EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 3rd May 2016 9.30 to 12.30

Module 3A1

FLUID MECHANICS I

Answer not more than ***five*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachments: 3A1 Data Sheet for Applications to External Flows (2 pages); Boundary Layer Data Card (1 page); Incompressible Flow Data Card (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Figure 1 shows a two-dimensional flowfield consisting of two sources, at $(\pm d, 0)$, and a sink at the origin. The (two-dimensional) volumetric flow rate of each source is m and that of the sink is km. The fluid is incompressible, and k is sufficiently large that all the source emissions are swallowed by the sink.

(a) What is the minimum value that k can take?

[5%]

(b) Calculate where are the stagnation points of the flow?

[10%]

(c) Sketch the flow streamlines in the first quadrant ($x \geq 0$, $y \geq 0$) for:

(i) k close to its minimum value;

(ii) k large; and

(iii) k intermediate between the values of (i) and (ii).

[25%]

(d) The flowfield is used to model a scheme for removing pollutant discharge from pipes at the source locations.

(i) What is the minimum value of k needed to ensure that no pollutant reaches the point $(d, 2d)$?

(ii) There are two wedge-shaped regions which remain pollutant-free irrespective of distance from the origin. Indicate these regions on a sketch and, for the value of k found in (i), calculate their angular extent at the origin.

[60%]

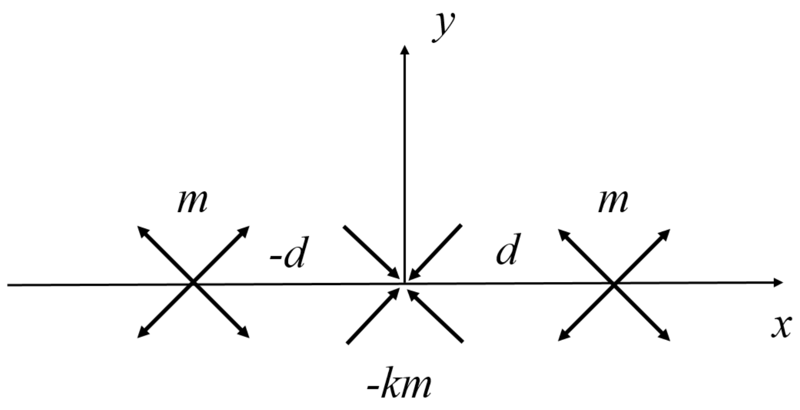

Fig.1

2 (a) A three-dimensional source at the origin has volumetric flow rate m.

(i) Show that the x-direction component of its velocity field on the plane $x = d$ is given by

$$\frac{m}{4\pi}\,\frac{d}{(d^2+l^2)^{3/2}}$$

where $$l^2 = y^2 + z^2 \quad .$$

(ii) Write down and evaluate the integral for the volumetric flow rate through the circular disc defined by $x = d$, $l \leq L$.

[30%]

(b) The flow past a three-dimensional "Rankine oval" is represented by the superposition of: a source of volumetric flow rate m at $(-d, 0, 0)$; a sink of the same strength at $(d, 0, 0)$; and a uniform flow with velocity U in the x-direction.

(i) The upstream stagnation point is at $x = -(d+s)$. Find an equation linking the dimensionless parameters $m/(\pi d^2 U)$ and s/d.

(ii) The Rankine oval represented by the flow has maximum cross-sectional radius L. State how the flow rate across this section is linked to the source strength m, and hence, using the result of part (a), find an equation relating $m/(\pi d^2 U)$ to L/d.

[40%]

(c) For the case where the Rankine oval of part (b) is slender (ie. $L/d \ll 1$), find the leading order approximation for:

(i) L/d in terms of $m/(\pi d^2 U)$;

(ii) the "aspect ratio" $L/(d+s)$ in terms of $m/(\pi d^2 U)$;

(iii) the minimum surface pressure in terms of the aspect ratio and the free stream flow properties.

[30%]

(TURN OVER

3 (a) The vorticity equation for an incompressible fluid with kinematic viscosity ν is

$$\frac{D\underline{\omega}}{Dt} = \underline{\omega}.\nabla\underline{u} + \nu\nabla^2\underline{\omega} \quad .$$

What physical processes are described by the three terms?

[20%]

(b) An unsteady, viscous, two-dimensional flow has streamlines that are straight and lie in the *x*-direction.

(i) Explain why the *x*-direction velocity component must be independent of x.

