EGT2
ENGINEERING TRIPOS PART IIA

Monday 24 April 2017 9.30 to 12.30

Module 3A1

FLUID MECHANICS I

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachments: 3A1 Data Sheet for Applications to External Flows (2 pages); Boundary Layer Data Card (1 page); Incompressible Flow Data Card (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 In polar coordinates the potential function, ϕ, for an incompressible flow must satisfy

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0\ .$$

(a) Show that the function $\phi = \mathrm{A}r^{\alpha} \cos \alpha\theta$, where α is a constant, is a valid potential function.

[25%]

(b) Show that this function can represent the flow in a corner. Find the angle of the corner.

[25%]

(c) An inviscid liquid flows downwards between two vertical, parallel plates. An air bubble is held stationary on one plate by the flow, as shown in Fig.1. The interface between bubble and liquid makes an angle b with the vertical. Using Bernoulli's equation, estimate b.

[25%]

(d) Calculate A at this value of b. Comment on the speed of the flow far upstream; comment also on the defects of this model.

[25%]

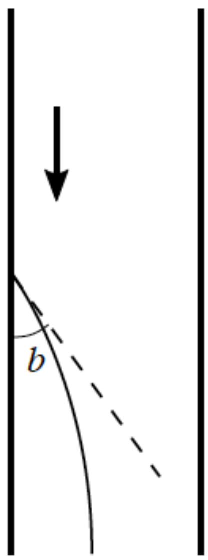

Fig.1

2 An inviscid two-dimensional flow with speed U in the x-direction flows over an infinitely long wall at $y = 0$. The wall contains a porous section from $x = -L$ to $x = +L$. Uniform suction is applied through the porous section, with volumetric flowrate V per unit x length.

(a) Show that the complex potential for this flow is

$$F(z) = Uz - \int_{-L}^{+L} \frac{V}{\pi} \ln (z - x')\, dx' \ .$$

[20%]

(b) Derive an expression for the velocity field and sketch the flow

(i) in the limit $|z| >> L$; and
(ii) in the limit $|z| << L$.

[20%]

(c) Find the position of any stagnation points in the flow in terms of U, V and L.

[20%]

(d) Hence sketch the streamlines of the flow in the domain $y > 0$ and $|z| \sim L$.

[20%]

(e) Without further calculations, sketch the streamlines when the fluid is viscous. Explain the features of your sketch, particularly for $|z| << L$.

[20%]

(TURN OVER

3 (a) The complex potential for a flow is $F(z) = U\,(z - a\mathrm{e}^{\mathrm{i}kz})$. Derive the streamfunction for this flow.

[20%]

(b) Hence, or otherwise, show that this represents the inviscid, irrotational flow across a corrugated surface at position $y = a \sin kx$, as long as $ka \ll 1$.

[30%]

(c) Inviscid fluid flows between a flat plate at $y = 0$ and a corrugated surface at $y = -d + a \sin kx$ where $ka \ll 1$ but kd is not small; the flow speed is U at $x = 0$ and $y = 0$. Derive the streamfunction for this flow by superposition, or otherwise, and hence derive an expression for the difference between the maximum and minimum velocity in this flow.

[50%]

4 (a) As defined on the data sheet, the incidence and camber solutions of thin aerofoil theory correspond to a vortex sheet of strength

$$\gamma(y) = -U\left[(2\alpha + g_0)\frac{1-\cos\phi}{\sin\phi} + \sum_{n=1}^{\infty} g_n \sin n\phi\right].$$

Explain why this expression implies a specific "design" incidence with minimal drag and give an expression for this angle.

[20%]

(b) A thin aerofoil with chord *c* has a camber line given by

$$\frac{y_c}{c} = Y\left[\left(\frac{x}{c}\right)^3 - 3\left(\frac{x}{c}\right)^2 + 2\left(\frac{x}{c}\right)\right]$$

where *Y* is a positive constant.

(i) Find the design incidence of the aerofoil.

