EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 21 April 2015 9.30 to 12.30

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Compressible Flow Data Book (38 pages)
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) State the conditions under which the stagnation enthalpy, h_0, is constant along a streamline in steady compressible flow, where:

$$h_0 = h + \frac{1}{2}V^2$$ [15%]

(b) For a perfect gas, obtain an expression for the ratio of stagnation to static temperature in terms of Mach number. Hence obtain an expression for the corresponding pressure ratio assuming isentropic flow. Show that Bernoulli's equation is recovered at low Mach number. [30%]

(c) An aircraft equipped for the calibration of airspeed indicators is flying at an altitude of 9000 m. The true airspeed is known to be 370 m/s. What stagnation pressure should be measured by a pitot probe located in the free stream? [40%]

(d) Draw an *h-s* diagram to illustrate the flow. [15%]

2 (a) A one-dimensional stationary hydraulic jump sits in a channel. The water depth upstream of the jump is h_1 while the depth downstream is h_2. Show that the Froude number upstream of the jump is given by:

$$Fr_1^2 = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1}+1\right)$$ [30%]

(b) Water is flowing steadily in a channel of uniform cross-sectional area. The depth of water is 1.2 m and the flow velocity is 1.7 m/s. The flow is controlled using a sluice gate at the entrance to the channel. Starting at a given instant of time, the gate is raised slowly allowing more water to flow into the channel until a new constant depth is reached just downstream of the gate. A fully-developed hydraulic jump is found to occur some distance down the channel, travelling with a velocity of 7 m/s. Calculate the new depth and velocity of the flow between the sluice gate and the hydraulic jump. [40%]

(c) Draw and label an *x-t* diagram to illustrate the process described above and its effects on the flow and wave motion in the channel. If it takes 30 seconds to raise the sluice gate, calculate the location where the hydraulic jump reaches its full strength. [30%]

3 (a) Consider steady adiabatic compressible flow with friction in a duct of constant cross-sectional area A. Starting from the principles of conservation of mass and energy, show that:

$$\delta\left(\frac{F}{A}\right) = P\left(M^2 - 1\right)\frac{\delta V}{V}$$

where F is the impulse function, P is the static pressure, M is the Mach number and V is the velocity. [30%]

(b) Show that in this duct a subsonic flow will accelerate and that a supersonic flow will decelerate. [10%]

(c) A large reservoir is filled with air (c_p = 1005 J/kg K) at a pressure of 20 bar and a temperature of 300 K. The reservoir is connected via a frictionless convergent-divergent nozzle to a pipe with a constant diameter of 0.2 m and a skin friction coefficient c_f = 0.0025. The pipe is 2.72 m long, both the nozzle and the exit is choked and there is no shock wave in the pipe.

(i) What is the Mach number at the entrance to the pipe? [15%]

(ii) What is the mass flow rate in the pipe? [15%]

(d) The pipe is extended in length with the diameter unchanged and the exit still choked. The entry conditions are the same as in (c). A shock wave is found to occur 0.438 m downstream of the pipe entrance. Calculate the strength of the shock and the new length of the pipe. [30%]

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4 An aircraft is designed for supersonic flight. It is equipped with a focused-design two-dimensional single 16° ramp intake as sketched in Figure 1. Conditions may be assumed to be uniform in the direction into the page.

(a) For the design Mach number of $M_\infty = 2.00$, draw a carefully labelled sketch of the shock system within the inlet. Briefly explain why there is a gap between the wedge and the side of the aircraft. [20%]

(b) Calculate the stagnation pressure ratio across the shock system. [15%]

(c) The aircraft slows to $M_\infty = 1.80$. Assuming that the wedge is allowed to translate parallel to the free stream direction in order to maintain a focused shock system, recalculate your answer to part (b). [15%]

(d) The aircraft slows further to $M_\infty = 1.60$. Sketch the resulting flow into and around the intake and briefly comment on the key features. Estimate the stagnation pressure ratio of the shock arrangement in this configuration, briefly stating your assumptions. [20%]

(e) The intake is modified such that the 16° wedge now runs on a curved (rather than straight) track. In this way it rotates by 4° as it translates, so as to reduce the effective turning angle of the wedge to 12° at $M_\infty = 1.60$, as sketched in Figure 2. You may assume that the turning angle varies linearly from 16° at $M_\infty = 2.00$ with the free stream Mach number and that a focused shock system is maintained throughout.