(ii) Show that, for this flow, the vorticity equation simplifies to

$$\frac{\partial \omega}{\partial t} = \nu\frac{\partial^2 \omega}{\partial y^2} \quad .$$

(iii) Confirm that the vorticity field

$$\omega = \frac{A}{\sqrt{\nu t}} e^{-y^2/4\nu t}$$

(with A constant) is a solution of the equation in part (b)(ii) and sketch profiles of ω against y for a succession of increasing times. (Three will be sufficient and only the region $y \geq 0$ need be shown.)

[40%]

(c) The flow boundary condition is suddenly altered so that there is no shear stress on the plane $y = 0$.

(i) What does this imply for the vorticity at $y = 0$?

(ii) Without further analysis, but giving justification on the basis of your answer to part (a), sketch the subsequent evolution of the vorticity profile.

[40%]

4 Consider a plane jet emerging into a stationary fluid from a narrow slit in a wall, as shown in Fig. 2. The jet spreads at constant pressure and is thin so that the velocity $\underline{u}$ varies much more rapidly across the jet than along it.

(a) Apply the arguments of boundary layer theory to deduce an approximate equation for the velocity in the jet and state the boundary conditions for the horizontal velocity $u(x, y)$ assuming the jet is symmetrical.

[15%]

(b) How does the momentum flux

$$J = \rho \int_{-\infty}^{+\infty} u^2 dy$$

evaluated at $x = constant$ depend on x.

[15%]

(c) Using the stream function

$$\Psi = \nu^{1/2} x^{1/3} f(\eta)$$

where ν is the kinematic viscosity and $\eta = \frac{y}{3\nu^{1/2}x^{2/3}}$, deduce the equation for f and state the boundary conditions.

[40%]

(d) Show that f satisfies the equation: $f' + \frac{1}{2} f^2 = constant$.

[30%]

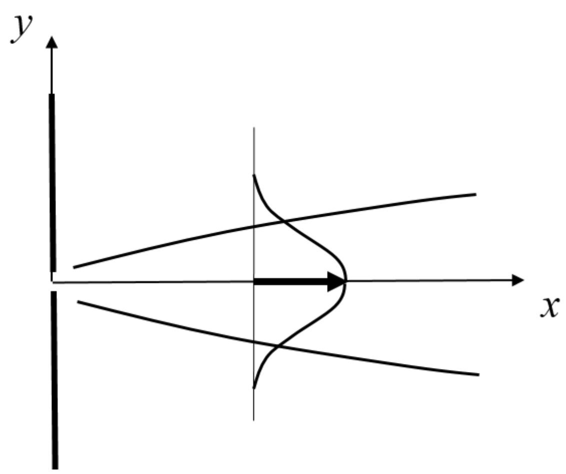

Fig.2

(TURN OVER

5 Consider turbulent duct and boundary-layer flows for which there are three regions near a wall as sketched in Fig. 3. Let u be the mean velocity parallel to the wall, τ_w the wall shear stress and U the velocity at the edge of the outer layer, $y = \delta$.

(a) In a region quite close to the wall (in the viscous wall layer and overlap layer), we are far enough away from the outer edge that we may assume the local flow has no information about the free-stream velocity U and the boundary layer thickness δ. Express u as a function of relevant variables, and use dimensional analysis to deduce the *law of the wall*. You may use the friction velocity $u^* = \sqrt{\tau_w/\rho}$.

[20%]

(b) The viscosity is important only very close to the wall (in the viscous wall layer). With increasing y (in the overlap layer) the viscosity ceases to play a role long before parameters other than those in (a) have an influence. Use dimensional analysis to deduce the logarithmic-law in the overlap layer

$$\frac{u}{u^*} = \frac{1}{\kappa} ln\left(\frac{u^* y}{\nu}\right) + B$$

where κ and B are the dimensionless constants.

[20%]

(c) Very close to the wall, the wall law is linear. Express this linear viscous relation using the friction velocity.

[10%]

(d) Assume that the logarithmic-law obtained in (b) correlates the local mean velocity $u(r)$ of a turbulent pipe flow all the way across a circular pipe of radius R. Deduce an equation between the mean velocity V and the friction velocity u^*. You may use:

$$\int ln\, y\, dy = y\, ln\, y - y \quad ; \quad \int y \ln y\, dy = \frac{1}{2} y^2 (\ln y) - \frac{1}{4} y^2 .$$

[30%]

(e) Deduce the formula relating the mean velocity to the maximum velocity u_{max}

$$V/u_{max} = 1/\left(1 + \frac{3}{2\kappa}\sqrt{\frac{f}{8}}\right)$$

where the Darcy friction factor $f = 8\tau_w/(\rho V^2)$.

