile

$$u_1(r) = U_1\left(1 - \frac{r^2}{{R_1}^2}\right)$$

Downstream the flow is again axisymmetric with

$$u_2(r) = U_2\left(1 - k\frac{r^2}{{R_2}^2}\right)$$

(i) What is the vorticity vector at each location?

(ii) A streamline originating upstream at $r = r_1$ is found at $r = r_2$ downstream. How is the vorticity at these points linked? Hence derive a relationship for k in terms of R_1 and R_2 and find an expression for U_2/U_1 .

[60%]

(c) Discuss qualitatively, using sketches, how the velocity profile would develop further downstream from the contraction.

[20%]

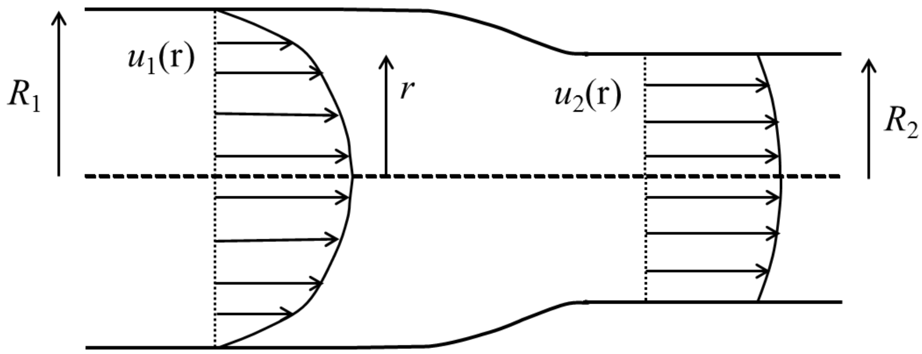

Fig.3

4 A flat plate at zero angle of attack has a constant free stream velocity U and x denotes the distance from the leading edge of the plate.

(a) Assume the velocity profile within the boundary layer is given by

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

where δ is the boundary layer thickness. Derive an expression for the momentum thickness θ and use the momentum integral equation to deduce how θ varies with x .

[20%]

(b) Now assume the velocity profile within the boundary layer is given by

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Again, derive an expression for the momentum thickness θ and use the momentum integral equation to deduce how θ varies with x .

[20%]

(c) The exact solution of the laminar boundary layer equation within this layer is the solution of the Blasius equation

$$f''' + ff'' = 0$$

with the boundary conditions

$$f(0) = f'(0) = 0\,,\ f'(\infty) = 1, f''(\infty) = 0$$

The solution has the velocity profile

$$\frac{u(x,y)}{U} = f'(\eta)\,,\ \eta = y\sqrt{\frac{U}{\nu x}}$$

Show that the corresponding momentum thickness satisfies

$$\theta\sqrt{\frac{U}{\nu x}} = f''(0)$$

You may use the fact: $f(\infty)(1 - f'(\infty)) = 0$.

[30%]

(d) Compare the approximate θ in Parts (a) and (b) with the exact solution from (c) and calculate the relative errors. You may use the fact that: $f''(0) = 0.4696$.

[10%]

(e) Which of the two assumed velocity profiles in Parts (a) and (b) is in fact not consistent with the assumed constant free stream velocity?

[20%]

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5 Consider the linear velocity field

$$u = Bx\,,\; v = -\,By$$

in the plane $y \geq 0$, where B is a positive constant. It describes an *inviscid* two-dimensional flow towards a stagnation point. An exact similarity solution of the Navier-Stokes equations can be found which: (i) satisfies the *no-slip* boundary conditions at the stationary, rigid boundary, $y = 0$; and (ii) takes the *inviscid* solution as the asymptotic solution far from the boundary; see Fig.4. Now, consider a stream function of the form

$$\psi = B\,x\,f\,(y)$$

where f is a function of y to be determined and $f\,(0) = 0$.

(a) If ψ represents the exact solution of the Navier-Stokes equations which satisfies the above two conditions, find the boundary condition for $f\,'$ at $y = 0$ and also for $f\,(y)$ as $y \to \infty$.

[20%]

(b) Substitute ψ into the y-component of the Navier-Stokes equations and deduce that $\partial p/\partial y$ is a function of y only – and hence that $\partial^2 p/\partial x \partial y = 0$.

[20%]

(c) Consider now the x-component, deduce that $x^{-1}\,\partial p/\partial x$ is a constant and use one of the boundary conditions to determine this constant. Hence deduce the differential equation for f .

[20%]

(d) The kinematic viscosity of the fluid is ν . What is the proper length scale g and velocity scale U for this flow?