Sketch a graph of the stagnation pressure ratio against Mach number for both this revised design and the original version, on the same axes for $1.60 \leq M_\infty \leq 2.00$. Briefly comment on any notable features. [30%]

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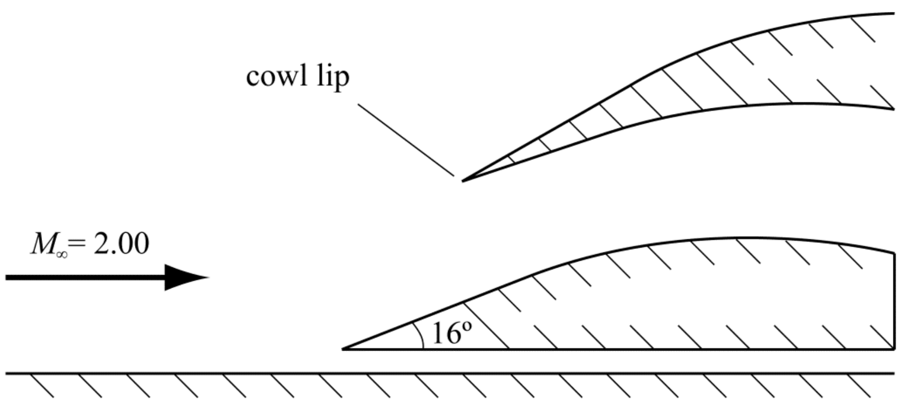

Fig. 1.

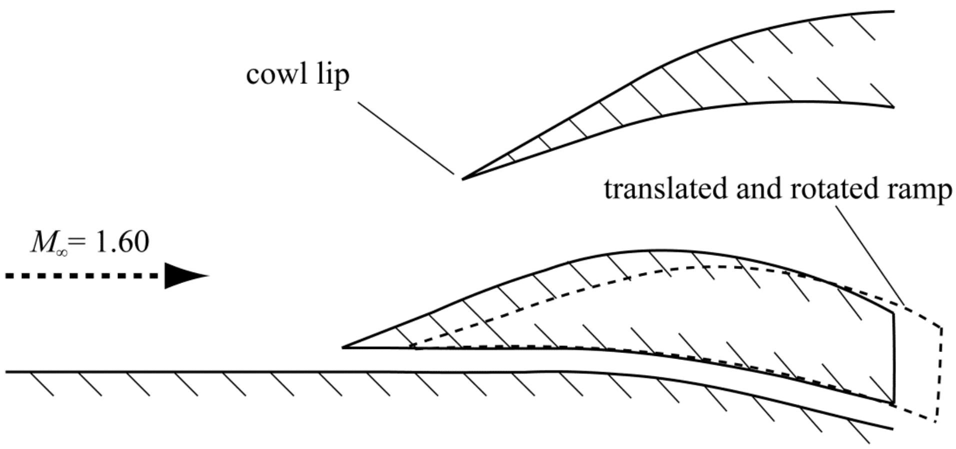

Fig. 2.

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5 (a) Show that the turning angle, θ, of an oblique shock wave in a perfect gas tends to:

$$\theta \to \arctan\left(\frac{\sin 2\beta}{(\gamma+1)-2\sin^2\beta}\right)$$

as the free-stream Mach number tends to infinity, where β is the shock angle and γ is the ratio of specific heat capacities. [20%]

(b) Calculate the maximum turning angle for $\gamma = 1.40$. [40%]

(c) A designer requires to turn a supersonic flow of $M_\infty = 2.00$ by a total of 12° using a number of wedges. Determine which of the shock-wave systems arising from the following combinations of wedges would give the greatest and the least stagnation pressure loss and give a careful physical explanation for this result.

(i) A single 12° wedge,

(ii) an 8° wedge followed by a 4° wedge,

(iii) a 4° wedge followed by an 8° wedge.