[20%]

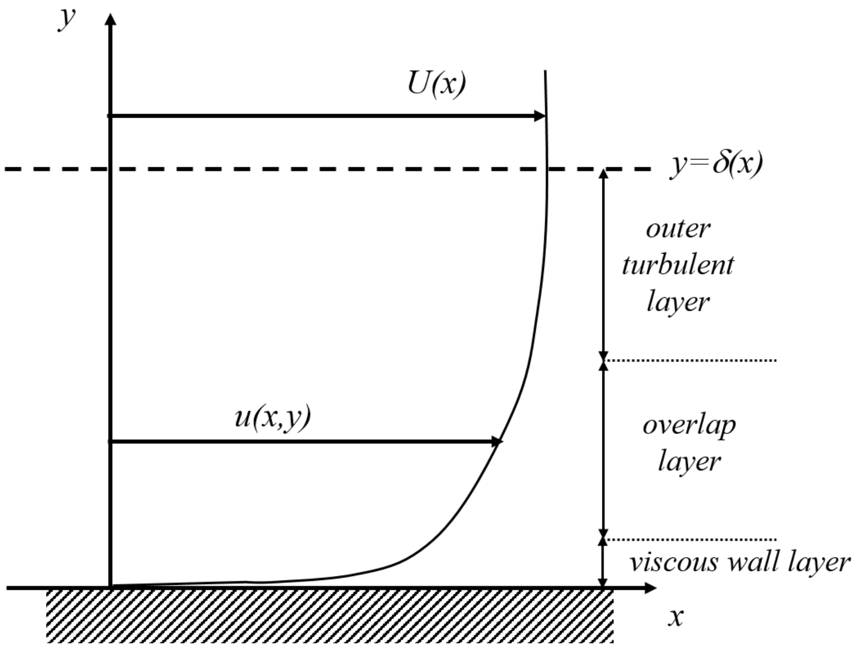

Fig.3

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6 (a) The camber of a smoothly deformable thin aerofoil is modelled by

$$y_c = \frac{h}{\kappa}\frac{x}{c}\left(1 - \frac{x}{c}\right)\left(\kappa - \frac{x}{c}\right)$$

and is shown in Fig.4 for $\kappa = 0.75$, 1.0 and 1.5. Show that the first three coefficients in the Fourier series for $-2dy_c/dx$ are:

$$g_0 = -\frac{1}{4\kappa}\frac{h}{c}\ ;\ g_1 = \left(2 - \frac{1}{\kappa}\right)\frac{h}{c}\ ;\ g_2 = -\frac{3}{4\kappa}\frac{h}{c}$$

[40 %]

(b) The lift coefficient, c_l , and pitching moment about the quarter chord point, c_{m0} , vary with κ. Sketch graphs of $c_l(\kappa)$ and $c_{m0}(\kappa)$. Explain the behaviour of $c_l(\kappa)$ and $c_{m0}(\kappa)$ with reference to the aerofoil shapes sketched in Fig.4.

[40%]

(c) The aerofoil is at zero incidence. Calculate the centre of pressure as a function of κ. Explain the behaviour of the centre of pressure around $\kappa = 0.75$ and discuss the usefulness of the concept of the centre of pressure.

[20 %]