(ii) Sketch the pressure distribution at this incidence using as axes $(-c_p)$ and x/c .

(iii) On separate sketches, show the pressure distributions at zero incidence and at twice the design incidence.

[60%]

(c) Your sketches for Part (b) apply to an aerofoil of zero thickness. Describe, annotating your sketches accordingly, how thickness modifies the pressure distributions, and discuss its aerodynamic merit.

[20%]

(TURN OVER

5 In Lifting Line theory a wing of semi-span s in a flow with free-stream velocity U is represented by a bound circulation Γ that varies with spanwise location y.

(a) If the wing has symmetrical aerofoil sections, the local lift coefficient at incidence α is given by

$$c_{\mathrm{l}}(y) = 2\pi\,[\alpha + \alpha_{\mathrm{t}}(y) - \alpha_{\mathrm{d}}(y)]$$

where α_{t} and α_{d} are, respectively, the twist and downwash angles. Explain, with the aid of a diagram, the derivation of this equation.

[20%]

(b) At its intended design point, a wing has bound circulation $\Gamma(y)$ given by

$$\frac{\Gamma}{Us} = G_1(\sin\theta + a\sin 3\theta)$$

in which G_1 and a are positive constants and θ defines spanwise position via $y = -s\cos\theta$. What is the local downwash angle as a function of θ?

[50%]

(c) The wing in Part (b) is to have symmetrical aerofoil sections and a spanwise uniform lift coefficient c_{ld} at its design point. What twist distribution (relative to the section at mid-span) and chord variation are required? Give your answers as functions of θ and sketch their approximate dependence on y with curves for $a = 0$ as comparators.

[30%]

6 Consider a two-dimensional high Reynolds number flow around an expansion corner with turning angle $\alpha\pi$, as shown in Fig.2.

(a) Use complex potential flow theory to find an inviscid solution for the flow around the corner such that the velocity at the wall is of power-law type $U = \mathrm{k}x^{\mathrm{m}}$. Determine m in terms of α.

[20%]

(b) A boundary layer develops from the origin. Consider a similarity solution to that boundary layer of the form

$$\psi(x, y) = F(x)\, f(\eta) \quad \text{with} \quad \eta = y/g(x)$$

where ψ denotes the streamfunction. Assume $\mathrm{d}f/\mathrm{d}\eta\,(\infty) = 1$ and hence show that

$$\psi = U(x)\, g(x)\, f(\eta).$$

[10%]

(c) Write down the laminar boundary layer equations and calculate expressions for u, v, $\partial u/\partial x$, $\partial v/\partial y$ and $\partial^2 u/\partial y^2$.

[20%]

(d) Substitute the above expressions into the boundary layer equations to derive a differential equation relating f and g.

[20%]

(e) Find the necessary conditions for the existence of a similarity solution and hence deduce the differential equation for f in the simplest form.

[20%]

(f) Discuss the physical validity of the solution found. [10%]

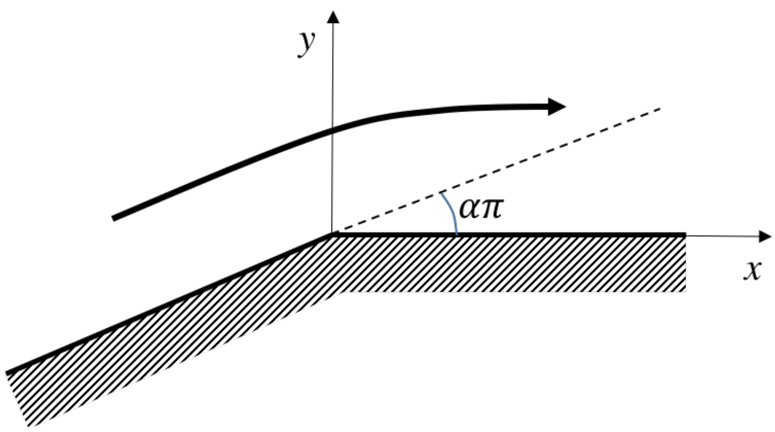

Fig.2

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7 Consider the boundary layer equations for the flow past a flat plate:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau}{\partial y} \quad ; \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,.$$

(a) Show that

$$\frac{\partial}{\partial x}\left[u(U-u)\right] + \frac{\partial}{\partial y}\left[v(U-u)\right] + (U-u)\frac{dU}{dx} = -\frac{1}{\rho}\frac{\partial \tau}{\partial y}$$

where $U = U(x)$ is the external flow over the plate.