What are the disadvantages of the system giving the best aerodynamic performance? [40%]

6 The one-dimensional scalar convection equation

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0$$

where A is a positive constant, is to be discretized using the upwind difference formula

$$u_i^{n+1} = u_i^n - c\left(u_i^n - u_{i-1}^n\right)$$

in which time t has been divided into n equal intervals of size δt and distance x has been divided into i equal intervals of size δx.

(a) (i) Explain the physical meaning of upwind differencing for hyperbolic equations. [10%]

(ii) Define the parameter c and explain its numerical significance. [10%]

(b) This difference formula is known to give rise to false diffusion. Show that the effective viscosity is given by

$$\upsilon_{eff} = \frac{A\delta x}{2}\left(1 - \frac{A\delta t}{\delta x}\right) \qquad [40\%]$$

(c) Determine the order of accuracy of the difference formula in time and space. [10%]

(d) For an initial condition consisting of a unit step profile in the variable u, draw diagrams to illustrate the development of the numerical solution for the cases $c = 0.5$, $c = 1.0$ and $c = 1.1$. [30%]

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7 Note that part (a) of this question is not related to part (b).

(a) The continuity equation may be stated as

$$\frac{\partial \rho}{\partial t} + \nabla . \rho \underline{u} = 0$$

(i) Use Gauss's theorem to derive a general two-dimensional finite volume approximation to this equation. [20%]

(ii) A triangular cell from a two-dimensional finite-volume mesh is defined by the vertices with coordinates (0,0), (5,2) and (2,4) where the unit of distance is 10^{-4} m. The velocity component v in the y - direction takes corresponding values of 2 m/s, 5 m/s and 3 m/s at these vertices. The velocity component u in the x direction is zero everywhere, and the density ρ is constant and equal to 1.0 kg/m^3.

Calculate the mass flow rate into the cell, using the trapezium rule to evaluate the integrals. [30%]

(b) The flow through a low-speed, single-stage, axial flow compressor can be assumed to be incompressible and both the mean radius and the annulus cross-sectional area are constant through the machine. There is no inlet swirl and the rotor exit relative flow angle is α_2^{rel}.

(i) Show that the stagnation enthalpy rise, Δh_0, across the compressor is given by:

$$\frac{\Delta h_0}{U^2} = 1 + \phi \tan \alpha_2^{rel}$$

where ϕ is the flow coefficient and U is the blade speed at the mean radius. [20%]

(ii) Stating carefully any assumptions made, show that the total-to-total stagnation pressure rise across the compressor can be expressed as:

$$\frac{\Delta p_0}{\rho U^2} = \eta_{tt} \frac{\Delta h_0}{U^2}$$

where η_{tt} is the isentropic total-to-total efficiency and ρ is the density. [15%]

(cont.

(iii) Sketch, on a combined figure that identifies the important features, how the stage loading coefficient and the non-dimensional pressure rise are likely to vary with flow coefficient. [15%]

8 A stream-tube that passes through an axial compressor stage is at a constant radius and the corresponding blade speed is 265 m/s. Upstream of the rotor there is no swirl, the axial velocity is 132 m/s and the stagnation temperature is 280 K. Downstream of the rotor (but upstream of the stator) the axial velocity is 118 m/s and the stagnation temperature is 308 K.

(a) Calculate the tangential velocity downstream of the rotor. [10%]

(b) Determine the flow turning through the rotor. [15%]

(c) The flow through the stream-tube is known to have a 95% total-to-total isentropic efficiency as it passes through the rotor. Calculate the total-to-total pressure ratio of the flow through the rotor. [15%]

(d) Determine the static-to-static pressure ratio of the flow through the rotor. [20%]

(e) Calculate the ratio of the stream-tube areas (axial projection) across the rotor and suggest any design change that may be appropriate. [25%]

(f) If the stagnation pressure loss coefficient of the stream-tube as it passes through the stator is 0.08, calculate the overall stagnation pressure ratio of the stage. [15%]

You may assume that the working fluid is a perfect gas with:

$$R = 287 \text{ J/kgK}, \gamma = 1.4 \text{ and } c_p = 1005 \text{ J/kgK}.$$

END OF PAPER

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