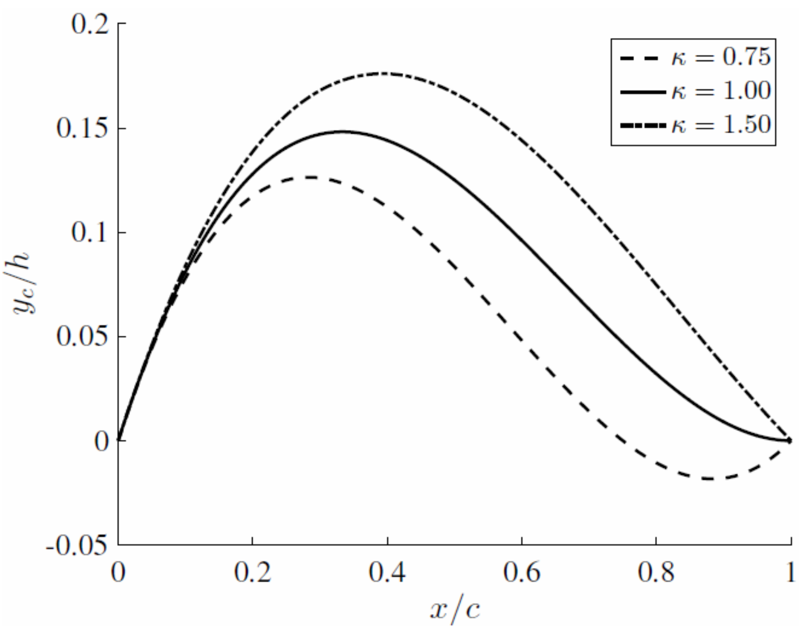

Fig.4

7 The F104 Starfighter shown in Fig.5 has a trapezoidal wing with $c_r = 3.75$ m, $c_t = 1.50$ m and $s = 3.18$ m. Its maximum take-off weight is 120,000 N at a take-off speed of 100 ms^{-1} in air of density 1.2 kgm^{-3}.You may assume that the wing has an elliptic circulation distribution:

$$\Gamma(y) = \Gamma_0\sqrt{1 - y^2/s^2}$$

(a) Use the substitution $y = -s\cos\theta$ and the Glauert integral to show that the downwash angle is uniform and with value $\alpha_d = \Gamma_0/(4Us)$.

[20%]

(b) Calculate Γ_0 at take-off.

[20%]

(c) The *lifting line* equation is:

$$\frac{\Gamma(y)}{\pi U\, c(y)} = \alpha - \alpha_0(y) - \alpha_d(y)$$

where $\Gamma(y)$ is the local circulation, $c(y)$ is the local chord, α_d is the local downwash angle, α_0 is the local zero lift angle and α is the aircraft angle of attack defined such that $\alpha_0 = 0$ at the wing tip, $y = s$. Calculate the angle of attack, α, at take-off. Sketch the spanwise distribution of α_0 that delivers elliptic loading at take-off.

[40%]

(d) Assuming that the wing has uniform camber and no twist, sketch the local lift coefficient $c_l(y)$ for $0 \le y \le s$ and calculate the value of y where it is a maximum. For good supersonic performance the F104 has a very thin wing (0.4 mm thick at the leading edge). Suggest why this aircraft was nicknamed the *aluminium death tube* by the Canadian Air Force which lost 110 of its 235 F104s to accidents and discuss possible mechanisms to improve its safety at take-off.

[20%]

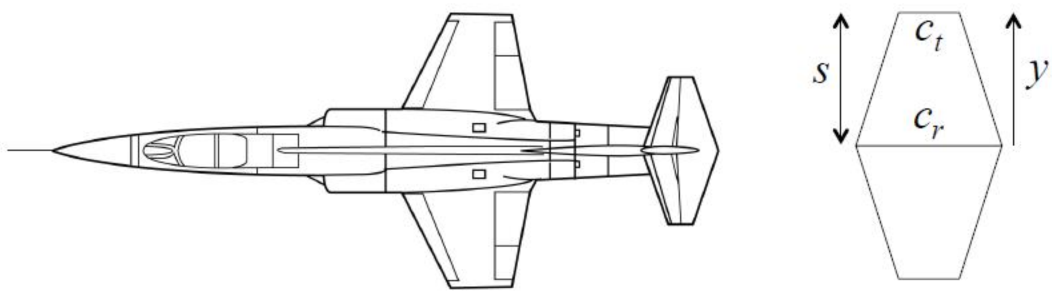

Fig.5

(TURN OVER

8 (a) Making sensible assumptions, derive from first principles an equation to approximate the additional drag caused by the engine cooling system of a vehicle. The engine is cooled through a radiator which has mass flow $\dot{m}_r$ and the vehicle is travelling at speed V. Explain your assumptions.

[25%]

(b) A prototype passenger vehicle travelling at 30 m/s requires a mass flow of 1 kg/s of air through its radiator. The vehicle's reference area is 2.5 m^2 and the ambient air density is 1.2 kg/m^3. Estimate the additional drag coefficient due to the radiator air flow. What approximate proportion of the overall aerodynamic drag is this? Does it matter where the radiator is located?

[30%]

(c) What is a Gurney flap? Using simple sketches, briefly explain its effect on an aerofoil. Where might you find such a device on a vehicle? Why might it be used?

[45%]

END OF PAPER

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