[20%]

(e) Use the above length and velocity scales to redefine the stream functions as

$$\psi = x\,F\,(\eta)\,U^{1/2}\,,\; \eta = y\,/\,g \quad .$$

Deduce the differential equation, as well as the boundary conditions, for F .

[20%]

Note: the steady Navier-Stokes equations are:

$$\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0\,, \\
u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\,, \\
u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\,.
\end{aligned}$$

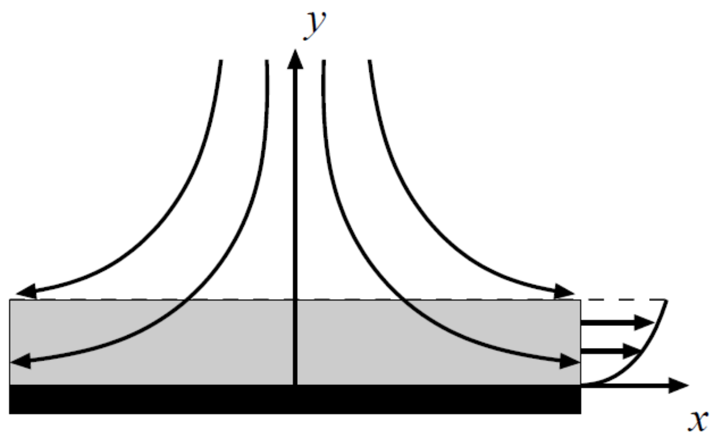

Fig.4

6 Consider a thin 2D airfoil with camber line:

$$y_c = h\,(x/c)\,[1 - (x/c)]\,[a + (x/c)]$$

where h , c and a are constants.

(a) Using the Data Book relationship between the camber slope and the chordwise vorticity distribution calculate the lift and moment coefficients in terms of h , c and a .

[50%]

(b) Write the expression from Part (a) for the moment coefficient to display the moment about the quarter chord point. Describe how this corresponds to experimental measurement.

[25%]

(c) Discuss the physical processes which at high angle of attack lead to differences between the analysis and measurement described in Part (b).

[25%]

(TURN OVER

7 (a) Sketch the flow near a finite wing which leads to the formation of a "horseshoe" vortex structure. Differentiate clearly between the bound vortex and the trailing vortical structures. Which of these structures gives rise to lift – and which to drag?

[40%]

(b) Using information from the Data Book derive an expression for the spanwise variation of downwash. Explain clearly the steps in your derivation.

[30%]

(c) Using information from the Data Book derive an expression for the induced drag coefficient. Explain clearly the steps in your derivation. Show that elliptically loaded wings have the minimum induced drag.

[30%]

8 Figure 5 shows the drag behaviour of passenger cars plotted as a function of an angle α . This angle defines a key geometric parameter at the rear of the vehicle.

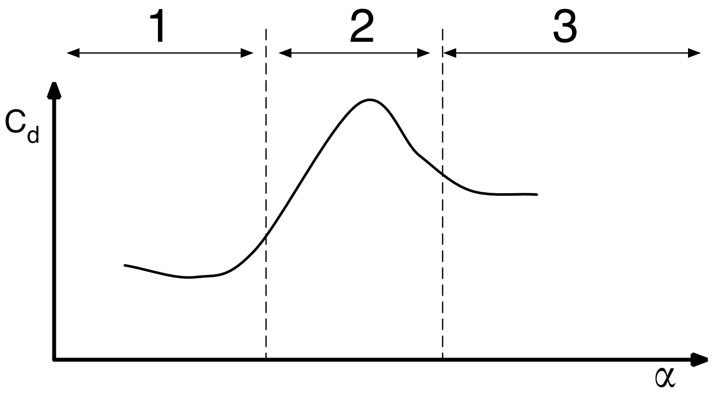

Fig.5

(a) Draw a simple sketch showing how this angle is defined.

[5%]

(b) Draw a sketch showing the flow regimes typically seen in regions 1 and 3 as marked on the above graph. Briefly explain why there is a difference in drag coefficient. For what numerical values of α would one typically find the drag minimum in region 1 ?

[40%]

(c) In region 2 the drag coefficient is at first seen to rise sharply with increasing values of α . Give physical reasoning for this drag rise and illustrate this with a sketch of the flowfield seen near the maximum drag value.

[20%]

(d) Using the above graph, explain the design choices typically made for:

(i) small cars;

(ii) large cars.

[20%]

(e) If a car is designed to lie in region 3 , what small improvements can be made to its basic rear shape to improve the drag behaviour (without changing the overall flow domain)?

[15%]

END OF PAPER

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