[30%]

(b) Integrate the equation in Part (a) from the plate wall to infinity to derive the von Karman momentum integral equation.

[20%]

(c) Assume that the external flow U is uniform. Consider the velocity profile

$$u = U\sin\left(\frac{\pi y}{2\delta}\right) \quad \text{for } y < \delta, \qquad u = U \quad \text{for } y \geq \delta$$

where δ is the boundary layer thickness, assumed to be zero at $x = 0$. Find the momentum thickness θ in terms of x, U and the kinematic viscosity υ.

[30%]

(d) It is observed that this sine-wave profile gives much better accuracy than the parabolic profile

$$u = U\left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right].$$

Explain this observation in terms of the boundary conditions implied by these profiles.

[20%]

8 A race car has an underbody diffuser with planform area A and with pressure recovery C_p. It is fed by flow from upstream as shown in Fig.3 with a core flow with wake deficits on either side.

(a) Derive an expression for the downforce generated by the diffuser. You may assume the whole area A experiences the recovered flow.

[30%]

(b) If now the underbody flow mixes out to be uniform before arriving at the diffuser derive a new expression for the downforce. Assume no flow enters or leaves around the floor edge.

[30%]

(c) What is the relative magnitude of the two downforce values. Comment on the difference.

[25%]

(d) How might the non-uniform flow be maintained in practice.

[15%]

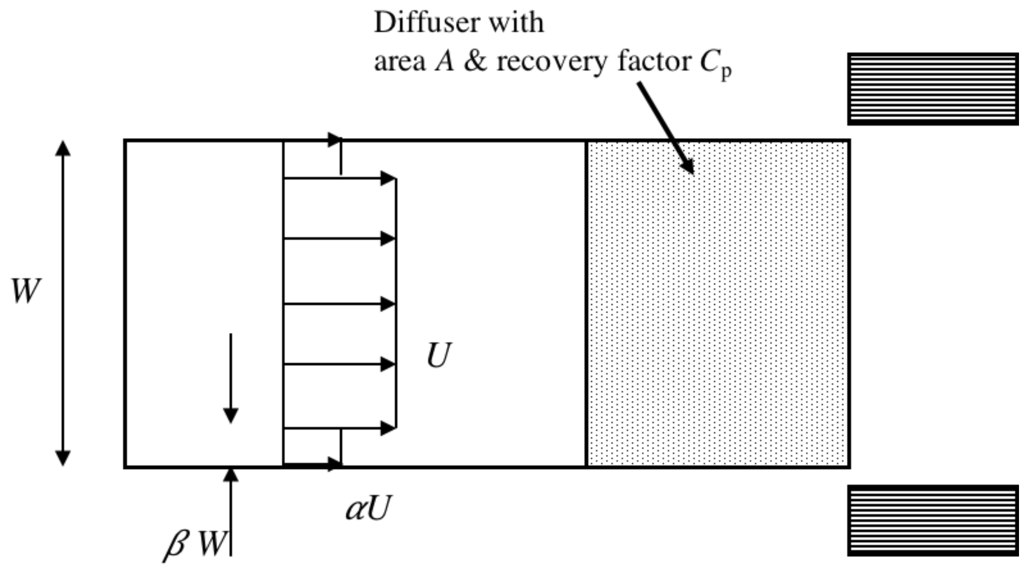

Fig.3

END OF PAPER

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